A1 DOUBLES ASSIGNMENT 24 – PART B

Skills 1

Find the Cartesian equation of the curves given by the following parametric equations.

a $x = \sin t, \quad y = \sin\left(t + \frac{\pi}{4}\right), \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$

b $x = 3\cos t, \quad y = 2\cos\left(t + \frac{\pi}{6}\right), \quad 0 < t < \frac{\pi}{3}$

c $x = \sin t, \quad y = 3\sin(t + \pi), \quad 0 < t < 2\pi$

Skills 2

Find the following integrals.

a $\int \frac{2(x^2 + 3x - 1)}{(x + 1)(2x - 1)}\,dx$ **b** $\int \frac{x^3 + 2x^2 + 2}{x(x + 1)}\,dx$ **c** $\int \frac{x^2}{x^2 - 4}\,dx$ **d** $\int \frac{x^2 + x + 2}{3 - 2x - x^2}\,dx$

PROBLEM SOLVING

1. Graph Transformations

a) Sketch the curve of $y = x^3 + x^2 - 6x$ showing clearly the coordinates of the points where the curve touches or crosses the axes.

b) The point with coordinates (2,0) lies on the curve with equation $y = (x + a)^3 - (x + a)^2 - 6(x + a)$ where a is a constant. Find the two possible values of a

2. Modulus graphs (inc inequality problems)

(a) Sketch the graph of $y = |2x + a|$, $a > 0$, showing the coordinates of the points where the graph meets the coordinate axes.

(b) On the same axes, sketch the graph of $y = \frac{1}{x}$.

(c) Explain how your graphs show that there is only one solution of the equation

$$x|2x + a| - 1 = 0.$$

(d) Find, using algebra, the value of x for which $x|2x + 1| - 1 = 0$.

a) And b) use graph sketching app c)1 d) ½

3. Graphical Inequalities

a) On a coordinate grid shade the region that satisfies the inequalities

$$y + 2x \le 4, y - 2x \le 2, \ x \ge 1$$

b) Find the coordinates of the vertices of the shaded region ABC.
c) Find the area of the shaded region

4. Co-ordinate Geometry –line

Relative to a fixed origin O the points A , B and C have respective coordinates $(1,3)$, $(1,11)$ and $(13, k)$, where k is a constant.
a) Find the length of AB , in the form $a\sqrt{17}$, where a is an integer.
b) Given the length of BC is $3\sqrt{17}$, , determine the possible values of k .
The actual value of k is in fact the smaller of the two values found in part (b).
c) Show clearly that angle $ABC = 90°$.
d) Calculate the area of the triangle ABC .

5. Kinematics. Variable Acceleration.

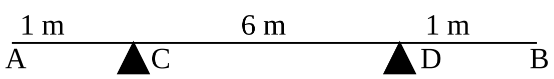
A swimmer C swims with velocity v ms^{-1} in a swimming pool. At time t seconds after starting,

$$v = 0.006\,t^2 - 0.18\,t + k$$

where k is a constant. C swims from one end of the pool to the other in 28.4 seconds.
(a) Find the acceleration of C in terms of ***t***.
(b) Given that the minimum speed of C is 0.65 ms^{-1}, show that k=2.
(c) Express the distance travelled by C in terms of ***t***, and calculate the length of the pool.

6. Moments

a) A uniform plank AB of length 8m and mass 20kg rests on supports at C and D, where AC = DB = 1m.

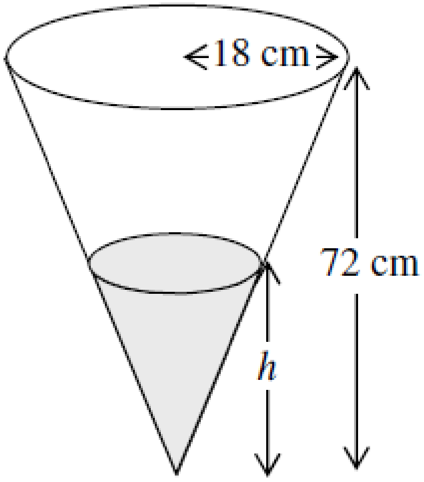


i. A girl of mass 45kg stands at B. Find the contact forces at C and D.
ii. What is the maximum mass that can be placed at B before the plank starts to tip?

b) A non-uniform rod AB of length 3 m and mass 5 kg is suspended in equilibrium in a horizontal position by vertical ropes attached to points P and Q of the rod. AP = 1 m and AQ = 2.5 m. The tensions in the ropes are equal. Find the distance of the centre of mass of the rod from A.

c) A uniform ladder AB of mass M kg and length 5m rests with end A on a smooth horizontal floor and end B against a smooth vertical wall. The ladder is held in equilibrium at an angle θ to the floor by a light horizontal string attached to the wall and to a point C on the ladder. If tanθ = 2, find the tension in the string when the length AC is 2m.

7. Connected Rates



Flowers at a florists are stored in vases which are in the shape of hollow inverted right circular cones with height 72 cm and radius 18 cm.
One such vase is initially empty and placed under a tap where the water is flowing into the vase at the constant rate of $6\pi\ cm^3 s^{-1}$

a) Show that the volume, Vcm^3, of the water in the vase is given by $V = \frac{1}{48}\pi h^3$, where h cm is the height of the water in the vase.

b) Find the rate at which h is rising when h = 4 cm.

c) Determine the rate at which h is rising 12.5 **minutes** after the vase was placed under the tap.

8. During a chemical reaction, a compound is being made from two other substances. At time *t* hours after the start of the reaction, *x* g of the compound has been produced. Assuming that *x* = 0 initially, and that

$$\frac{dx}{dt} = 2(x-6)(x-3)$$

(a) Show that it takes approximately 7 minutes to produce 2 g of the compound.

(b) Explain why it is not possible to produce 3 g of the compound.

9. Given that *y* = 2 at $x = \frac{\pi}{8}$, solve the differential equation

$$\frac{dy}{dx} = \frac{3y^2}{2\sin^2 2x}$$

giving your answer in the form *y* = f(*x*).

10. A curve is given by the parametric equations $x = 2t^2 - 1$, $y = 3(t + 1)$. Find the points of intersection of this curve and the line with equation $3x - 4y = 3$

11.

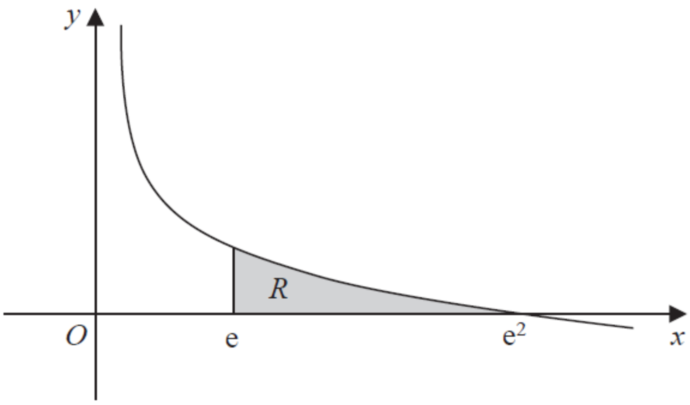


Figure 5

Figure 5 shows a sketch of part of the curve with equation $y = 2 - \ln x$, $x > 0$

The finite region R, shown shaded in Figure 5, is bounded by the curve, the x-axis and the line with equation $x = \mathrm{e}$.

The table below shows corresponding values of x and y for $y = 2 - \ln x$

x	e	$\frac{\mathrm{e}+\mathrm{e}^2}{2}$	e^2
y	1		0

(*a*) Complete the table giving the value of y to 4 decimal places.

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 3 decimal places.

(*c*) Show that $\int (\ln x)^2 \,\mathrm{d}x = x(\ln x)^2 - 2x\ln x + 2x + c$

12. a) using a suitable substitution of your choosing $\int \frac{1}{1-x^{\frac{1}{2}}}dx$

b) *using integration by parts, find the exact integral*: $\int_2^3 x^3 e^{x^2} dx$

c) using the substitution $x = 2sinu$, $\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}dx$

d) $\int 2x(ln3x)^2 dx$