BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 4B

Skills 1

Draw a labelled mathematical diagram to model each of the following situations.

Note – Use capital letters for forces, e.g. ***W*** for weight, ***R*** for normal reaction, ***T*** for tension, ***F*** for friction. An unknown force of indeterminate cause is often called ***P*** or ***X***.

(a) A football of mass 0.5 kg resting on horizontal ground.
(b) A box of mass 12kg hanging from a vertical rope.
(c) A piece of space debris, drifting in interstellar space.
(d) A cup of tea of mass 100 g resting on a rough table which is sloping at an angle 30^o to the horizontal.
(e) A heavy box, being dragged at a constant speed along a rough horizontal floor by a rope which is at an angle 45^o to the horizontal.
(f) A book being pushed from rest across a smooth horizontal table by a finger which is at angle 30^o to the horizontal.
(g) A smooth ring hanging on a taut string either end of which is attached to two points equal distance from the ground.

TAP FOR ANSWERS

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Skills 2

Find the equation of the normal to the curve at the point where x=1

(a) $y = x^2 - 3x$

(b) $y = \frac{7}{x^3}$

(c) $y = \frac{4-3x^2}{x}$

(d) Find the equation of the normal to $y = 3x^2 - x + 1$ at $x = 0$

(e) Find the equation of the normal to $y = 2x + \frac{1}{x}$ at $x = \frac{1}{2}$

(f) Find the equation of the normal to $y = x^3 + x^2$ at $x = 1$

TAP FOR ANSWERS

Skills 1 - Answers

Checked by your teacher

TAP TO RETURN

BHASVIC MαTHS
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Skills 2 – Answers

(a) $x - 3 - y = 0$

(b) $x - 21y + 146 = 0$

(c) $x - 7y + 6 = 0$

(d) $x - y + 1 = 0$

(e) $2x - 4y + 11 = 0$

(f) $x + 5y - 11 = 0$

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 4B

1

A quadratic function is defined by $f(x) = x^2 + kx + 9$ where k is a constant. It is given that the equation $f(x) = 0$ has two distinct real roots.

(a) Find the set of values k can take.

For the case where $k = 4\sqrt{3}$

(b) Express $f(x)$ in the form $(x + a)^2 + b$ stating the values of a and b, and hence write down the least value taken by $f(x)$

(c) solve the equation $f(x) = 0$ expressing your answer in terms of surds simplified as far as possible.

TAP FOR ANSWERS

2

The points A and B have coordinates $(-2, -7)$ and $(3, 8)$ respectively.

(a) Find the coordinates of the point at which the line through AB crosses the x-axis.

The mid-point of AB lies on the line with equation $y = kx$, where k is a constant.

(b) Find the value of k.

TAP FOR ANSWERS

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3

NEW TECHNIQUES!

A car is moving along a horizontal level road. The car's engine provides a constant driving force. The motion of the car is opposed by a constant resistance.

(a) Modelling the car as a particle, draw a force diagram to show the forces acting on the car.

(b) Given that the resultant force acting on the car is 4200N in the direction of motion, and that the magnitude of the driving force is eight times the magnitude of the resistance force, calculate the magnitude of the resistance.

TAP FOR ANSWERS

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4

NEW TECHNIQUES!

Three people are trying to move a skip. Two of the people are pushing horizontally with forces of magnitude 120N and 150N and one person is pulling with a horizontal force of magnitude X N. The frictional force resisting motion is 385N. Given that the skip does not move, find the value of X.

TAP FOR ANSWERS

BHASVIC MαTHS
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5

NEW TECHNIQUES!

Draw a labelled diagram, form labelled equations and re-arrange the equations to find the unknowns.

Two particles A and B, 7.5kg and 10kg respectively are connected by a light inextensible string. They are being pulled up a slope inclined at an angle of 30^o at a constant speed. A is above B. By considering particle B only, what is the tension in the string?

TAP FOR ANSWERS

BHASVIC MαTHS
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6

A child playing cricket on horizontal ground hits the ball towards a fence 10 m away. The ball moves in a vertical plane which is perpendicular to the fence. The ball just passes over the top of the fence, which is 2 m above the ground, as shown in Figure 3.

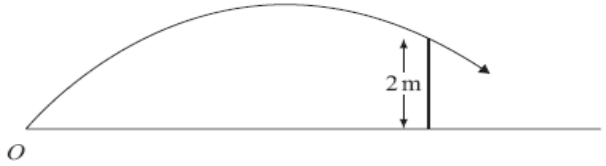

Figure 3

The ball is modelled as a particle projected with initial speed u m s^{-1} from point O on the ground at an angle α to the ground.

(a) By writing down expressions for the horizontal and vertical distances, from O of the ball t seconds after it was hit, show that $2 = 10\tan\alpha - \frac{50g}{u^2\cos^2\alpha}$

Given that $\alpha = 45°$,

(b) find the speed of the ball as it passes over the fence.

TAP FOR ANSWERS

BHASVIC MαTHS
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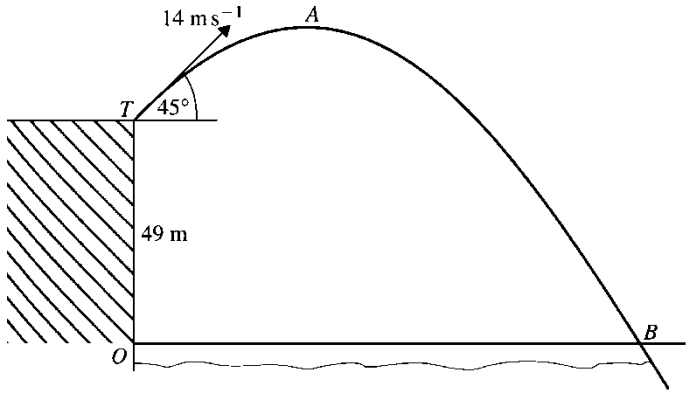
7

A golf ball is struck from the point T, at the top of a cliff which is 49m above sea level, with a speed of 14m/s at an angle of 45^o to the horizontal, as shown in the diagram. The point O is at sea level and vertically below T. The point A is the highest point reached by the ball in its motion. The ball strikes the sea at the point B.

(a) Find the height A above sea level.

(b) Find the distance OB.



TAP FOR ANSWERS

BHASVIC MαTHS
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8

A curve has the equation $y = x + \frac{3}{x}, x \neq 0$.

The point P on the curve has x coordinate 1.

(a) Show that the gradient of the curve at P is – 2.

(b) Find an equation for the normal to the curve at P, giving your answer in the form $y = mx + c$.

(c) Find the coordinates of the point where the normal to the curve at P intersects the curve again

TAP FOR ANSWERS

9

A cylinder of radius r cm has a volume of 100 cm^2.

(a) Find an expression for the height of the cylinder in terms of π and r.

(b) Show that the surface area of the cylinder is given by $A = 2\pi r^2 + 200r^{-1}$.

(c) Find the minimum surface area of the cylinder (you do not have to prove that it is a minimum).

TAP FOR ANSWERS

BHASVIC MαTHS
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10

A cuboid has base of width x cm, length $2x$ cm and height h cm. Its volume is 72 cm^2.

(a) Show that is surface area is given by $SA = 4x^2 + \frac{216}{x}$.

b) Find the value of x for which the surface area is a minimum.

c) Prove that the answer to part (b) gives a minimum surface area.

TAP FOR ANSWERS

BHASVIC MαTHS
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11

$$f(x) = \frac{1}{x}$$

(a) Given that $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$, show that $f'(x) = \lim_{h \to 0} \frac{-1}{x^2+xh}$

(b) Deduce that $f'(x) = -\frac{1}{x^2}$

TAP FOR ANSWERS

12

(a) Given that 2 + i is a root of the equation $z^2 + bz + c = 0$, where b and c are real constants,

(i) write down the other root of the equation,
(ii) find the value of b and the value of c.

(b) Given that 2 + i is a root of the equation $z^3 + mz^2 + nz - 5 = 0$, where m and n are real constants, find the value of m and the value of n.

Hint for (b) use the factor theorem and the real/imaginary parts lemma (i.e. if $a = bi = c + di$, then $a=c$ and $b=d$).

TAP FOR ANSWERS

1 - Answers

(a) $k < -6$ or $k > 6$ (you must include a shaded sketch)

(b) $a = -2\sqrt{3}, b = -3$ hence least value is -3

(c) $-\sqrt{3}, -3\sqrt{3}$

TAP TO RETURN

2 - Answers

(a) line AB is $3x - y - 1 = 0$ so coordinate is $\left(\frac{1}{3}, 0\right)$

(b) $k = 1$

TAP TO RETURN

3 - Answers

(b) 600N

TAP TO RETURN

4 - Answers

X=115N

TAP TO RETURN

5 - Answers

5g N

TAP TO RETURN

6 - Answers

9.13 m s^{-1}

TAP TO RETURN

7 - Answers

(a) 54m

(b) 43m

TAP TO RETURN

8 - Answers

(b) $y = \frac{1}{2}x + \frac{7}{2}$

(c) $\left(6, \frac{13}{2}\right)$

TAP TO RETURN

9 - Answers

(a) $h = \frac{100}{\pi r^2}$

(c) 119 cm^2 (3 s.f.)

TAP TO RETURN

10 - Answers

(b) $x = 3$

TAP TO RETURN

11 - Answers

Proof

TAP TO RETURN

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12 - Answers

(a) (i) $2 - i$

(a) (ii)

(b) $m = -5, n = 9$

TAP TO RETURN