A1 DOUBLES ASSIGNMENT 22 – PART B

SKILLS 1

Find particular solutions to the following differential equations using the given boundary conditions.

a $\frac{dy}{dx} = \sin x \cos^2 x;\ y = 0,\ x = \frac{\pi}{3}$

b $\frac{dy}{dx} = \sec^2 x \sec^2 y;\ y = 0,\ x = \frac{\pi}{4}$

c $\frac{dy}{dx} = 2\cos^2 y \cos^2 x;\ y = \frac{\pi}{4},\ x = 0$

d $\sin y \cos x \frac{dy}{dx} = \frac{\cos y}{\cos x},\ y = 0,\ x = 0$

SKILLS 2

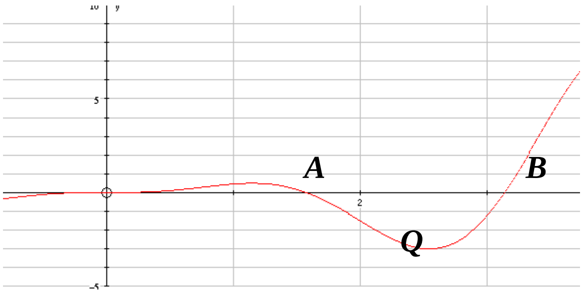
Integrate the following with respect to *x*:

(a) $\int \frac{\sec^2 x}{(1+\tan x)^3}\,dx$ (b) $\int 2\sin x \cos^3 x\, dx$ (c) $\int \frac{x}{(1-x^2)^5}\,dx$

PROBLEM SOLVING

1. Graph Transformations

The graph shown is the curve of $y = f(x)$. The curve crosses the *x* axis at $A\left(\frac{8}{5}, 0\right)$ and $B\left(\frac{16}{5}, 0\right)$ and has a turning point at $Q\left(\frac{7}{2}, -3\right)$



Sketch, showing the new coordinates of *A*, *B* and *C*: (a)) $f(2x)$ (b) $3f(x)$ (c) $f(x)+3$

2. Forces and Newton's laws: single particles slopes

A 6N weight rests on a rough 25° incline. The perpendicular reaction is measured to be 10N. A horizontal force H pushes the weight so that it is just on the point of slipping up the plane.

(a) Complete a force diagram
(b) Find the force H
(c) Find μ, the coefficient of friction.

(d) Force H is now removed. Showing all your calculations clearly, justify whether the 6N weight will slide down the plane, or remain in equilibrium.

3. Newton's Laws. Connected Particles

A train engine of mass 6400kg is pulling a carriage of mass 1600kg along a straight horizontal railway track. The engine is connected to the carriage by a shunt which is parallel to the direction of motion of the coupling. The shunt is modelled as a light rod. The engine provides a constant driving force of 12000N. The resistances to the motion of the engine and the carriage are modelled as constant forces of magnitude R N and 2000N respectively.

Given that the acceleration of the engine and the carriage is 0.5 ms^{-2}.

(a) find the value of R
(b) show that the tension in the shunt is 2800N

4. Kinematics. Variable Acceleration.

A particle P of mass 0.5 kg moves under the action of a single force $\mathbf{F}$ Newtons. At time t seconds, the velocity $\mathbf{v}$ m s^{-1} of P is given by;

$$\mathbf{v} = 3t^2\mathbf{i} + (1 - 4t)\mathbf{j}.$$

Find;
(*a*) the acceleration of P at time t seconds,
(*b*) the magnitude of $\mathbf{F}$ when $t = 2$.

5.
Moments

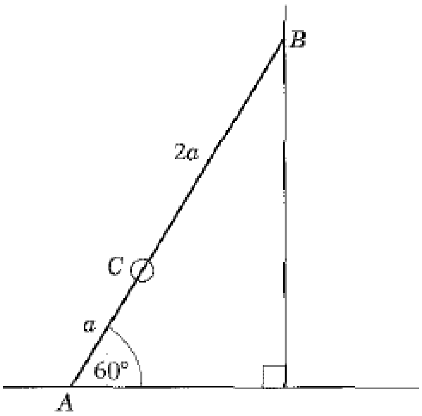
When a rigid body is in equilibrium, then:

A) There is zero resultant force in any direction
B) The sum of the moments about any point is zero

Firstly, always draw a complete and clear force diagram.

For a 2-D horizontal see-saw / bridge problem, two equations to solve:
force up = force down, anticlockwise moments = clockwise moments

For a 3-D ladder problem, three equations to solve:
force up = force down, forces left = forces right, anticlockwise moments = clockwise moments



A ladder AB, of mass m and length $3a$, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass $2m$ is fixed on the ladder at the point C, where $AC = a$.
The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium at an angle of 60° with the ground.

Find the coefficient of friction between the ladder and the ground.

For the above problem, draw a complete and clear force diagram, including F_{max} and R at A, perpendicular reaction S at B, the weight of the ladder, and the weight of the load at C.
Friction is limiting so you can use $F_{max} = \mu R$
You can take moments about any point you like. Point A is easy then the equation will not have F_{max} or R.
However, you can take moments about a point not even on the ladder – if you choose the point where R and S intersect, these forces will not then be in the equation, and you can find F_{max} directly.
Construct three equations, then solve to find the coefficient of friction.

6. Projectiles & equations of flight paths

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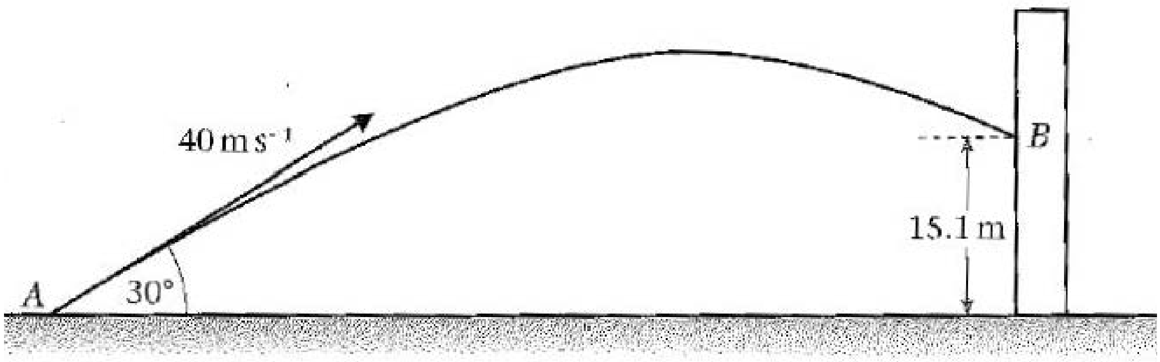


A golf ball is driven from a point A with a speed of 40 m s^{-1} at an angle of elevation of 30°. On its downward flight, the ball hits an advertising hoarding at a height 15.1 m above the level of *A*, as shown in the diagram above. Find

a) the time taken by the ball to reach its greatest height above *A*,

b) the time taken by the ball to travel from *A* to *B*,

c) the speed with which the ball hits the hoarding.

7. Mechanics Vectors

[*In this question, the unit vectors* **i** *and* **j** *are horizontal vectors due east and north respectively.*]

At time $t = 0$, a football player kicks a ball from the point A with position vector (2**i** + **j**) m on a horizontal football field. The motion of the ball is modelled as that of a particle moving horizontally with constant velocity (5**i** + 8**j**) m s^{-1}. Find

(*a*) the speed of the ball,
(*b*) the position vector of the ball after t seconds.

The point B on the field has position vector (10**i** + 7**j**) m.

(*c*) Find the time when the ball is due east of B.
(d) Find the time when the ball is due north of B.

8. Differentiation - proof of sinx and cosx derivatives from 1st principles
Use first principles to show that the derivative of cosec x is –cosec x cot x.

9. Differentiation Techniques: chain, product, quotient rules inc e^x and lnx and trig derivatives and dx/dy problems

The variables x, y and z satisfy the following relationships:
$x = \ln(z+1) \quad and \quad \frac{d^2y}{dz^2} = \frac{y}{e^{2x}}$
Show that $\frac{d^2y}{dx^2} = \frac{dy}{dx} + y$

10. A curve C has parametric equations

$$x = 6\cos 2t, \qquad y = 2\sin t, \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

(*a*) Show that $\frac{dy}{dx} = \lambda \operatorname{cosec} t$, giving the exact value of the constant λ.

(*b*) Find an equation of the normal to C at the point where $t = \frac{\pi}{3}$

Give your answer in the form $y = mx + c$, where m and c are simplified surds.

The cartesian equation for the curve C can be written in the form

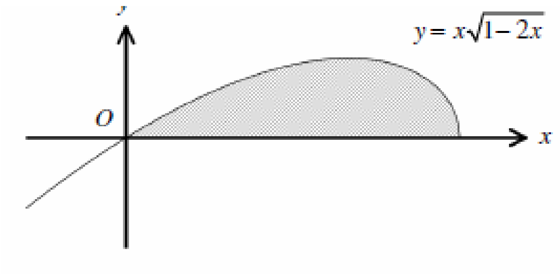
$$x = f(y), \qquad -k < y < k$$

where f(y) is a polynomial in y and k is a constant.

(*c*) Find f(y)

(*d*) State the value of k.

11. Definite Integration



The figure above shows the graph of the curve with equation

$$y = x\sqrt{1-2x}, \qquad x \le \frac{1}{2}$$

Use integration by parts and reverse chain rule to find the area of the finite region bounded by the curve and the x axis.

12.

$$f(x) = \frac{5x^2 - 8x + 1}{2x(x-1)^2}$$

a Given that $f(x) = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$, find the values of the constants A, B and C.

b Hence find $\int f(x)\,dx$.

c Hence show that $\int_4^9 f(x)\,dx = \ln\left(\frac{32}{3}\right) - \frac{5}{24}$