BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 21B

Skills 1

PART A:

Integrate the following functions with respect to x:

(a) $\cos^2 2x$

(b) $\tan^2 3x$

(c) $\frac{2}{3x-1}$

(d) $\frac{2\ \text{cosec}^2\ 2x}{2-3\cot 2x}$

PART B:

Find:

(a) $\int\left(1-\frac{1}{x}\right)^2 \mathrm{d}x$

(b) $\int(\sin x+2\cos x)^2\ \mathrm{d}x$

(c) $\int \tan 3x\ \mathrm{d}x$

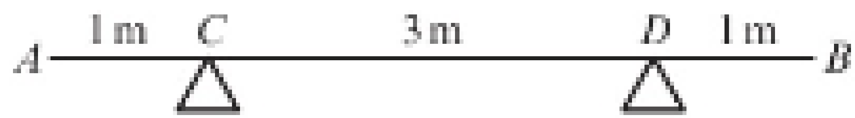
(d) $\int \frac{2cosec^2 2x}{(2-3cot2x)^5}dx$

TAP FOR ANSWERS

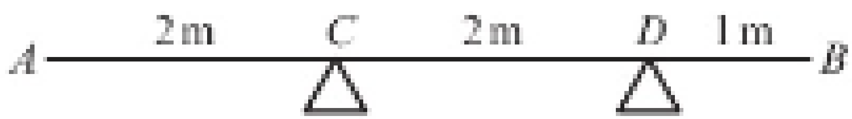
Skills 2

AB is a uniform rod of length 5 m and weight 20 N. In these diagrams *AB* is resting in a horizontal position on supports at *C* and *D*. In each case, find the magnitudes of the reactions at *C* and *D*.

(a)



(b)



(c)

(d)

TAP FOR ANSWERS

Skills 1 - Answers

PART A:

(a) $\frac{1}{2}x + \frac{1}{8}sin4x + c$

(b) $\frac{1}{3}tan3x - x + c$

(c) $\frac{3}{2}\ln|3x - 1| + c$

(d) $\frac{1}{3}\ln|2 - 3cot2x| + c$

PART B:

(a) $x - \frac{1}{x} - 2\ln|x| + c$

(b) $\frac{5}{2}x + \frac{3}{4}sin2x - cos2x + c$

(c) $-\frac{1}{3}\ln|cos3x| + c$

(d) $-\frac{1}{12}(2 - 3cot2x)^{-4} + c$

TAP TO RETURN

Skills 2 – Answers

(a) 10 N, 10 N

(b) 15 N, 5N

(c) 12 N, 8 N

(d) 12.6 N, 7.4 N

TAP TO RETURN

1

Using a suitable trigonometric substitution for x, find $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} x^2\sqrt{1-x^2}\,dx$

TAP FOR ANSWERS

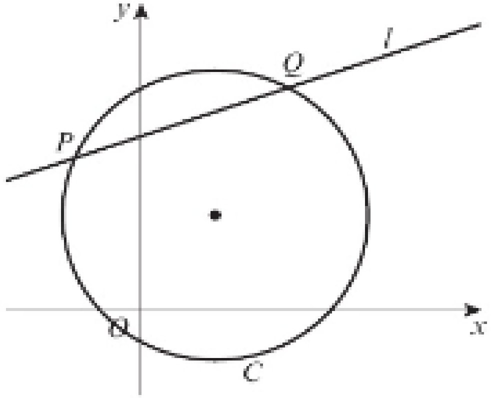
2

On a coordinate grid shade the region that satisfies the inequalities

$$y + x > 4, y < 2x - 3, y \geq 2 \ and \ x < 3$$

TAP FOR ANSWERS

3

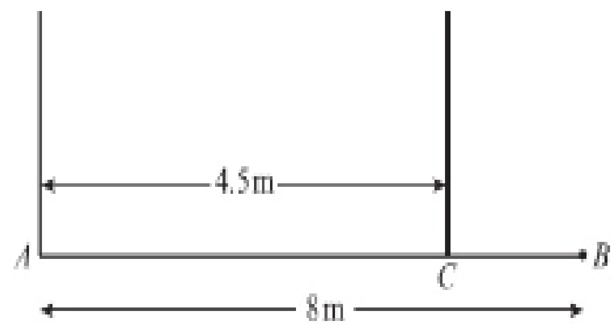


The circle C has equation $x^2 - 4x + y^2 - 6y = 7$
The line l with equation $x - 3y + 17 = 0$ intersects the circle at the points P and Q.

(a) find the coordinates of the point P and the point Q.
(b) find the equation of the tangent at the point P and the point Q.
(c) find the equation of the perpendicular bisector of the chord PQ.
(d) Show that the two tangents and the perpendicular bisector intersect at a single point and find the coordinates of the point of intersection.

TAP FOR ANSWERS

4



A uniform bean *AB* has weight *w* N and length 8 m. The beam is held in a horizontal position in equilibrium by two vertical light inextensible wires attached to the beam at the points *A* and *C* where *AC* = 4.5 m, as shown in the diagram. A particle of weight 30 N is attached to the beam at *B*.

(a) Show that the tension in the wire attached to the beam at *C* is $\left(\frac{8}{9}W + \frac{160}{3}\right)$ N.

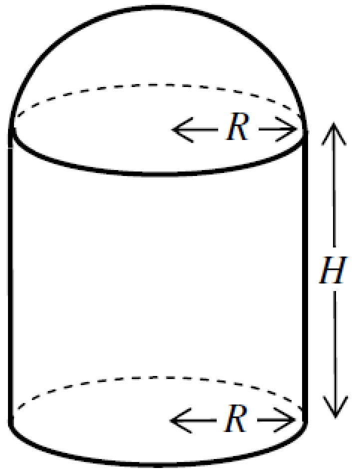
(b) Find, in terms of *W*, the tension in the wire attached to the beam at *A*.

Given that the tension in the wire attached to the beam at *C* is twelve times the tension in the wire attached to the beam at *A*.

(c) Find the value of *W*.

TAP FOR ANSWERS

5



The figure above shows a hollow container consisting of a right circular cylinder of radius R and height H joined to a hemisphere of radius R.
The cylinder is open on one of the circular ends and the hemisphere is also open on one of its circular ends so that the resulting object is completely sealed.
Given that the volume of the container is V, show that the surface area of the container is minimised when R=H, and hence show further that this minimum surface area is $\sqrt[3]{\frac{5}{3}\pi V}$

TAP FOR ANSWERS

6

The radius, R, of a circle, in cm, at time t seconds is given by $R = 10\left(1 - e^{-kt}\right)$, where k is a positive constant and t>0.

Show that if A is the Area of the circle, in cm^2, then $\frac{dA}{dt} = 200\pi k(e^{-kt} - e^{-2kt})$

TAP FOR ANSWERS

7

A curve C has equation $3^x = y - 2xy$

Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point on C with coordinates (2, -3).

TAP FOR ANSWERS

8

(a) Find $\int \sec^2 3x \, \mathrm{d}x$

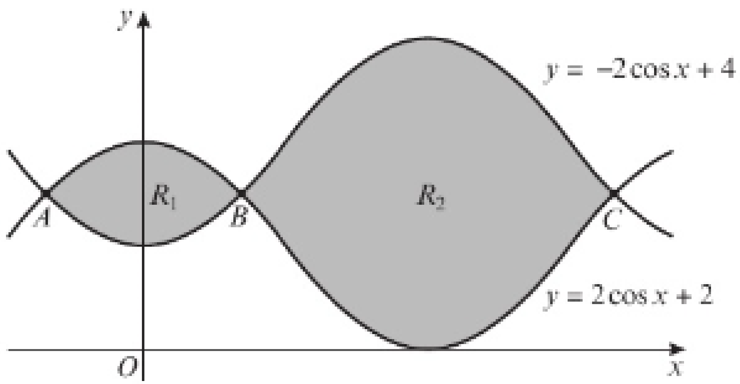
(b) Using integration by parts, or otherwise, find $\int x \sec^2 3x \, \mathrm{d}x$

(c) Hence show that $\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} x \sec^2 3x \, \mathrm{d}x = p\pi - q \ln 3$, finding the exact values of the constants p and q.

TAP FOR ANSWERS

9

The diagram shows a sketch of part of the curves with equations $y = s\cos x + 2$ and $y = -2\cos x + 4$



(a) Find the coordinates of the points A, B and C.

(b) Find the area of region R_1 in the form $a\sqrt{3} + \frac{b\pi}{c}$, where a, b and c are integers to be found.

(c) Show that the ratio of $R_2 : R_1$ can be expressed as $\left(3\sqrt{3} + 2\pi\right) : \left(3\sqrt{3} - \pi\right)$

TAP FOR ANSWERS

10

Rabbits were introduced onto an island. The number of rabbits, *P*, *t* years after they were introduced is modelled by the equation

$$P = 80\mathrm{e}^{\frac{1}{5}t}, \quad t \in \mathbb{R}, \quad t \geq 0.$$

(a) Write down the number of rabbits that were first introduced to the island.

(b) Find the number of years it would take for the number of rabbits to first exceed 1000.

(c) Find $\frac{\mathrm{d}P}{\mathrm{d}t}$.

(d) Find *P* when $\frac{\mathrm{d}P}{\mathrm{d}t} = 50$.

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 21B

11

Two functions *f* and *g* are defined by

$$f : x \mapsto \frac{25}{3x-2},\ x \in \mathbb{R},\ 1 < x \leq 9 \quad \text{and} \quad g : x \mapsto x^2,\ x \in \mathbb{R},\ 1 < x \leq 3\ .$$

(a) the range of *f* *(using a sketch to illustrate your answer)*

(b) the inverse function f^{-1}, stating its domain

(c) the composite function *fg*, stating its domain

(d) the solutions to the equation $fg(x) = \dfrac{2}{x-1}$

TAP FOR ANSWERS

12

(a) $M = \frac{e^t}{1+e^t}$

(b) $\frac{2}{3}$

(c) M approaches 1

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 21B

1 - Answers

$$\frac{2\pi+3\sqrt{3}}{96}$$

TAP TO RETURN

2 - Answers

Use graph sketching app

TAP TO RETURN

3 - Answers

(a) $P(-2,5)$ and $Q(4,7)$

(b) $y = 2x + 9$ and $y = -\frac{1}{2}x + 9$

(c) $y = -3x + 9$

(d) (0, 9)

TAP TO RETURN

4 - Answers

(a) $\frac{9}{2}T_c = 4W + 8 \times 30$

$\frac{9}{2}T_c = 4W + 240$

$9T_c = 4W + 480$

$T_c = \frac{8}{9}W + \frac{160}{3}$

(b) $T_A = \frac{w}{6} - \frac{70}{3}$

(c) 750 N

TAP TO RETURN

5 - Answers

Proof

TAP TO RETURN

6 - Answers

Proof

TAP TO RETURN

7 - Answers

$2 - 3\ln 3$

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 21B

8 - Answers

(a) $\frac{1}{3}\tan 3x + c$

(b) $\frac{1}{3}x\tan 3x - \frac{1}{9}\ln|\sec 3x| + c$

(c) $\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} x\sec^2 3x = \left[\frac{1}{3}x\tan 3x - \frac{1}{9}\ln|\sec 3x|\right]_{\frac{\pi}{18}}^{\frac{\pi}{9}}$

$= \left(\frac{\sqrt{3}\pi}{27} - \frac{1}{9}\ln 2\right) - \left(\frac{\sqrt{3}\pi}{162} - \frac{1}{9}\ln\frac{2}{\sqrt{3}}\right)$

$= \frac{5\sqrt{3}\pi}{162} - \frac{1}{9}\ln 2 + \frac{1}{9}\ln 2 - \frac{1}{9}\ln\sqrt{3}$

$= \frac{5\sqrt{3}\pi}{162} - \frac{1}{18}\ln 3 \Rightarrow p = \frac{5\sqrt{3}\pi}{162}$ and $q = \frac{1}{18}$

TAP TO RETURN

9 - Answers

(a) $A\left(-\frac{\pi}{3}, 3\right), B\left(\frac{\pi}{3}, 3\right)$ and $c\left(\frac{5\pi}{3}, 3\right)$

(b) $a = 4, b = -4, c = 3$ (or $a = 4, b = 4, c = -3$)

(c) $R_2 = \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (-2\cos x + 4)\, dx - \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (2\cos x + 2)\, dx$

$= \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (-4\cos x + 2) dx = [-4\sin x + 2x]_{\frac{\pi}{3}}^{\frac{5\pi}{3}}$

$= \left(2\sqrt{3} + \frac{10\pi}{3}\right) - \left(-2\sqrt{3} + \frac{2\pi}{3}\right) = 4\sqrt{3} + \frac{8\pi}{3}$

$R_2 : R_1 \Rightarrow 4\sqrt{3} + \frac{8\pi}{3} : 4\sqrt{3} - \frac{4\pi}{3} \Rightarrow 3\sqrt{3} + 2\pi : 3\sqrt{3} - \pi$

TAP TO RETURN

10 - Answers

(a) 80

(b) 12.6286…

(c) $\frac{dP}{dt} = 16e^{\frac{t}{5}}$

(d) 250

TAP TO RETURN

11 - Answers

(a) $1 \leq f(x) < 25$

(b) $f^{-1} : x \mapsto \dfrac{25+2x}{3x} \quad x \in \mathbb{R} \quad 1 \leq x < 25$

(c) $fg : x \mapsto \dfrac{25}{3x^2 - 2} \quad x \in \mathbb{R} \quad 1 < x \leq 3$

(d) $x = \dfrac{7}{6}$ or $x = 3$

TAP TO RETURN

12 - Answers

The mass M at time t of the leaves of a certain plant varies according to the differential equation $\frac{dM}{dt} = M - M^2$

(a) Given that at time $t = 0$, $M = 0.5$, find an expression for M in terms of t.

(b) Find a value of M when $t = \ln 2$.

(c) Explain what happens to the value of M as t increases.

TAP TO RETURN