Skills 1

(a) By completing the square, find the centre and radius of the circle $x^2 + y^2 + 4x - 6y + 10 = 0$

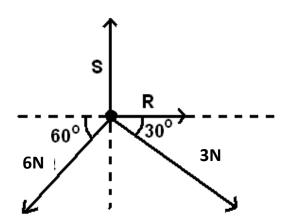
(b) $(x-2)^2 + (y-4)^2 = 25$, show on a sketch the circle and the tangents from the point (8, 2)

(c) Find the length of the tangents to the circle $(x + 3)^2 + (y + 5)^2 = 30$ from the point (-2, 3)

Skills 2

A particle at the origin is in equilibrium under the action of forces shown. Find the forces R and S

R +



TAP FOR ANSWERS

Skills 1 - Answers

(a) (-2, 3) $\sqrt{3}$

(b) Sketch

(c) $\sqrt{35}$

Skills 2 – Answers

 $R = 6 - \sqrt{3}N, S = 3\sqrt{3} + \frac{3}{2}N$

1

A triangle has vertices (2, 3), (4, 9), (5, 2)

(a) Find the exact perimeter of the triangle

(b) Prove that the triangle is right angled

2

The circle *C* has equation $x^2 + 3x + y^2 + 6y = 3x - 2y - 7$

- (a) Find the centre and radius of the circle.
- (b) Find the points of intersection of the circle and the y-axis
- (c) Show that the circle does not intersect the *x*-axis

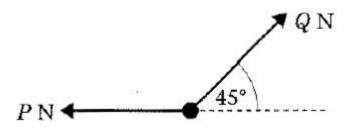
3

STATICS

A particle of mass 3kg is held in equilibrium by two light inextensible strings. One string is horizontal. The other string is inclined at 45° to the horizontal, as shown in the figure. The tension in the horizontal string is P N and the tension in the other string is QN. Find

(a) the value of Q

(b) The value of P



4

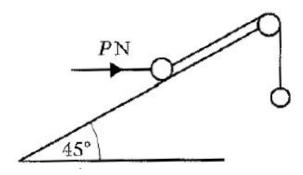
An astronaut weighs 735 N on earth and 120 N on the moon. Work out the value of acceleration due to the gravity on the moon.

5

A mass of 3kg rests on the surface of a smooth plane which is inclined at an angle of 45° to the horizontal. The mass is attached to a cable which passes up the plane along the line of greatest slope and then passes over a smooth pulley at the top of the plane. The cable carries a mass of 1kg freely suspended at the other end. The masses are modelled as a particle, and the cable as a light inextensible string. There is a force of *P* N acting horizontally on the 3kg mass and the system is in equilibrium. Calculate

(a) the magnitude of P

(b) the normal reaction between the mass and the plane.



6

(a) Find an equation of the tangent and the normal at the point where x = 2 on the curve with equation $y = \frac{8}{r} - x + 3x^2$, x > 0.

(b) The normals to the curve $2y = 3x^3 - 7x^2 + 4x$, at the points O(0, 0) and A(1,0), meet at the point N.

(i) Find the coordinates of *N*.

(ii) Calculate the area of triangle OAN.

7

 $f(x) = x^2 - 2x - 8$

- (a) Sketch the graph of y = f(x)
- (b) On the same set of axes, sketch the graph of y = f'(x)
- (c) Explain why the *x*-coordinate of the turning point of y = f(x) is the same as the *x*-coordinate of the point where the graph of y = f'(x) crosses the *x*-axis

8

(a) Given that $P = 2x + \frac{100}{x}$ (x > 0), find the value of x for which P is minimum.

Use the second derivative to justify your answer.

(b) $f(x) = x^3 + 4x^2 - 3x + 7$

Find the set of values of x for which f(x) is increasing

9

NEW TECHNIQUES!

Integration is the reverse of differentiation, so in terms of basic polynomials you add one the function and then divide by the new power. Every time we integrate, we need to put a +c at the end. Why might that be do you think?

For example if $\frac{dy}{dx} = x^2$, $y = \frac{x^3}{3} + c$ and if $\frac{dy}{dx} = 4x^5$ $y = \frac{2x^3}{3} + c$. In general when $\frac{dy}{dx} = ax^n$, $y = \frac{ax^{n+1}}{n+1} + c$

We set out an integration like this $\int x^2 dx = \frac{x}{3} + 6$. Integrate the following: (a) $\int x^3 dx =$

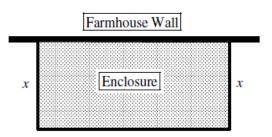
(b) $\int 3x^2 dx =$

(c) $\int \frac{2}{5} x^4 dx =$

(d) $\int 5x^{\frac{3}{2}}dx =$

(e) $\int 2x^{-2} dx =$ You can differentiate your answers to check they are correct.

10

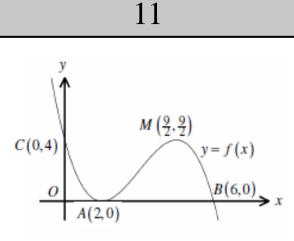


The figure above shows the plan of a rectangular enclosure to be built next to a farmhouse. One of the farmhouse's walls will form one of the sides of the enclosure and 25 metres of fencing will form the other three sides.

The width of the enclosure is x metres, as shown in the figure.

The area of the enclosure must be at most $75 m^2$.

Given further that the width of the enclosure must be at least 3 metres but no more than 9 metres, determine the range of possible values of x.



The figure above shows the graph of the curve with equation y = f(x). The curve meets the x axis at A(2,0) crosses the x axis a the point B(6,0), and crosses the y axis at the point C(0,4). The curve has a maximum at $M\left(\frac{9}{2},\frac{9}{2}\right)$ Sketch on separate diagrams the graphs of

(a) y = 2f(x)

(b)
$$y = f(x + 2)$$

(c)
$$y = f\left(\frac{1}{2}x\right)$$

12

Given that $x = -\frac{1}{2}$ is the real solution of the equation $2x^3 - 11x^2 + 14x + 10 = 0$, find the two complex solutions of this equation.

1 - Answers

(a) $3\sqrt{10} + 5\sqrt{2}$

(b) Use Pythagoras or perpendicular gradients

2 - Answers

- (a) Centre (0, -4) and radius = 3
- (b) (0, -1) and (0, -7)
- (c) Students' own work. Equation $x^2 = -7$ has no real solutions.

3 - Answers

(a)
$$R(\uparrow)$$

 $Q \sin 45 - 3g = 0$
 $\therefore Q = \frac{3g}{\sin 45}$
 $= 3\sqrt{2}g$
 $= 42 (2 \text{ s.f.})$
(b) $R(\rightarrow)$
 $Q \cos 45 - p = 0$
 $\therefore P = 3\sqrt{2}g \cos 45^{\circ}$
 $= 3g$
 $= 29 (2 \text{ s.f.})$

4 - Answers

1.6 m s⁻²

5 - Answers

(a) Consider the 1kg mass: $R(\uparrow)$ T - 1g = 0 $\therefore T = g = 9.8$

Consider the 3kg mass: $R(\nearrow)$ $T + P \cos 45 - 3g \sin 45 = 0$ $\therefore P \cos 45 = 3g \sin 45 - T$ But T = g $\therefore P \cos 45 = 3g \sin 45 - g$ $\therefore P \cos 45 = 3g \sin 45 - g$ $\therefore P = 3g - \frac{g}{\cos 45}$ $= 3g - g\sqrt{2}$ = 16 (s.f.)

5 - Answers

(b)
$$R(\)$$

 $R - P \sin 45 - 3g \cos 45 = 0$
 $\therefore R = P \sin 45 + 3g \cos 45$
 $= 6g \frac{\sqrt{2}}{2} - g$
 $= 32 (2 \text{ s.f.})$

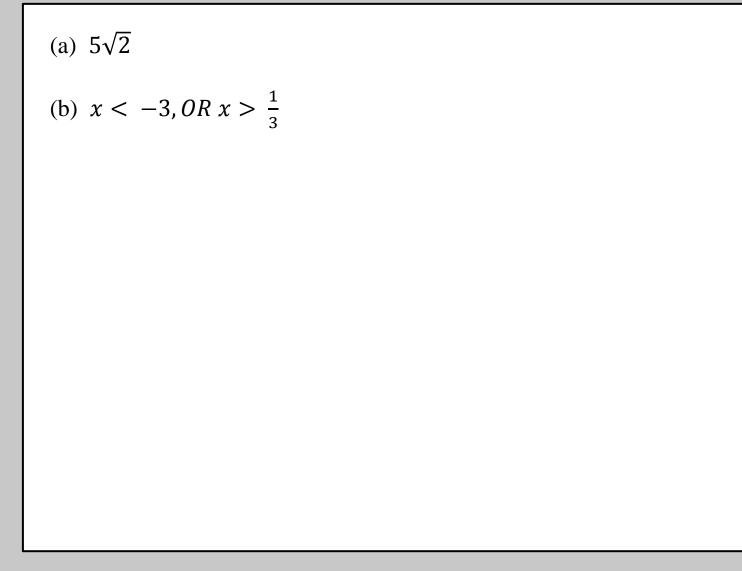
6 - Answers

(a) 9x - y - 4 = 0 and $9y + x \equiv 128 = 0$ (b) (i) $\left(\frac{4}{5}, -\frac{2}{5}\right)$ (ii) $\frac{1}{5}$

7 - Answers

- (a) Sketches
- (b) Discuss in class
- (c) Discuss in class

8 - Answers



9 - Answers

(a)	$\frac{x^4}{4} + C$
(b)	$x^3 + c$
(c)	$\frac{2}{25}x^5 + c$
(d)	$2x^{\frac{5}{2}} + c$
(e)	$-2x^{-1} + c$

10 - Answers

$$3 < x < 5$$
 and $\frac{15}{2} < x < 9$

11 - Answers

Sketches – discuss in class

12 - Answers

