# BHASVIC MαTHS
# A1 DOUBLES ASSIGNMENT 16B

## Skills 1

Find:

(a) $\int 3sec4x\ tan4x\ dx$

(b) $\int -2\sec^4 2x\ tan2x\ dx$

(c) $\int \frac{2x-1}{x^2-x} dx$

(d) $\int \frac{2}{3-x} + \frac{6}{4x+1} dx$

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## Skills 2

For each of the following functions, whose domain is the set of **positive** real numbers, sketch the
function and hence state the range.

For each function find its inverse

(a) $f(x) = \frac{1}{x+1}$

(b) $f(x) = (x+1)^2 - 1$

(c) $f(x) = x^2 + 4x + 5$

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## Skills 1 - Answers

(a) $\frac{3}{4}sec4x + c$

(b) $-\frac{1}{4}sec^4 2x + c$

(c) $\ln|x^2 - x| + c$

(d) $-2\ln|3 - x| + \frac{3}{2}\ln|4x + 1| + c$

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## Skills 2 – Answers

(a) $0 < f(x) < 1;\ f^{-1}(x) = \frac{1-x}{x}$

(b) $f(x) \geq 0;\ f^{-1}(x) = (x+1)^{\frac{1}{2}} - 1$

(c) $f(x) \geq 5;\ f^{-1}(x) = (x-1)^{\frac{1}{2}} - 2$

TAP TO RETURN

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1

(a)Sketch the two inequalities $y < (2 - x)(3 + x)$ $and$ $y - x \geq 2$

(b) shade the region that satisfies both inequalities

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2

Corinne and her brother Dermot are lifted by their parents onto the two ends of a rope which is slung over a large horizontal branch. When their parents let go of them Dermot, whose mass is 54kg, begins to descend with an acceleration of $1\text{ms}^{-2}$. By modelling the children as a pair of particles connected by a light inextensible string, and the branch as a smooth pulley,

(a) Show that Corinne's mass is 44kg

(b) Calculate the tension in the rope

(c) Find the force on the branch

(d) In a more sophisticated model, the branch is assumed to be rough. Explain what effect this would have on the initial acceleration of the children.

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3

A particle is projected from a point on level ground with speed $u$ m s$^{-1}$ and angle of elevation $\alpha$. The maximum height reached by the particle is 42 m above the ground and the particle hits the ground 196 m from its point of projection.

Find the value of $\alpha$ and the value of $u$.

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4

In this question **i** and **j** are the unit vectors due east and due north respectively, and **k** is the unit vector acting vertically upwards.

A BASE jumper descending with a parachute is modelled as a particle of mass 50 kg subject to forces describing the wind, **W**, and air resistance, **F**, where:

$\mathbf{W} = (20\mathbf{i} + 16\mathbf{j})$ N
$\mathbf{F} = (-4\mathbf{i} - 3\mathbf{j} + 45\mathbf{k})$ N

(a) With reference to the model, suggest a reason why the k component of F is greater than the other components.

(b) Taking $g = 9.8$ m s$^{-2}$, find the resultant force acting on the BASE jumper.

(c) Given that the BASE jumper starts from rest and travels a distance of 180 m before landing, find the total time of the descent.

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5

For each of the following functions, find the interval on which the function is:
(i) convex (ii) concave

(a) $f(x) = x^3 - 3x^2 + x - 2$

(b) $f(x) = x^4 - 3x^3 + 2x - 1$

(c) $f(x) = \sin x, 0 < x < 2\pi$

(d) $f(x) = x^2 + 3x - 7$

(e) $f(x) = e^x - x^2$

(f) $f(x) = \ln x, x > 0$

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6

Differentiate tan x from first principles

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7

Given that $e^{2x} + e^{2y} = xy$, find $\frac{dy}{dx}$ in terms of $x$ and $y$.

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8

Given that $\int_0^{\theta} 4 \sin 2x \cos^4 2x \, dx = \frac{4}{5}$ where $0 < \theta < \pi$, find the exact value of $\theta$.

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9

Evaluate the following

$$\lim_{\delta x \to 0} \sum_{x=1}^{2} \left(x^{\frac{3}{2}} - 8x^{-\frac{3}{2}}\right)^2 dx$$

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10

Find:

(a) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 4sin^2\left(\frac{x}{2}\right) dx$

(b) $\int \frac{2}{\cos^2\left(\frac{x}{2}\right)} dx$

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11

(a) Sketch the graph of $y = |2x + a|, a > 0$, showing the coordinates of the points where the graph meets the coordinate axes.

(b) On the same axes, sketch the graph of $y = \frac{1}{x}$

(c) Explain how your graphs show that there is only one solution of the equation $x|2x + a| - 1 = 0$

(d) Find, using algebra, the value of $x$ for which $x|2x + a| - 1 = 0$.

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12

Three of the roots of the equation $az^5 + bz^4 + cz^3 + dz^2 + ez + f = 0$ are $-2, 2i$ and $1 + i$. Find the values of $a$, $b$, $c$, $d$, $e$ and $f$, given that they are all real.

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1 - Answers

Proof

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# 2 - Answers

(b) T=475N (3sf)

(c) 950N (3sf)

(d) Friction opposes motion, therefore accelerates less.

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## 3 - Answers

$\alpha = 40.6°$ (nearest 0.1°)  $u = 44$ (2 s.f.)

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# 4 - Answers

(a) Air resistance acts in opposition to the motion of the BASE jumper. The motion downwards will be greater than the motion in the other directions.

(b) $(16\mathbf{i} + 13\mathbf{j} - 40\mathbf{k})$ N

(c) 20 seconds

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## 5 - Answers

(a) (i) $(1, \infty)$ (ii) $(-\infty, 1)$

(b) (i) $(-\infty, 0) \cup \left(\frac{3}{2}, \infty\right)$ (ii) $\left(0, \frac{3}{2}\right)$

(c) (i) $(\pi, 2\pi)$ (ii) $(0, \pi)$

(d) (i) nowhere (ii) $(-\infty, \infty)$

(e) (i) $(\ln 2, \infty)$ (ii) $(-\infty, \ln 2)$

(f) (i) nowhere (ii) $(0, \infty)$

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# 6 - Answers

$\sec^2 x$

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# 7 - Answers

$\frac{dy}{dx} = \frac{y-2e^{2x}}{2e^{2y}-x}$

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# 8 - Answers

$\theta = \frac{\pi}{2}$

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9 - Answers

$\frac{47}{4}$

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## 10 - Answers

(a) $\frac{\pi}{6} - \sqrt{3} + \sqrt{2}$

(b) $4\tan\left(\frac{x}{2}\right) + c$

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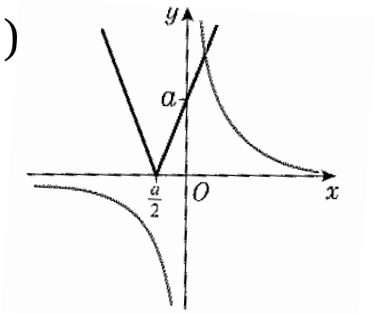
# 11 - Answers

(a& b) )




(c) One intersection point

(d) $x = \frac{-a+\sqrt{a^2+8}}{4}$

TAP TO RETURN

# 12 - Answers

$a = 1$, $b = 0$, $c = 2$, $d = 4$, $e = -8$ and $f = 16$.

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