BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 20B

Skills 1

Use the substitutions given to find:

(a) $\int x\sqrt{1+x}\,\mathrm{d}x; u = 1+x$

(b) $\int \frac{1+\sin x}{\cos x}\,\mathrm{d}x; u = \sin x$

(c) $\int \sin^3 x\,\mathrm{d}x; u = \cos x$

(d) $\int \frac{2}{\sqrt{x}(x-4)}\,\mathrm{d}x; u = \sqrt{x}$

(e) $\int \sec^2 x \tan x\sqrt{1+\tan x}\,\mathrm{d}x;\ u^2 = 1+\tan x$

(f) $\int \sec^4 x\,\mathrm{d}x; u = \tan x$

TAP FOR ANSWERS

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Skills 2

Find the following integrals

(a) $\int 3 \ln x \, dx$

(b) $\int x \ln x \, dx$

(c) $\int \frac{\ln x}{x^3}$

(d) $\int (\ln x)^2 dx$

(e) $\int (x^2 + 1) \ln x \, dx$

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Skills 1 - Answers

(a) $\frac{2}{5}(1+x)^{\frac{5}{2}}-\frac{2}{3}(1+x)^{\frac{3}{2}}+c$

(b) $-\ln|1-\sin x|+c$

(c) $\frac{\cos^3 x}{3}-\cos x+c$

(d) $\ln\left|\frac{\sqrt{x}-2}{\sqrt{x}+2}+c\right|$

(e) $\frac{2}{5}(1+\tan x)^{\frac{5}{2}}-\frac{2}{3}(1+\tan x)^{\frac{3}{2}}+c$

(f) $\tan x+\frac{1}{3}\tan^3 x+c$

TAP TO RETURN

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Skills 2 – Answers

(a) $3x \ln x - 3x + c$

(b) $\frac{x^2}{2} \ln x - \frac{x^2}{4} + c$

(c) $-\frac{\ln x}{2x^2} - \frac{1}{4x^2} + c$

(d) $x(\ln x)^2 - 2x \ln x + 2x + c$

(e) $\frac{x^3}{3} \ln x - \frac{x^3}{9} + x \ln x - x + 3$

TAP TO RETURN

BHASVIC MαTHS
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1

(a) Find the set of values of u that satisfy $\frac{3}{u^2} + 2 \leq \frac{-7}{u}, u \neq 0$

(b) Hence find the set of values of u that satisfy $\frac{3}{(u-1)^2} + 2 \leq \frac{-7}{(u-1)}, u \neq 0$

TAP FOR ANSWERS

2

A ball is thrown in the air. After t seconds, its height, s, in metres above the ground is given by the equation $2s = -10t^2 + 16t + 3$.

(a) Find t when the ball is 4.5 metres above the ground.

(b) Show that $s = a(t + b)^2 + c$ where a, b and c are constants to be found.

(c) Hence find the maximum height of the ball and the value of t for which this occurs.

TAP FOR ANSWERS

3

Solve the following inequalities

(a) $\left|\frac{x-1}{x-2}\right| \geq 4$

(b) $\left|\frac{2p^2}{3p+2}\right| < 1$

TAP FOR ANSWERS

4

The points A, B, C and D have co-ordinates (-5,6) and (5,1) and (8,3) and (k,-13), respectively, where k is a constant.

(a) Find an equation of the straight line through A and B.

(b) Given that CD is perpendicular to AB, find the value of k

TAP FOR ANSWERS

5

A smooth bead Y is threaded on a light inextensible string. The ends of the string are attached to two fixed points X and Z on the same horizontal level. The bead is held in equilibrium by a horizontal force of magnitude 8N acting parallel to ZX. The bead Y is vertically below X and angle XZY = 30°
Find (a) the tension in the string, (b) the weight of the bead.

TAP FOR ANSWERS

6

A particle P is projected with velocity $(3u\mathbf{i} + 4u\mathbf{j})$ m s^{-1} from a fixed point O on a horizontal plane. Given that P strikes the plane at a point 750 m from O,

(a) show that $u = 17.5$,

(b) calculate the greatest height above the plane reached by P,

(c) find the angle the direction of motion of P makes with $\mathbf{i}$ when $t = 5$.

TAP FOR ANSWERS

7

The curve C has equation $y = \frac{x}{x^2+1}$

(a) Show that there is no point on C where the gradient is -1

(b) Find the co-ordinates of the points on C where the gradient is $\frac{12}{25}$

TAP FOR ANSWERS

8

(a) A curve has parametric equations $x = t^2 - 1, \quad y = t - t^3$. Draw this curve for when $-2 \leq t \leq 2$.

(b) Find the Cartesian equation of the curve when t > 0

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 20B

9

The curve C shown in Figure 4 has parametric equations

$$x = 1 + \sqrt{3}\tan\theta, \qquad y = 5\sec\theta, \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

The curve C crosses the y-axis at A and has a minimum turning point at B, as shown in Figure 4.

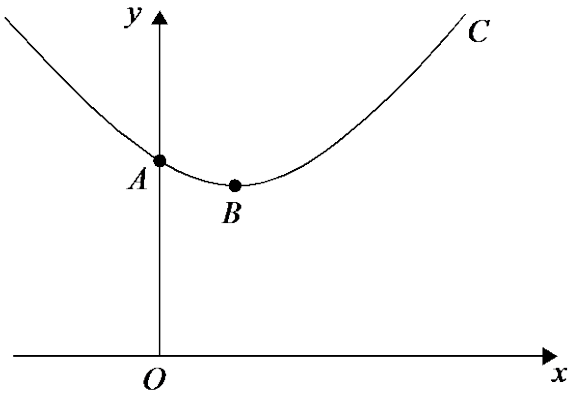


Figure 4

(a) Find the exact coordinates of A.

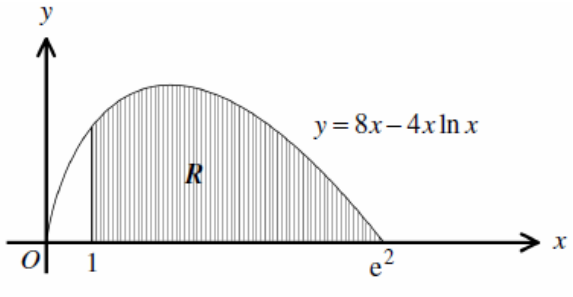
(b) Show that $\frac{dy}{dx} = \lambda \sin\theta$, giving the exact value of the constant λ

(c) Find the coordinates of B.

(d) Show that the Cartesian equation for the curve C can be written in the form $y = k\sqrt{(x^2 - 2x + 4)}$ where k is a simplified surd to be found.

TAP FOR ANSWERS

10



The figure above shows the graph of the curve with equation

$$y = 8x - 4x \ln x \, , 0 < x \leq e^2$$

The region R is bounded by the curve, the x axis and the line with equation $x = 1$

Determine the exact area of R

TAP FOR ANSWERS

11

Using a suitable trigonometric substitution for x, find $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} x^2\sqrt{1-x^2}\,\mathrm{d}x$

TAP FOR ANSWERS

12

Find:

$$\int \frac{3-x}{2x^3-x^2}\,dx$$

TAP FOR ANSWERS

1 - Answers

(a) $-3 \leq u \leq \frac{-1}{2}$ (you must include a sketch)

(b) $-2 \leq u \leq \frac{1}{2}$

TAP TO RETURN

2 - Answers

(a) $t = \frac{3}{5}$ or $t = 1$

(b) $a = -5, b = \frac{4}{5}, c = 4.7$

(c) Max is 4.7 metres when $t = 0.8$ seconds.

TAP TO RETURN

3 - Answers

(a) $\frac{9}{5} \leq x \leq \frac{7}{3}$

(b) $-\frac{1}{2} < p < 2$

TAP TO RETURN

4 - Answers

(a) $x + 2y = 7$

(b) $k = 0$

TAP TO RETURN

5 - Answers

(a) T=9.24N (3sf)

(b) 13.9N (3sf)

TAP TO RETURN

6 - Answers

(b) 250 m

(c) 21.8°

TAP TO RETURN

7 - Answers

(b) $\left(\frac{1}{2}, \frac{2}{5}\right)$ and $\left(-\frac{1}{2}, -\frac{2}{5}\right)$

TAP TO RETURN

8 - Answers

(a) Check desmos

(b) $y = \sqrt{x+1} - \left(\sqrt{x+1}\right)^3$

TAP TO RETURN

9 - Answers

(a) $A\left(0, \frac{10\sqrt{3}}{3}\right)$

(b) $\lambda = \frac{5}{\sqrt{3}}$

(c) $B(1, 5)$

(d) $k = \frac{5\sqrt{3}}{3}$

TAP TO RETURN

10 - Answers

$e^4 - 5$

TAP TO RETURN

11 - Answers

$\frac{2\pi+3\sqrt{3}}{96}$

TAP TO RETURN

12 - Answers

$-5\ln|x| + 3x^{-1} + 5\ln|2x-1| + c$

TAP TO RETURN