

Paper 2: Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1	Sets $f(-2) = 0 \Rightarrow 2 \times (-2)^3 - 5 \times (-2)^2 + a \times -2 + a = 0$	M1	3.1a
	Solves linear equation $2a - a = -36 \Rightarrow a =$	dM1	1.1b
	$\Rightarrow a = -36$	A1	1.1b
(3 marks)			
Notes:			
<p>M1: Selects a suitable method given that $(x + 2)$ is a factor of $f(x)$ Accept either setting $f(-2) = 0$ or attempted division of $f(x)$ by $(x + 2)$</p> <p>dM1: Solves linear equation in a. Minimum requirement is that there are two terms in 'a' which must be collected to get $..a = .. \Rightarrow a =$</p> <p>A1: $a = -36$</p>			

Question	Scheme	Marks	AOs
2(a)	Identifies an error for student A: They use $\frac{\cos \theta}{\sin \theta} = \tan \theta$ It should be $\frac{\sin \theta}{\cos \theta} = \tan \theta$	B1	2.3
		(1)	
(b)	(i) Shows $\cos(-26.6^\circ) \neq 2 \sin(-26.6^\circ)$, so cannot be a solution	B1	2.4
	(ii) Explains that the incorrect answer was introduced by squaring	B1	2.4
		(2)	
(3 marks)			
Notes:			
<p>(a)</p> <p>B1: Accept a response of the type 'They use $\frac{\cos \theta}{\sin \theta} = \tan \theta$. This is incorrect as $\frac{\sin \theta}{\cos \theta} = \tan \theta$' It can be implied by a response such as 'They should get $\tan \theta = \frac{1}{2}$ not $\tan \theta = 2$' Accept also statements such as 'it should be $\cot \theta = 2$'</p>			
<p>(b)</p> <p>B1: Accept a response where the candidate shows that -26.6° is not a solution of $\cos \theta = 2 \sin \theta$. This can be shown by, for example, finding both $\cos(-26.6^\circ)$ and $2 \sin(-26.6^\circ)$ and stating that they are not equal. An acceptable alternative is to state that $\cos(-26.6^\circ) = +ve$ and $2 \sin(-26.6^\circ) = -ve$ and stating that they therefore cannot be equal.</p> <p>B1: Explains that the incorrect answer was introduced by squaring Accept an example showing this. For example $x = 5$ squared gives $x^2 = 25$ which has answers ± 5</p>			

Question	Scheme	Marks	AOs
3	Attempts the product and chain rule on $y = x(2x+1)^4$	M1	2.1
	$\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$	A1	1.1b
	Takes out a common factor $\frac{dy}{dx} = (2x+1)^3 \{(2x+1)+8x\}$	M1	1.1b
	$\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$	A1	1.1b
(4 marks)			
Notes:			
M1:	Applies the product rule to reach $\frac{dy}{dx} = (2x+1)^4 + Bx(2x+1)^3$		
A1:	$\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$		
M1:	Takes out a common factor of $(2x+1)^3$		
A1:	The form of this answer is given. Look for $\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$		

Question	Scheme	Marks	AOs
4 (a)	$gf(x) = 3 \ln e^x$	M1	1.1b
	$= 3x, (x \in \mathbb{R})$	A1	1.1b
		(2)	
(b)	$gf(x) = fg(x) \Rightarrow 3x = x^3$	M1	1.1b
	$\Rightarrow x^3 - 3x = 0 \Rightarrow x =$	M1	1.1b
	$\Rightarrow x = (+)\sqrt{3}$ only as $\ln x$ is not defined at $x = 0$ and $-\sqrt{3}$	M1	2.2a
		(3)	
(5 marks)			
Notes:			
(a)			
M1: For applying the functions in the correct order			
A1: The simplest form is required so it must be $3x$ and not left in the form $3 \ln e^x$ An answer of $3x$ with no working would score both marks			
(b)			
M1: Allow the candidates to score this mark if they have $e^{3 \ln x} =$ their $3x$			
M1: For solving their cubic in x and obtaining at least one solution.			
A1: For either stating that $x = \sqrt{3}$ only as $\ln x$ (or $3 \ln x$) is not defined at $x = 0$ and $-\sqrt{3}$ or stating that $3x = x^3$ would have three answers, one positive one negative and one zero but $\ln x$ (or $3 \ln x$) is not defined for $x \leq 0$ so therefore there is only one (real) answer. Note: Student who mix up fg and gf can score full marks in part (b) as they have already been penalised in part (a)			

Question	Scheme	Marks	AOs
5(a)	Substitutes $t = 0.5$ into $m = 25e^{-0.05t} \Rightarrow m = 25e^{-0.05 \times 0.5}$	M1	3.4
	$\Rightarrow m = 24.4\text{g}$	A1	1.1b
		(2)	
(b)	States or uses $\frac{d}{dt}(e^{-0.05t}) = \pm C e^{-0.05t}$	M1	2.1
	$\frac{dm}{dt} = -0.05 \times 25e^{-0.05t} = -0.05m \Rightarrow k = -0.05$	A1	1.1b
		(2)	
(4 marks)			
Notes:			
(a)			
M1: Substitutes $t = 0.5$ into $m = 25e^{-0.05t} \Rightarrow m = 25e^{-0.05 \times 0.5}$			
A1: $m = 24.4\text{g}$ An answer of $m = 24.4\text{g}$ with no working would score both marks			
(b)			
M1: Applies the rule $\frac{d}{dt}(e^{kx}) = k e^{kx}$ in this context by stating or using $\frac{d}{dt}(e^{-0.05t}) = \pm C e^{-0.05t}$			
A1: $\frac{dm}{dt} = -0.05 \times 25e^{-0.05t} = -0.05m \Rightarrow k = -0.05$			

Question	Scheme	Marks	AOs
6(i)	$x^2 - 6x + 10 = (x - 3)^2 + 1$	M1	2.1
	Deduces "always true" as $(x - 3)^2 \geq 0 \Rightarrow (x - 3)^2 + 1 \geq 1$ and so is always positive	A1	2.2a
		(2)	
(ii)	For an explanation that it need not (always) be true This could be if $a < 0$ then $ax > b \Rightarrow x < \frac{b}{a}$	M1	2.3
	States 'sometimes' and explains if $a > 0$ then $ax > b \Rightarrow x > \frac{b}{a}$ if $a < 0$ then $ax > b \Rightarrow x < \frac{b}{a}$	A1	2.4
		(2)	
(iii)	Difference $= (n + 1)^2 - n^2 = 2n + 1$	M1	3.1a
	Deduces "Always true" as $2n + 1 = (\text{even} + 1) = \text{odd}$	A1	2.2a
		(2)	

(6 marks)

Notes:

(i)

M1: Attempts to complete the square or any other valid reason. Allow for a graph of $y = x^2 - 6x + 10$ or an attempt to find the minimum by differentiation

A1: States always true with a valid reason for their method

(ii)

M1: For an explanation that it need not be true (sometimes). This could be if

$$a < 0 \text{ then } ax > b \Rightarrow x < \frac{b}{a} \text{ or simply } -3x > 6 \Rightarrow x < -2$$

A1: Correct statement (sometimes true) and explanation

(iii)

M1: Sets up the proof algebraically.

For example by attempting $(n + 1)^2 - n^2 = 2n + 1$ or $m^2 - n^2 = (m - n)(m + n)$ with $m = n + 1$

A1: States always true with reason and proof

Accept a proof written in words. For example

If integers are consecutive, one is odd and one is even

When squared odd \times odd = odd and even \times even = even

The difference between odd and even is always odd, hence always true

Score M1 for two of these lines and A1 for a good proof with all three lines or equivalent.

Question	Scheme	Marks	AOs
7(a)	$\sqrt{(4-x)} = 2\left(1 - \frac{1}{4}x\right)^{\frac{1}{2}}$	M1	2.1
	$\left(1 - \frac{1}{4}x\right)^{\frac{1}{2}} = 1 + \frac{1}{2}\left(-\frac{1}{4}x\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{1}{4}x\right)^2 + \dots$	M1	1.1b
	$\sqrt{(4-x)} = 2\left(1 - \frac{1}{8}x - \frac{1}{128}x^2 + \dots\right)$	A1	1.1b
	$\sqrt{(4-x)} = 2 - \frac{1}{4}x - \frac{1}{64}x^2 + \dots \text{ and } k = -\frac{1}{64}$	A1	1.1b
		(4)	
(b)	The expansion is valid for $ x < 4$, so $x = 1$ can be used	B1	2.4
		(1)	
(5 marks)			
Notes:			
(a)			
M1: Takes out a factor of 4 and writes $\sqrt{(4-x)} = 2(1 \pm \dots)^{\frac{1}{2}}$			
M1: For an attempt at the binomial expansion with $n = \frac{1}{2}$			
Eg. $(1+ax)^{\frac{1}{2}} = 1 + \frac{1}{2}(ax) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(ax)^2 + \dots$			
A1: Correct expression inside the bracket $1 - \frac{1}{8}x - \frac{1}{128}x^2 + \dots$ which may be left unsimplified			
A1: $\sqrt{(4-x)} = 2 - \frac{1}{4}x - \frac{1}{64}x^2 + \dots$ and $k = -\frac{1}{64}$			
(b)			
B1: The expansion is valid for $ x < 4$, so $x = 1$ can be used			

Question	Scheme	Marks	AOs
8 (a)	Gradient $AB = -\frac{2}{5}$	B1	2.1
	y coordinate of A is 2	B1	2.1
	Uses perpendicular gradients $y = +\frac{5}{2}x + c$	M1	2.2a
	$\Rightarrow 2y - 5x = 4$ *	A1*	1.1b
		(4)	
(b)	Uses Pythagoras' theorem to find AB or AD Either $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2 + 2^2}$	M1	3.1a
	Uses area $ABCD = AD \times AB = \sqrt{29} \times \sqrt{\frac{116}{25}}$	M1	1.1b
	area $ABCD = 11.6$	A1	1.1b
		(3)	
(7 marks)			
Notes:			
(a) It is important that the student communicates each of these steps clearly			
B1: States the gradient of AB is $-\frac{2}{5}$			
B1: States that y coordinate of $A = 2$			
M1: Uses the form $y = mx + c$ with $m =$ their adapted $-\frac{2}{5}$ and $c =$ their 2			
Alternatively uses the form $y - y_1 = m(x - x_1)$ with $m =$ their adapted $-\frac{2}{5}$ and $(x_1, y_1) = (0, 2)$			
A1*: Proceeds to given answer			
(b)			
M1: Finds the lengths of AB or AD using Pythagoras' Theorem. Look for $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2 + 2^2}$			
Alternatively finds the lengths BD and AO using coordinates. Look for $\left(5 + \frac{4}{5}\right)$ and 2			
M1: For a full method of finding the area of the rectangle $ABCD$. Allow for $AD \times AB$			
Alternatively attempts area $ABCD = 2 \times \frac{1}{2} BD \times AO = 2 \times \frac{1}{2} '5.8' \times '2'$			
A1: Area $ABCD = 11.6$ or other exact equivalent such as $\frac{58}{5}$			

Question	Scheme	Marks	AOs	
9	$\int (3x^{0.5} + A) dx = 2x^{1.5} + Ax(+c)$	M1 A1	3.1a 1.1b	
	Uses limits and sets $= 2A^2 \Rightarrow (2 \times 8 + 4A) - (2 \times 1 + A) = 2A^2$	M1	1.1b	
	Sets up quadratic and attempts to solve	Sets up quadratic and attempts $b^2 - 4ac$	M1	1.1b
	$\Rightarrow A = -2, \frac{7}{2}$ and states that there are two roots	States $b^2 - 4ac = 121 > 0$ and hence there are two roots	A1	2.4
(5 marks)				

Notes:

M1: Integrates the given function and achieves an answer of the form $kx^{1.5} + Ax(+c)$ where k is a non-zero constant

A1: Correct answer but may not be simplified

M1: Substitutes in limits and subtracts. This can only be scored if $\int A dx = Ax$ and not $\frac{A^2}{2}$

M1: Sets up quadratic equation in A and either attempts to solve or attempts $b^2 - 4ac$

A1: Either $A = -2, \frac{7}{2}$ and states that there are two roots

Or states $b^2 - 4ac = 121 > 0$ and hence there are two roots

Question	Scheme	Marks	AOs
10	Attempts $S_{\infty} = \frac{8}{7} \times S_6 \Rightarrow \frac{a}{1-r} = \frac{8}{7} \times \frac{a(1-r^6)}{1-r}$	M1	2.1
	$\Rightarrow 1 = \frac{8}{7} \times (1-r^6)$	M1	2.1
	$\Rightarrow r^6 = \frac{1}{8} \Rightarrow r = \dots$	M1	1.1b
	$\Rightarrow r = \pm \frac{1}{\sqrt{2}}$ (so $k = 2$)	A1	1.1b
(4 marks)			

Notes:

M1: Substitutes the correct formulae for S_{∞} and S_6 into the given equation $S_{\infty} = \frac{8}{7} \times S_6$

M1: Proceeds to an equation just in r

M1: Solves using a correct method

A1: Proceeds to $r = \pm \frac{1}{\sqrt{2}}$ giving $k = 2$

Question	Scheme	Marks	AOs
11 (a)	$f(x) \geq 5$	B1	1.1b
		(1)	
(b)	Uses $-2(3-x)+5 = \frac{1}{2}x+30$	M1	3.1a
	Attempts to solve by multiplying out bracket, collect terms etc $\frac{3}{2}x = 31$	M1	1.1b
	$x = \frac{62}{3}$ only	A1	1.1b
		(3)	
(c)	Makes the connection that there must be two intersections. Implied by either end point $k > 5$ or $k \leq 11$	M1	2.2a
	$\{k : k \in \mathbb{R}, 5 < k \leq 11\}$	A1	2.5
		(2)	
(6 marks)			
Notes:			
(a)			
B1: $f(x) \geq 5$ Also allow $f(x) \in [5, \infty)$			
(b)			
M1: Deduces that the solution to $f(x) = \frac{1}{2}x + 30$ can be found by solving $-2(3-x)+5 = \frac{1}{2}x+30$			
M1: Correct method used to solve their equation. Multiplies out bracket/ collects like terms			
A1: $x = \frac{62}{3}$ only. Do not allow 20.6			
(c)			
M1: Deduces that two distinct roots occurs when $y = k$ intersects $y = f(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k > 5$ or $k \leq 11$			
A1: Correct solution only $\{k : k \in \mathbb{R}, 5 < k \leq 11\}$			

Question	Scheme	Marks	AOs
12(a)	Uses $\cos^2 x = 1 - \sin^2 x \Rightarrow 3\sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$	M1	3.1a
	$\Rightarrow 12\sin^2 x + \sin x - 1 = 0$	A1	1.1b
	$\Rightarrow (4\sin x - 1)(3\sin x + 1) = 0$	M1	1.1b
	$\Rightarrow \sin x = \frac{1}{4}, -\frac{1}{3}$	A1	1.1b
	Uses arcsin to obtain two correct values	M1	1.1b
	All four of $x = 14.48^\circ, 165.52^\circ, -19.47^\circ, -160.53^\circ$	A1	1.1b
		(6)	
(b)	Attempts $2\theta - 30^\circ = -19.47^\circ$	M1	3.1a
	$\Rightarrow \theta = 5.26^\circ$	A1ft	1.1b
		(2)	
(8 marks)			
Notes:			
(a)			
M1:	Substitutes $\cos^2 x = 1 - \sin^2 x$ into $3\sin^2 x + \sin x + 8 = 9\cos^2 x$ to create a quadratic equation in just $\sin x$		
A1:	$12\sin^2 x + \sin x - 1 = 0$ or exact equivalent		
M1:	Attempts to solve their quadratic equation in $\sin x$ by a suitable method. These could include factorisation, formula or completing the square.		
A1:	$\sin x = \frac{1}{4}, -\frac{1}{3}$		
M1:	Obtains two correct values for their $\sin x = k$		
A1:	All four of $x = 14.48^\circ, 165.52^\circ, -19.47^\circ, -160.53^\circ$		
(b)			
M1:	For setting $2\theta - 30^\circ = \text{their } '-19.47^\circ'$		
A1ft:	$\theta = 5.26^\circ$ but allow a follow through on their $'-19.47^\circ'$		

Question	Scheme	Marks	AOs
13(a)	$R = \sqrt{109}$	B1	1.1b
	$\tan \alpha = \frac{3}{10}$	M1	1.1b
	$\alpha = 16.70^\circ$ so $\sqrt{109} \cos(\theta + 16.70^\circ)$	A1	1.1b
		(3)	
(b)	(i) e.g $H = 11 - 10 \cos(80t)^\circ + 3 \sin(80t)^\circ$ or $H = 11 - \sqrt{109} \cos(80t + 16.70)^\circ$	B1	3.3
	(ii) $11 + \sqrt{109}$ or 21.44 m	B1ft	3.4
		(2)	
(c)	Sets $80t + "16.70" = 540$	M1	3.4
	$t = \frac{540 - "16.70"}{80} = (6.54)$	M1	1.1b
	$t = 6$ mins 32 seconds	A1	1.1b
		(3)	
(d)	Increase the '80' in the formula For example use $H = 11 - 10 \cos(90t)^\circ + 3 \sin(90t)^\circ$		3.3
		(1)	
(9 marks)			
Notes:			
(a)			
B1: $R = \sqrt{109}$ Do not allow decimal equivalents			
M1: Allow for $\tan \alpha = \pm \frac{3}{10}$			
A1: $\alpha = 16.70^\circ$			
(b)(i)			
B1: see scheme			
(b)(ii)			
B1ft: their 11+ their $\sqrt{109}$ Allow decimals here.			
(c)			
M1: Sets $80t + "16.70" = 540$. Follow through on their 16.70			
M1: Solves their $80t + "16.70" = 540$ correctly to find t			
A1: $t = 6$ mins 32 seconds			
(d)			
B1: States that to increase the speed of the wheel the 80's in the equation would need to be increased.			

Question	Scheme	Marks	AOs
14(a)	Sets $500 = \pi r^2 h$	B1	2.1
	Substitute $h = \frac{500}{\pi r^2}$ into $S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r \times \frac{500}{\pi r^2}$	M1	2.1
	Simplifies to reach given answer $S = 2\pi r^2 + \frac{1000}{r}$ *	A1*	1.1b
		(3)	
(b)	Differentiates S with both indices correct in $\frac{dS}{dr}$	M1	3.4
	$\frac{dS}{dr} = 4\pi r - \frac{1000}{r^2}$	A1	1.1b
	Sets $\frac{dS}{dr} = 0$ and proceeds to $r^3 = k$, k is a constant	M1	2.1
	Radius = 4.30 cm	A1	1.1b
	Substitutes their $r = 4.30$ into $h = \frac{500}{\pi r^2} \Rightarrow$ Height = 8.60 cm	A1	1.1b
		(5)	
(c)	States a valid reason such as <ul style="list-style-type: none"> The radius is too big for the size of our hands If $r = 4.3$ cm and $h = 8.6$ cm the can is square in profile. All drinks cans are taller than they are wide The radius is too big for us to drink from They have different dimensions to other drinks cans and would be difficult to stack on shelves with other drinks cans 	B1	3.2a
		(1)	
9 marks			
Notes:			
(a)			
B1: Uses the correct volume formula with $V=500$. Accept $500 = \pi r^2 h$			
M1: Substitutes $h = \frac{500}{\pi r^2}$ or $rh = \frac{500}{\pi r}$ into $S = 2\pi r^2 + 2\pi r h$ to get S as a function of r			
A1*: $S = 2\pi r^2 + \frac{1000}{r}$ Note that this is a given answer.			
(b)			
M1: Differentiates the given S to reach $\frac{dS}{dr} = Ar \pm Br^{-2}$			
A1: $\frac{dS}{dr} = 4\pi r - \frac{1000}{r^2}$ or exact equivalent			
M1: Sets $\frac{dS}{dr} = 0$ and proceeds to $r^3 = k$, k is a constant			
A1: $R =$ awrt 4.30cm			
A1: $H =$ awrt 8.60 cm			
(c)			
B1: Any valid reason. See scheme for alternatives			

Question	Scheme	Marks	AOs
15	$\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$	M1 A1	3.1a 1.1b
	Substitutes $x = 4 \Rightarrow \frac{dy}{dx} = 6$	M1	2.1
	Uses (4, 15) and gradient $\Rightarrow y - 15 = 6(x - 4)$	M1	2.1
	Equation of l is $y = 6x - 9$	A1	1.1b
	Area $R = \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11 \right) - (6x - 9) dx$	M1	3.1a
	$= \left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x(+c) \right]_0^4$	A1	1.1b
	Uses both limits of 4 and 0 $\left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x \right]_0^4 = 2 \times 4^{\frac{5}{2}} - \frac{15}{2} \times 4^2 + 20 \times 4 - 0$	M1	2.1
	Area of $R = 24$ *	A1*	1.1b
	Correct notation with good explanations	A1	2.5
	(10)		
(10 marks)			

Question 15 continued**Notes:**

M1: Differentiates $5x^{\frac{3}{2}} - 9x + 11$ to a form $Ax^{\frac{1}{2}} + B$

A1: $\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$ but may not be simplified

M1: Substitutes $x = 4$ in their $\frac{dy}{dx}$ to find the gradient of the tangent

M1: Uses their gradient and the point (4, 15) to find the equation of the tangent

A1: Equation of l is $y = 6x - 9$

M1: Uses Area $R = \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11 \right) - (6x - 9) dx$ following through on their $y = 6x - 9$

Look for a form $Ax^{\frac{5}{2}} + Bx^2 + Cx$

A1: $= \left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x(+c) \right]_0^4$ This must be correct but may not be simplified

M1: Substitutes in both limits and subtracts

A1*: Correct area for $R = 24$

A1: Uses correct notation and produces a well explained and accurate solution. Look for

- Correct notation used consistently and accurately for both differentiation and integration
- Correct explanations in producing the equation of l . See scheme.
- Correct explanation in finding the area of R . In way 2 a diagram may be used.

Alternative method for the area using area under curve and triangles. (Way 2)

M1: Area under curve $= \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11 \right) = \left[Ax^{\frac{5}{2}} + Bx^2 + Cx \right]_0^4$

A1: $= \left[2x^{\frac{5}{2}} - \frac{9}{2}x^2 + 11x \right]_0^4 = 36$

M1: This requires a full method with all triangles found using a correct method

Look for Area $R =$ their $36 - \frac{1}{2} \times 15 \times \left(4 - \text{their } \frac{3}{2} \right) + \frac{1}{2} \times \text{their } 9 \times \text{their } \frac{3}{2}$

Question	Scheme	Marks	AOs
16(a)	Sets $\frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{(11-2P)}$	B1	1.1a
	Substitutes either $P=0$ or $P=\frac{11}{2}$ into $1 = A(11-2P) + BP \Rightarrow A$ or B	M1	1.1b
	$\frac{1}{P(11-2P)} = \frac{1/11}{P} + \frac{2/11}{(11-2P)}$	A1	1.1b
		(3)	
(b)	Separates the variables $\int \frac{22}{P(11-2P)} dP = \int 1 dt$	B1	3.1a
	Uses (a) and attempts to integrate $\int \frac{2}{P} + \frac{4}{(11-2P)} dP = t + c$	M1	1.1b
	$2 \ln P - 2 \ln(11-2P) = t + c$	A1	1.1b
	Substitutes $t=0, P=1 \Rightarrow t=0, P=1 \Rightarrow c = (-2 \ln 9)$	M1	3.1a
	Substitutes $P=2 \Rightarrow t = 2 \ln 2 + 2 \ln 9 - 2 \ln 7$	M1	3.1a
	Time = 1.89 years	A1	3.2a
		(6)	
(c)	Uses ln laws $2 \ln P - 2 \ln(11-2P) = t - 2 \ln 9$ $\Rightarrow \ln\left(\frac{9P}{11-2P}\right) = \frac{1}{2}t$	M1	2.1
	Makes 'P' the subject $\Rightarrow \left(\frac{9P}{11-2P}\right) = e^{\frac{1}{2}t}$ $\Rightarrow 9P = (11-2P)e^{\frac{1}{2}t}$ $\Rightarrow P = f\left(e^{\frac{1}{2}t}\right)$ or $\Rightarrow P = f\left(e^{-\frac{1}{2}t}\right)$	M1	2.1
	$\Rightarrow P = \frac{11}{2 + 9e^{-\frac{1}{2}t}} \Rightarrow A = 11, B = 2, C = 9$	A1	1.1b
		(3)	
(12 marks)			

Question 16 continued**Notes:****(a)**

B1: Sets $\frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{(11-2P)}$

M1: Substitutes $P=0$ or $P=\frac{11}{2}$ into $1 = A(11-2P) + BP \Rightarrow A$ or B

Alternatively compares terms to set up and solve two simultaneous equations in A and B

A1: $\frac{1}{P(11-2P)} = \frac{1/11}{P} + \frac{2/11}{(11-2P)}$ or equivalent $\frac{1}{P(11-2P)} = \frac{1}{11P} + \frac{2}{11(11-2P)}$

Note: The correct answer with no working scores all three marks.

(b)

B1: Separates the variables to reach $\int \frac{22}{P(11-2P)} dP = \int 1 dt$ or equivalent

M1: Uses part (a) and $\int \frac{A}{P} + \frac{B}{(11-2P)} dP = A \ln P \pm C \ln(11-2P)$

A1: Integrates both sides to form a correct equation including a 'c' Eg
 $2 \ln P - 2 \ln(11-2P) = t + c$

M1: Substitutes $t=0$ and $P=1$ to find c

M1: Substitutes $P=2$ to find t . This is dependent upon having scored both previous M's

A1: Time = 1.89 years

(c)

M1: Uses correct log laws to move from $2 \ln P - 2 \ln(11-2P) = t + c$ to $\ln\left(\frac{P}{11-2P}\right) = \frac{1}{2}t + d$ for their numerical 'c'

M1: Uses a correct method to get P in terms of $e^{\frac{1}{2}t}$

This can be achieved from $\ln\left(\frac{P}{11-2P}\right) = \frac{1}{2}t + d \Rightarrow \frac{P}{11-2P} = e^{\frac{1}{2}t+d}$ followed by cross multiplication and collection of terms in P (See scheme)

Alternatively uses a correct method to get P in terms of $e^{-\frac{1}{2}t}$ For example

$\frac{P}{11-2P} = e^{\frac{1}{2}t+d} \Rightarrow \frac{11-2P}{P} = e^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \frac{11}{P} - 2 = e^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \frac{11}{P} = 2 + e^{-\left(\frac{1}{2}t+d\right)}$ followed by division

A1: Achieves the correct answer in the form required. $P = \frac{11}{2+9e^{-\frac{1}{2}t}} \Rightarrow A=11, B=2, C=9$ oe