BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 19B

Skills 1

Find

(a) $\int 3e^{4x+2}dx$

(b) $\int (4e^{4-x} + 2)dx$

(c) $\int \frac{e^{2x}+1}{4e^{-x}}$

TAP FOR ANSWERS

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Skills 2

The functions f and g are defined by

$$f : x \to 5x + 2, \; x \in \mathbb{R} \qquad\qquad g : x \to \frac{1}{x}, \; x \in \mathbb{R}, x \neq 0$$

(a) Find the following functions, stating the domain in each case.

(i) $f^{-1}(x)$ (ii) $fg(x)$ (iii) $(fg)^{-1}(x)$

(b) Solve the equation $f^{-1}(x) = fg(x)$, giving your answers to 2 decimal places.

TAP FOR ANSWERS

Skills 1 - Answers

(a) $\frac{3}{4}e^{4x+2} + c$

(b) $-4e^{4-x} + 2x + c$

(c) $\frac{1}{12}e^{3x} + \frac{1}{4}e^{x} + c$

TAP TO RETURN

Skills 2 – Answers

(a) (i) $f^{-1}: x \to \frac{x-2}{5}, x \in \mathbb{R}$

(ii) $fg: x \to \frac{5}{x} + 2, x \in \mathbb{R}, x \neq 0$

(iii) $(fg)^{-1}: x \to \frac{5}{x-2}, x \in \mathbb{R}, x \neq 2$

(b) 13.81, -1.81

TAP TO RETURN

1

A parachutist drops from a helicopter *H* and falls vertically from rest towards the ground. Her parachute opens 2 s after she leaves *H* and her speed then reduces to 4 m s^{-1}. For the first 2 s her motion is modelled as that of a particle falling freely under gravity. For the next 5 s the model is motion with constant deceleration, so that her speed is 4 m s^{-1} at the end of this period. For the rest of the time before she reaches the ground, the model is motion with constant speed of 4 m s^{-1}.

(a) Sketch a speed-time graph to illustrate her motion from *H* to the ground.
(b) Find her speed when the parachute opens.

A safety rule states that the helicopter must be high enough to allow the parachute to open and for the speed of a parachutist to reduce to 4 m s^{-1} before reaching the ground. Using the assumptions made in the above model,

(c) find the minimum height of *H* for which the woman can make a drop without breaking this safety rule.

Given that *H* is 125 m above the ground when the woman starts her drop,

(d) find the total time taken for her to reach the ground.
(e) State one way in which the model could be refined to make it more realistic.

TAP FOR ANSWERS

2

(a) A particle rests in limiting equilibrium on a plane inclined at 30º to the horizontal.

Determine the acceleration with which the particle will slide down the plane when the angle of inclination is increased to 45º.

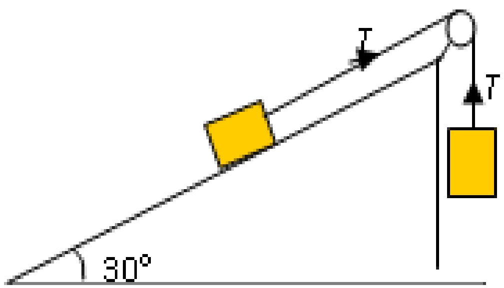
(b) A lift is accelerating upwards at 1.5 m s^{-2}. A girl of mass 30 kg is standing in the lift. Modeling the girl as a particle, find the force between her and the floor of the lift.

TAP FOR ANSWERS

3

Draw a force diagram and resolve forces for each block in appropriate directions.

A block of 5kg rests on a slope which is 30° to the horizontal it is connected by a light inextensible string which passes through a frictionless pulley to a second block of 1kg. Find the tension in the string, the acceleration of the blocks and the direction of travel.



TAP FOR ANSWERS

4

An object moves in a straight line from a point *O*. at time *t* seconds the object has acceleration, *a*, where

$$a = -\cos 4\pi t \text{ m s}^{-2}, 0 \leq t \leq 4$$

When $t = 0$, the velocity of the object is 0 m s^{-1} and its displacement is 0 m.
Find:

(a) An expression for the velocity at time *t* seconds.

(b) The maximum speed of the object

(c) An expression for the displacement of the object at time *t* seconds.

(d) The maximum displacement of the object from *O*

(e) The number of times the object changes direction during its motion.

TAP FOR ANSWERS

5

Two cyclists, C and D, are travelling with constant velocities $(5\mathbf{i} - 2\mathbf{j})$ m s^{-1} and $8\mathbf{j}$ m s^{-1} respectively relative to a fixed origin O.

(a) Find the velocity of C relative to D.

At noon, the position vectors of C and D are $(100\mathbf{i} + 300\mathbf{j})$ m and $(150\mathbf{i} + 100\mathbf{j})$ m respectively, referred to O. At t seconds after noon, the position vector of C relative to D is $\mathbf{s}$ metres.

(b) Show that $\mathbf{s} = (-50 + 5t)\,\mathbf{i} + (200 - 10t)\,\mathbf{j}$.

(c) By considering $|\mathbf{s}|^2$, or otherwise, find the value of t for which C and D are closest together.

TAP FOR ANSWERS

6

P is the point (5, 6, -2), Q is the point (2, -1, 1) and R is the point (2, -3, 6).

(a) Find the vectors $\overrightarrow{PQ}, \overrightarrow{PR}$ and $\overrightarrow{QR}$

(b) Hence, or otherwise, find the area of triangle PQR.

TAP FOR ANSWERS

7

Prove that the derivative of cos x is $-sin\,x$ from first principles

TAP FOR ANSWERS

8

Liquid dye is poured onto a large flat cloth and forms a circular stain, the area of which grows at a steady rate of 1.5 $cm^2\ s^{-1}$

Calculate, correct to 3 s.f.,

(a) the radius, in cm, of the stain 4 seconds after it started forming

(b) the rate, in $cm\ s^{-1}$, of increase of the radius of the stain after 4 seconds

TAP FOR ANSWERS

9

A curve has equation $2x^2 + xy + y^2 = 14$

(a) Show clearly that $\frac{dy}{dx} = -\frac{4x+y}{x+2y}$

(b) hence find the co-ordinates of the turning points of the curve.

TAP FOR ANSWERS

10

Evaluate the following

$$\lim_{\delta x \to 0} \sum_{x=0}^{2} \frac{1}{\sqrt{4x+1}} dx$$

TAP FOR ANSWERS

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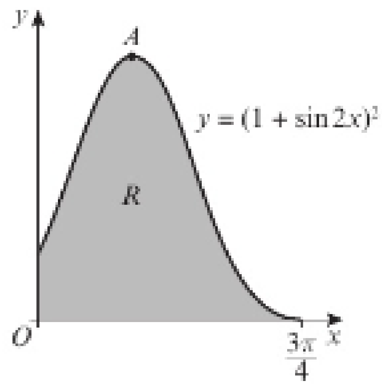
11



The Diagram shows the graph of $y = (1 + \sin 2x)^2, 0 \leq x \leq \frac{3\pi}{4}$

(a) Show that $(1 + 2\sin 2x)^2 \equiv \frac{1}{2}(3 + 4\sin 2x - \cos 4x)$.

(b) Hence find the area of the shaded region R.

(c) Find the coordinates of A, the turning point on the graph.

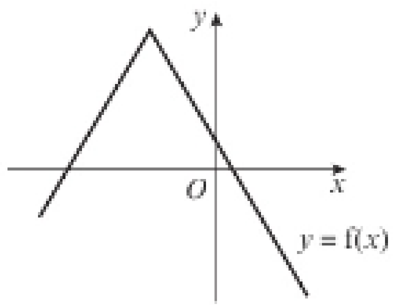
TAP FOR ANSWERS

12

The function f is defined by

$$f(x) = -\frac{5}{3}|x+4| + 8, x \in \mathbb{R}$$

The diagram shows a sketch of the graph $y = f(x)$



(a) State the range of f.
(b) Give a reason why $f^{-1}(x)$ does not exist.
(c) Solve the inequality $f(x) > \frac{2}{3}x + 4$.
(d) State the range of values of k for which the equation $f(x) = \frac{5}{3}x + k$ has no solutions

TAP FOR ANSWERS

1 - Answers

(b) 19.6ms^{-1}

(c) 78.6m

(d) 18.6 seconds

TAP TO RETURN

2 - Answers

(a) 2.0 ms^{-2} (2sf)

(b) 340 N (2sf)

TAP TO RETURN

3 - Answers

T=36.5N (3sf), a= 2.45 ms^{-2} down the plane for the 5kg block,

TAP TO RETURN

4 - Answers

(a) $v = -\frac{\sin 4\pi t}{4\pi}$

(b) $\frac{1}{4\pi}$

(c) $s = \frac{\cos 4\pi t}{16\pi^2} - \frac{1}{16\pi^2}$

(d) $\frac{1}{8\pi^2}$

(e) 16

TAP TO RETURN

5 - Answers

(a) $\mathbf{i} - 3\mathbf{j}$ m s^{-1}

(b) $\sqrt{10}$

(c) $5\mathbf{i} - 10\mathbf{j}$ m s^{-1}

(d) $t = 18$

TAP TO RETURN

6 - Answers

(a) $\overrightarrow{PQ} = -3\mathbf{i} - 8\mathbf{j} + 3\mathbf{k}, \overrightarrow{PR} = 3\mathbf{i} - 9\mathbf{j} + 8\mathbf{k}, \overrightarrow{QR} = -\mathbf{j} + 5\mathbf{k}$

(b) 20.0

TAP TO RETURN

7 - Answers

Proof

TAP TO RETURN

8 - Answers

(a) 1.38 cm

(b) 0.173 $cm\ s^{-1}$

TAP TO RETURN

9 - Answers

(1,-4) and (-1,4)

TAP TO RETURN

10 - Answers

1

TAP TO RETURN

11 - Answers

(a) $(1 + \sin 2x)^2 \equiv 1 + 2\sin 2x + \sin^2 2x$

$\equiv 1 + 2\sin 2x + \frac{1-\cos 4x}{2} \equiv \frac{3}{2} + 2\sin 2x - \frac{\cos 4x}{2}$

$\equiv \frac{1}{2}(3 + 4\sin 2x - \cos 4x)$

(b) $\frac{9\pi}{8} + 1$

(c) $\left(\frac{\pi}{4}, 4\right)$

TAP TO RETURN

12 - Answers

(a) $f(x) \leq 8$

(b) The function is not one-to-one

(c) $-\frac{32}{3} < x < -\frac{8}{7}$

(d) $k > \frac{44}{3}$

TAP TO RETURN