BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 18B

Skills 1

(a) $\int \sec^2 y \tan^5 y \, dy$

(b) $\int \operatorname{cosec} 3u \cot 3u \, du$

(c) $\int 4x(3x^2 + 1)^6 \, dt$

(d) $\int \frac{\sec^2 3x}{2+\tan 3x} \, dy$

(e) $\int \frac{4-x}{(x-2)(x-3)} \, dx$

TAP FOR ANSWERS

Skills 2

Use partial fractions to integrate the following:

(a) $\frac{3x+5}{(x+1)(x+2)}$

(b) $\frac{3x-1}{(2x+1)(x-2)}$

(c) $\frac{2x-6}{(x+3)(x-1)}$

(d) $\frac{3}{(2+x)(1-x)}$

TAP FOR ANSWERS

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Skills 1 - Answers

(a) $\frac{1}{6}\tan^6 y + c$

(b) $-\frac{1}{3}\operatorname{cosec}3u + c$

(c) $\frac{2}{21}\left(3x^2+1\right)^7 + c$

(d) $\frac{1}{3}ln(2 + tan3x) + c$

(e) $\ln|x-3| - 2\ln|x-2| + c$

TAP TO RETURN

Skills 2 – Answers

(a) $\ln\left|(x+1)^2(x+2)\right|+c$

(b) $\ln\left|(x-2)\sqrt{2x+1}\right|+c$

(c) $\ln\left|\frac{(x+3)^3}{x-1}\right|+c$

(d) $\ln\left|\frac{2+x}{1-x}\right|+c$

TAP TO RETURN

BHASVIC MαTHS
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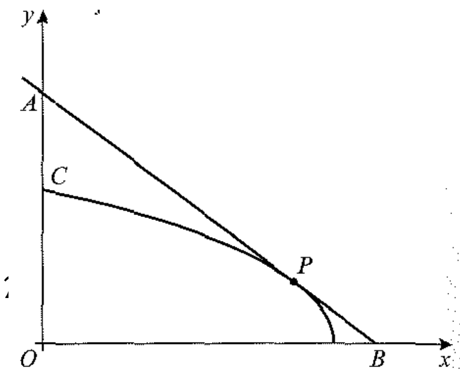
1

The diagram shows the curve C with parametric equations

$$x = a\sin^2 t\,, \qquad y = a\cos t, \qquad 0 \leq t \leq \frac{1}{2}$$



where a is a positive constant. The point P lies on C and has coordinates $\left(\frac{3}{4}a, \frac{1}{2}a\right)$.

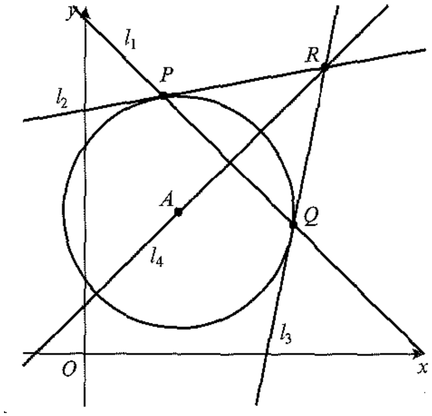
(a) Find $\frac{dy}{dx}$, giving your answer in terms of t.
Find an equation of the tangent to C at P.

(b) The tangent to C at P cuts the coordinate axes at points A and B.

(c) Show that the triangle AOB has area ka^2 where k is a constant to be found.

TAP FOR ANSWERS

2



The circle C has a centre at (6, 9) and a radius of $\sqrt{50}$.

The line l_1 with equation $x + y - 21 = 0$ intersects the circle at the points P and Q.

(a) Find the coordinates of the point P and the point Q.

(b) Find the equations of l_2 and l_3, the tangents at the points P and Q respectively.

(c) Find the equation of l_4, the perpendicular bisector of the chord PQ.

(d) Show that the two tangents and the perpendicular bisector intersect and find the coordinates of R, the point of intersection.

(e) Calculate the area of the kite $APRQ$.

TAP FOR ANSWERS

3

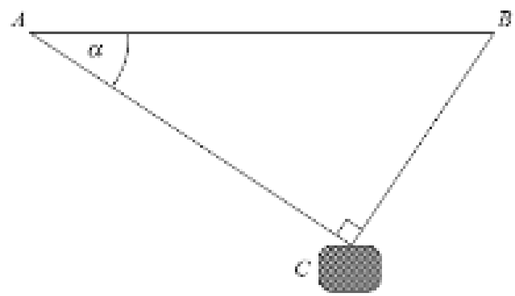


Fig. 2

Figure 2 shows a cable car C of mass 1 tonne which has broken down. The cable car is suspended in equilibrium by two cables AC and BC *perpendicular to each other* and attached to fixed points A and B, at the same horizontal level on either side of a valley. The cable AC is inclined at an angle α to the horizontal where $\tan\alpha = \frac{3}{4}$.

Show that the tension in the cable AC is 5900 N (2sf) and find the tension in the cable BC.

TAP FOR ANSWERS

4

NEW TECHNIQUES!

Given that the velocity $\boldsymbol{v}$ for a particle of mass 3kg at time $\boldsymbol{t}$ seconds moving in a straight line is given by; $\boldsymbol{v = 4\,t\,(\,8 - t\,)}$

Find;

(a) the maximum velocity,

(b) sketch the velocity-time graph for $0 \leq t \leq 8$,

(c) find the resultant force acting on the particle when $t = 2$.

Only zero and positive values of t should be considered.

TAP FOR ANSWERS

5

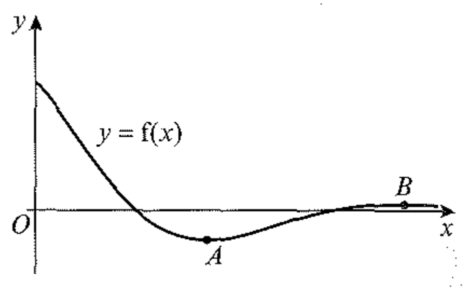
A ball is thrown from a window above a horizontal lawn. The velocity of projection is 15 m s^{-1} and the angle of elevation is α, where $\tan \alpha = \frac{4}{3}$. The balls takes 4 s to reach the lawn. Find

(a) the horizontal distance between the point of projection and the point where the ball hits the lawn,

(b) the vertical height above the lawn from which the ball was thrown.

TAP FOR ANSWERS

6



The curve C with equation $y = \mathrm{f}(x)$ is shown in the diagram, where

$$\mathrm{f}(x) = \frac{\cos 2x}{e^x}, 0 \leq x \leq \pi$$

The curve has a local minimum at A and a local maximum at B.

(a) Show that the x-coordinates of A and B satisfy the equation $\tan 2x = -0.5$ and hence find the coordinates of A and B.
(b) Using your answer to part (a), find the coordinates of the maximum and minimum turning points on the curve with equation $y = 2 + 4f(x - 4)$
(c) Determine the range of values for which $\mathrm{f}(x)$ is concave.

TAP FOR ANSWERS

7

The side of a cube of length x cm, is increasing at the constant rate of
1.5 $cm\ s^{-1}$

Find the rate at which the volume of the cube is increasing when its side is 6 cm

TAP FOR ANSWERS

8

A curve has implicit equation $x^3 + y^3 + 3y^2 + 3y - 6x = 50 + 2xy$
Find an equation of the normal to the curve at the point P(4,2)

TAP FOR ANSWERS

9

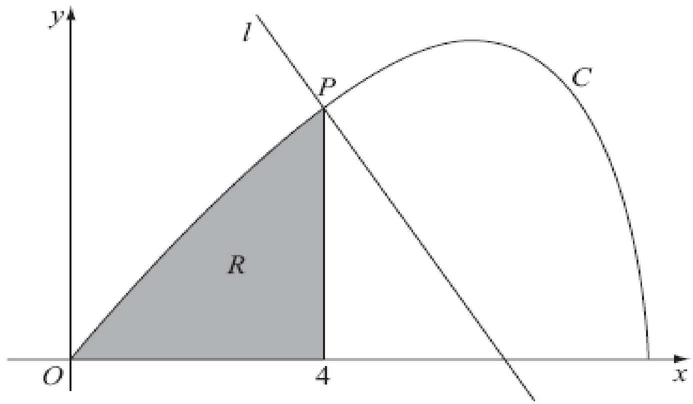


Figure 3 shows the curve C with parametric equations

$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leq t \leq \frac{\pi}{2}$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.
(a) Find the value of t at the point P.

The line l is a normal to C at P.
(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

TAP FOR ANSWERS

10

$$\mathrm{f}(x) = \frac{9x^2+4}{9x^2-4}, x \neq \pm\frac{2}{3}$$

(a) Given that $\mathrm{f}(x) = A + \frac{B}{3x-2} + \frac{C}{3x+2}$, find the values of the constants A, B and C.

(b) Hence find the exact value of $\int_{-\frac{1}{3}}^{\frac{1}{3}} \frac{9x^2+4}{9x^2-4}\,dx$, writing your answer in the form $a + b\ln c$, where a. b and c are rational numbers to be found.

TAP FOR ANSWERS

11

The figure above shows the graph of the curve with equation

$$y = 1 + \sin 2x\,, \qquad x \in \mathbb{R}$$

The point P lies on the curve where $x = \frac{\pi}{3}$

Show that the area of the finite region bounded by the curve, the y axis and the straight line segment OP is exactly

$$\frac{1}{12}\left(2\pi + 9 - \pi\sqrt{3}\right)$$

TAP FOR ANSWERS

BHASVIC MαTHS
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12

(a) Show that $\sin^2 x + 3\cos^2 x \equiv 2 + \cos 2x$.

(b) Hence evaluate $\int_{\pi/12}^{\pi/4} (\sin^2 x + 3\cos^2 x \equiv 2 + \cos 2x$ *check using your calculator to see if you're right*

(c) Show that $\frac{4\cos 2x}{\sin^2 2x} \equiv \operatorname{cosec}^2 x - \sec^2 x$

(d) Hence evaluate $\int_{\pi/6}^{\pi/3} \frac{4\cos 2x}{\sin^2 2x}\,dx$ *check using your calculator to see if you're right*

TAP FOR ANSWERS

1 - Answers

(a) $-\frac{1}{2}\sec t$

(b) $4y + 4x = 5a$

(c) Tangent crosses the x-axis at $x = \frac{5}{4}a$, and crosses the y-axis at $y = \frac{5}{4}a$. So area $AOB = \frac{1}{2}\left(\frac{4}{5}a\right)^2 = \frac{25}{32}a^2, k = \frac{25}{32}$

TAP TO RETURN

2 - Answers

(a) $P(5, 16)$ and $Q(13, 8)$

(b) $l_2: y = \frac{1}{7}x + \frac{107}{7}$ and $l_3: y = 7x - 83$

(c) $l_4: y = x + 3$

(d) All 3 equations have solution $x = \frac{43}{3}, y = \frac{52}{3}$ so $R(15, 18)$

(e) $\frac{200}{3}$

TAP TO RETURN

3 - Answers

$800g = 7800N$ (2sf)

TAP TO RETURN

4 - Answers

(a) 64 ms^{-1}

(c) 48N

TAP TO RETURN

5 - Answers

(a) 36 m

(b) 30 m (2 s.f.)

TAP TO RETURN

6 - Answers

(a) $f'(x) = -\frac{2\sin 2x + \cos 2x}{e^x}$

$f'(x) = 0 \Leftrightarrow 2\sin 2x + \cos 2x = 0 \Leftrightarrow \tan 2x = -0.5$

A (1.34, -0.234), *B* (2.91, 0.0487)

(b) Maximum (6.91, 2.20); minimum (5.34, 1.06) to 3 s.f.

(c) $0 < x \leq 0.322, 1.89 \leq x < \pi$

TAP TO RETURN

7 - Answers

(a) $162\ cm^3\ s^{-1}$

TAP TO RETURN

8 - Answers

$y = \frac{1}{2}x$

TAP TO RETURN

9 - Answers

(a) $t = \frac{\pi}{3}$

(b) proof

TAP TO RETURN

10 - Answers

(a) $A = 1$, $B = 2$, $C = -2$

(b) $a = \frac{2}{3}, b = -\frac{4}{3}, c = 3$

TAP TO RETURN

11 - Answers

(a) Proof

TAP TO RETURN

12 - Answers

(b) $\frac{\pi}{3} + \frac{1}{4}$

(d) 0

TAP TO RETURN