# Skills 1

For each of the following circles, find the lengths along the tangents from the given point to the circle:

- (a)  $(x+2)^2 + (y-3)^2 = 3$  from the point (0, 0)
- (b)  $(x-2)^2 + (y-4)^2 = 25$  from the point (8, 2)

(c) 
$$(x+3)^2 + (y+5)^2 = 30$$

from the point (-2, 3)

# Skills 2

(a) Find the coordinates of the stationary points of  $y = x^4 + 3x^3$ 

(b) Find the coordinates of the stationary points of  $y = \frac{1-27x^2}{x^3}$ 

(c) Find the coordinates of the stationary points of  $y = 20 + 15x - x^2 - \frac{x^3}{3}$ 

TAP FOR ANSWERS

### Skills 1 - Answers



### Skills 2 – Answers

(a) 
$$(0, 0), \left(\frac{-9}{4}, \frac{-2187}{256}\right)$$
  
(b)  $\left(\frac{1}{3}, -54\right), \left(\frac{-1}{3}, -54\right)$   
(c)  $\left(-5, \frac{-115}{3}\right), (3, 47)$ 

TAP TO RETURN

1

(a) Find the value of s if  $x^2 + (2s + 10)x + s^2 + 5 = 0$  has equal roots

(b) Find the values of p if  $2(p-1)x^2 + 2px + p - 1 = 0$  has real roots. Give your answer in surd form

2

The specification for a new rectangular car park states that the length is to be 5m more than the breadth. The perimeter of the car park is to be greater than 32m and the area of the car park is to be less than  $104cm^2$ 

(a) Form a linear inequality for the perimeter and solve it to find the range of values of *x*.

(b) Form a quadratic inequality for the area and solve it to find the range of values for *x*.

(c) Determine the set of possible values for *x*.

3

The triangle with vertices (0, 0), (1, p), (10, 0) is right angled. Find the two possible values of p.

#### 4

The point (6, 1) lies on the circle  $x^2 + y^2 - 8x - 4y + 15 = 0$ .

(a) Find the equation of the normal to the circle at this point

(b) Find the coordinates of the second point at which the normal cuts the circle

#### 5

A particle moves along a straight line. It accelerates from rest to a speed of  $12 \text{ ms}^{-1}$ . It then moves at a constant speed of  $12 \text{ ms}^{-1}$  for a period of time. Then the particle decelerates uniformly in twice the time for which it was accelerating, coming to rest after a total time of 19 seconds. Given that the total distance travelled by the particle is 174 m,

(a) Sketch a speed-time graph to illustrate the motion of the particle.

(b) find the time for which the particle is travelling at constant speed.

(c) find the time for which to particle is accelerating, and the acceleration of the particle.

6

A woman of mass 60kg is in a lift.

(a) Draw a diagram showing the forces acting on the woman when the lift is stationary

Find the normal reaction of the floor of the lift on the woman in each of the following cases:

(b) The lift is moving upwards at a constant speed of 3 ms<sup>-1</sup>

(c) The lift is moving upwards with an acceleration of 2 ms<sup>-2</sup> upwards

(d) The lift is moving downwards at an acceleration of 2 ms<sup>-2</sup> downwards

(e) The lift is moving downwards and slowing down with a deceleration of 2ms<sup>-2</sup>

In order to calculate the maximum number of people that can be safely carried in the lift, the following assumptions are made: the lift has mass 300kg, all resistances to motion may be considered negligible and ignored, the mass of each occupant is 75kg and the tension in the supporting cable should not exceed 12,000N.

(f) What is the maximum number of people who can be safely carried if the magnitude of the acceleration does not exceed  $3 \text{ ms}^{-2}$ .

7

Draw a labelled diagram, form labelled equations and re-arrange the equations to find the unknowns.

A stone that is modelled as a particle of mass m kg sits on a smooth slope of angle 60° to the horizontal, it is held at rest by a light inextensible string parallel to the slope that is fixed at point A at the top of the slope. The tension in the string is 8N. Find the mass of the particle and the size of the reaction force between the particle and the slope force, giving your answers to 3sf.

A brick that is modelled as a particle of mass 5 kg sits on a smooth slope of angle  $\alpha$  to the horizontal, it is held at rest by a light inextensible string parallel to the slope that is fixed at point A at the top of the slope. Given that the reaction force between the slope and the particle is 21N. Find the angle  $\alpha$ , and the tension in the string.

#### 8

#### Draw labelled diagrams and form equations for the following situations:-

(a) Two particles have masses 4 kg and *m* kg, where m < 4. They are attached to the ends of a light inextensible string which passes over a smooth fixed pulley with the masses hanging freely. The model is released from rest and accelerates at 1.4 ms<sup>-2</sup>

- (i) Find the tension in the string
- (ii) Calculate m

(b) Two particles A and B have mass 4kg and 7 kg respectively. They are attached to the ends of a light inextensible string with passes over a smooth pulley. Particle A lies on a smooth horizontal table and particle B hangs freely over the edge and is 1m from the ground.

- (i) What is the acceleration
- (ii) What is the tension in the string?
- (iii) The system is released from rest. What speed is B travelling at when it hits the ground?

9

[In this question, the unit vectors **i** and **j** are in a vertical plane, **i** being horizontal and **j** being vertical]

A particle *P* is projected from a point *A* with position vector 20 m with respect to a fixed origin *O*. The velocity of projection is  $(5ui + 4uj)m s^{-1}$ . The particle moves freely under gravity, passing through a point *B*, which has position vector (ki + 12j)m, where *k* is a constant, before reaching the point *C* on the *x*-axis, as shown in the figure below. The particle takes 4 s to move from *A* to *B*. Find

a) the value of *u*,

b) the value of *k*,

c) the angle the velocity of *P* makes with the *x*-axis as it reaches *C*.



#### 10

NEW TECHNIQUES!

The coordinates of *A* and *B* are (7, -1, 2) and (k, 0, 4) respectively. Given that the distance from *A* to *B* is 3 units, find the possible values of *k*.

#### 11

A circular pipe has outer diameter 4 cm and thickness *t* cm.

(a) Show that the area of the cross-section,  $A \text{ cm}^2$ , is given by  $A = \pi(4t - t^2)$ .

(b) Find the rate of increase of A with respect to t when  $t = \frac{1}{4}$  and when  $t = \frac{1}{2}$ .

A piece of wire 16 cm long is cut into two pieces. Once piece is 8x cm long and is bent to form a rectangle measuring 3x by x cm. The other piece is bent to form a square.

Find in terms of *x*:

c) the length of a side of the square;

d) the area of the square.

e) show that the combined area of the rectangle and the square is  $A \text{ cm}^2$  where  $A = 7x^2 - 16x + 16$ .

### 11

Find:

f) The value of x for which A has its minimum value;

g) the minimum value of *A*.

12

(a) Find the value of x for which the curve  $y = 800x + \frac{2}{x}$ , x > 0, has a stationary point.

(b) Using the second derivative, determine whether this point is a local maximum or minimum point.

#### 1 - Answers

(a) *s* = -2

(b)  $p \ge 2 + \sqrt{2} \text{ or } p \le 2 - \sqrt{2}$  (you must include a shadedsketch )

#### 2 - Answers

- (a) 4x 10 > 32, x > 10.5
- (b) x(x-5) < 104, -8 < x < 13
- (c) 10.5 < x < 13

### 3 - Answers

 $p = \pm 3$ TAP TO RETURN

#### 4 - Answers

(a) x + 2y - 8 = 0

(b) (2, 3)

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### 5 - Answers

(b) 10 seconds

(c) 3 seconds,  $a = 4ms^{-2}$ 

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#### 6 - Answers

(0) Joon	(b)	588N
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(c) 708N

(d) 468N

(e) 708

(f) 8 people

#### 7 - Answers

(a) m = 0.943 kg (3 sf) R = 4.62 N (3 sf)

(b)  $\alpha = 64.6^{\circ}$  (3sf), T = 44.3 N (3sf)

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8 - Answers



#### 9 - Answers

(a) 4.4	
(b) 88	
(c) 50° (2 s.f.)	TAP TO RETURN

#### 10 - Answers

k = 5 or k = 9

#### 11 - Answers

(b) $\frac{7}{2}\pi$ ; $3\pi$
(c) $4 - 2x$
(d) $16 - 16x + 4x^2$

(f) x = 1.143

(g) A = 6.857

### 12 - Answers

(a)  $\frac{1}{20}$ 

(b) Minimum (you need to give a reason)

TAP TO RETURN