BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 3B

Skills 1

1. Use the rule that if $y = ax^n$, then the gradient is given by $\frac{dy}{dx} = na\,x^{n-1}$ to find the gradient of the following graphs at the point where $x = 3$

(a) $y = x^2$
(b) $y = 3x^2$
(c) $y = 4x^3$
(d) $y = 8x^5$

2. In each of the following y is given as a function of x. Find the derived function $\frac{dy}{dx}$:

(a) $y = (2x^2 + 3)(x + 1)$ *(Hint: Expand first!)*
(b) $y = \sqrt[5]{x}$
(c) $y = 2x^{-5}$
(d) $y = x^2 - x^{-2}$
(e) $y = x(x^2 - 3)$
(f) $y = \frac{x^3 - 1}{2x}$

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Skills 2

Sketch the following, stating the *x* and *y* intercepts and the equations of any asymptotes:

(a) $y = (x-3)(x+3)(x-2)(x+2)$

(b) $y = x(x+3)^2(x-2)$

(c) $y = \frac{1}{x-1} + 2$

(d) $y = 64x - 9x^3$

(e) $y = \frac{2}{x^2}$

(f) $y = \sin 3x$

(g) $y = \cos\left(\frac{x}{2}\right)$

(h) $y = \tan\left(x + \frac{3\pi}{2}\right)$

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Skills 1 - Answers

1.

(a) 6

(b) 18

(c) 108

(d) 3240

2.

(a) $6x^2 + 4x + 3$

(b) $\frac{1}{5}x^{-\frac{4}{5}}$

(c) $-10x^{-6}$

(d) $2x + 2x^{-3}$

(e) $3x^2 - 3$

(f) $x + \frac{1}{2}x^{-2}$

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Skills 2 - Answers

In the library computers you can plot the graphs on ‘autograph’. On your phone you could use the free app ‘desmos’. Or, use your graphical calculator to check. It is important you try these yourself first, don’t go straight to the answers! And **don’t forget to label the intercepts & asymptotes**.

TAP TO RETURN

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1

Find the possible values of k for the quadratic equation $2kx^2 + 5kx + 5k - 3 = 0$ to have real roots.

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2

A quadratic graph $y = ax^2 + bx + c$ has a minimum point at $(4, -3)$ and passes through the point $(5,0)$. Find the values of a, b and c.

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(a) Sketch the curve of $y = x^3 - 6x^2 + 9x$ showing clearly the coordinates of any point where the curve touches or crosses the axes.

(b) The point with coordinates (-4,0) lies on the curve with equation $y = (x - k)^3 - 6(x - k)^2 + 9(x - k)$ where k is a constant. Find the two possible values of k.

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(a) A line passes through points $(p, 3)$, $(p + 2, 5)$ and $(1, 2)$ where $p > 0$. Find the value of p.

(b) The midpoint of $(5, p)$ and $(q, 10)$ is $(6, 6)$. Find the value of p and q.

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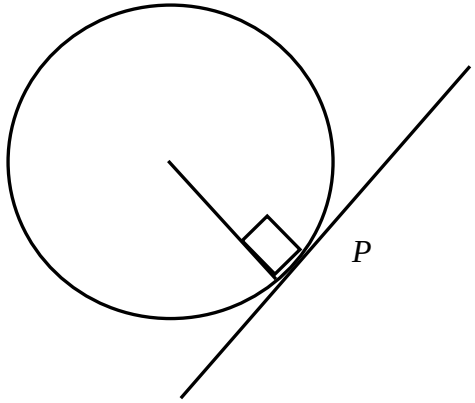
NEW TECHNIQUES!!

Remember that the tangent to a circle at P is always perpendicular to the radius joining P to the centre of the circle.

Use this information to find the equation of the tangent to the circle $(x + 2)^2 + (y - 2)^2 = 73$ at the point (1,-6), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.



TAP FOR ANSWERS

NEW TECHNIQUES!

Draw labelled mathematical diagram(s) to model each of the following situations.

Note – Use capital letters for forces, e.g. ***W*** for weight, ***R*** for normal reaction, ***T*** for tension, ***F*** for friction. An unknown force of indeterminate cause is often called ***P*** or ***X***.

- You'll need to use the fact that an object with mass *m* kg will have a weight of *mg* N, where *g* is the acceleration due to gravity.

(a) A book of mass 400 g resting on a horizontal table

(b) A fish of mass M kg dangling from a vertical fishing line

(c) An ice hockey puck gliding across the ice at a constant velocity

(d) A box of mass 75 g resting on a rough table which is sloping at an angle 30^o to the horizontal

(e) A dog, being dragged along by its lead (at an angle 45^o to the horizontal), from rest

(f) The head of a mop being pushed across the floor at a constant speed by its handle (which is at angle 30^o to the horizontal)

(g) A wet jumper hanging by a smooth hanger on a washing line.

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NEW TECHNIQUES!

Draw a labelled diagram and form an equation for the following scenario. The system is in equilibrium, so there is no resultant force in any direction. (E.g. the sum of the forces going up = the sum of the forces going down)

A fish of mass 5 kg is stationary and suspended from a vertical fishing line. What is the tension in the fishing line?

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NEW TECHNIQUES!

Particles *A* and *B* each of mass *20kg* are joined by a light inextensible string which passes over a smooth pulley so that the string hangs vertically on both sides. The system is in **equilibrium**.

(a) Draw a force diagram to model this situation.

(b) By considering each particle separately, find the tension T in the string.

(c) How have you used the fact that the string is light in your model?

(d) How have you used the fact that the string is inextensible in your model?

(e) How have you used the fact that the pulley is smooth in your model?

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A particle P is projected from a point O on level ground with speed 50 ms^{-1} at an angle Θ where
$\sin\Theta = \left(\frac{7}{25}\right)$ above the horizontal. Find:

(a) the height of P at the point where its horizontal displacement from 0 is 120 m,

(b) the speed of P two seconds after projection,

(c) the times after projection at which P is moving at an angle of $\tan^{-1}\left(\frac{1}{3}\right)$ to the ground

TAP FOR ANSWERS

10

Prove, from first principles, that the derivative of $10x^2$ is $20x$.

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11

NEW TECHNIQUES!

Determining the nature of a stationary point **(which has $f'(x) = 0$)**:

Second derivative test

If $f''(x) > 0$ then it's a local **minimum**

If $f''(x) < 0$ then it's a local **maximum**

If $f''(x) = 0$ then more investigation is required (you will see this in class).

$$f(x) = x^4 + 3x^3 - 5x^2 - 3x + 1$$

(a) Find the coordinates of the stationary points of *f(x)*, and determine the nature of each.

(b) Sketch the graph of $y = f(x)$.

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The normals to the curve $2y = 3x^3 - 7x^2 + 4x$, at the points $O(0,0)$ and $A(1,0)$, meet at the point N.

(a) Find the coordinates of N.

(b) Calculate the area of triangle OAN.

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1 - Answers

$0 \leq k \leq \frac{8}{5}$

TAP TO RETURN

2 - Answers

$a = 3, b = -24, c = 45$

TAP TO RETURN

3 - Answers

(a) Check on desmos

(b) -4 and -7

TAP TO RETURN

4 - Answers

(a) $p = 2$,

(b) $p = 2$, $q = 7$

TAP TO RETURN

5 - Answers

$3x - 8y - 51 = 0$

TAP TO RETURN

6 - Answers

Checked by your teacher.

TAP TO RETURN

7 - Answers

5g N

TAP TO RETURN

8 - Answers

(a) Diagram checked by your teacher.
(b) T = 20g N
(c) “tension at A is the same magnitude as tension at B”
(d) “acceleration of A is same magnitude as acceleration of B” (in this case there’s no acceleration as it’s in equilibrium)
(e) “tensions in two parts of the string are the same” (if there was friction in the pulley it makes sense that this would NOT be true, right?)

TAP TO RETURN

9 - Answers

(a) 4.4m

(b) 48 ms^{-1}

(c) 0.61s and 9.2s

TAP TO RETURN

10 - Answers

Proof

TAP TO RETURN

11 - Answers

(a) (1,-3) local minimum; (-3,-35) local minimum; $(-\frac{1}{4}, \frac{357}{256})$ local maximum

(b) Check Desmos

TAP TO RETURN

12 - Answers

(a) $(\frac{4}{5}, -\frac{2}{5})$

(b) $\frac{1}{5}$

TAP TO RETURN