BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 15B

Skills 1

Normally we work with equations in the form y = f(x) or x + y + z = 10 etc. These types of equations are called 'Cartesian Equations' – all the variables are grouped together into one equation, and each variable has its own axis on a graph.

Parametric Equations, however, look like this:

$$x = cost$$
$$y = sint$$

This is just another way of describing the circle $x^2 + y^2 = 1$. Can you see why? Sometimes it is more convenient to work with circular curves in Parametric form than Cartesian form.

See if you can find the Cartesian Forms of the Parametric Equations below by eliminating t.

(a) $x = 4cost$ $\quad y - 1 = sint$ *hint - $\sin^2 x + \cos^2 x \equiv 1$ *

(b) $x = 2t - 1$ $\quad y = 4t + 5$ *hint – make t the subject in one, and sub into the other*

(c) $x = \frac{t}{2t-1}$ $\quad y = \frac{t}{2t-3}$

TAP FOR ANSWERS

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Skills 2

DO NOT CHANGE THE EQUATIONS INTO CARTESIAN FORM.

(a) A curve is given by the parametric equations $x = 2t, \quad y = \frac{5}{t} \quad where\ t \neq 0.$
Complete the table below and draw the graph of the curve for $-5 \leq t \leq 5$

t	-5	-4	-3	-2	-1	-0.5	0.5	1	2	3	4	5
$x = 2t$	-10	-8				-1						
$y = \frac{5}{t}$	-1	-1.25					10					

(b) A curve is given by the parametric equations $x = t^2, \quad y = \frac{t^3}{5}$. Complete the table below and draw the graph of the curve for $-4 \leq t \leq 4$

t	-4	-3	-2	-1	0	1	2	3	4
$x = t^2$	16								
$y = \frac{t^3}{5}$	-12.8								

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Skills 1 - Answers

(a) $\frac{x^2}{16} + (y-1)^2 = 1$

(b) $y = 2x + 7$

(c) $y = \frac{x}{3-4x}$

TAP TO RETURN

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Skills 2 – Answers

(a) and (b) check using desmos or graphical calculator

TAP TO RETURN

1

Assuming standard results for sin x and cos x, prove that:

(a) The derivative of arccos x is $-\frac{1}{\sqrt{1-x^2}}$

(b) The derivative of arctan x is $\frac{1}{1+x^2}$

TAP FOR ANSWERS

2

The curve C has equation $y = \frac{1}{\cos x \sin x}, 0 < x \leq \pi$

(a) Find $\frac{dy}{dx}$

(b) Determine the number of stationary points of the curve C.

(c) Find the equation of the tangent at the point where $x = \frac{\pi}{3}$, giving your answer in the form $ax + by + c = 0$, where a, b and c are exact constants to be determined.

TAP FOR ANSWERS

3

A circle with centre C, has equation $x^2 + y^2 + 8x - 12y = 12$

(a) Find the co-ordinates of C and the radius of the circle

(b) The points P and Q lie on the circle. The origin is the midpoint of the chord PQ. Show that PQ has length $n\sqrt{3}$ where n is an integer.

TAP FOR ANSWERS

4

A train starts from a station X and moves with constant acceleration 0.6m s^{-2} for 20 s. The speed it has reached after 20 s is then maintained for T seconds. The train then decelerates from this speed to rest in a further 40 s, stopping at a station Y.

(a) Sketch a speed-time graph to illustrate the motion of the train.

Given that the distance between the stations is 4.2km, find

(b) The value of T,

(c) The distance travelled by the train while it is moving with constant speed.

TAP FOR ANSWERS

5

Draw labelled diagrams and form equations for the following situations

(a) A 30kg weight is sitting on a slanted rough roof. The angle between the roof and the horizontal is 30^o. Given that the weight is at the point of slipping down the roof, what is the reaction force and the coefficient of friction?

(b) A particle of mass 5 kg is resting on a rough slope inclined at 15^o to the horizontal with a coefficient of friction of 0.45. A horizontal force P is also applied to the particle. The particle is at the point of moving up the slope.

(i) What is the magnitude of the force P?

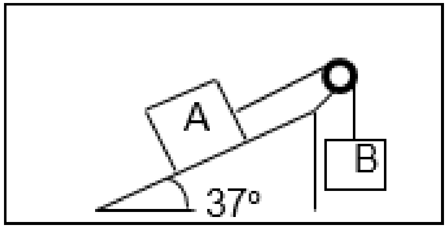
(*ii)* The force P is removed. Does the particle move? Explain your answer?

(Draw another model, and make new equations)

(c) Two particles A and B of mass 5 kg and m kg respectively ($5 < m$), are connected by a light inextensible string. A is held resting on a rough fixed plane inclined at 20º to the horizontal. The string passes over a smooth pulley P fixed at the top of the plane. The portion AP of the string lies along a line of greatest slope of the plane and B hangs freely from the pulley. Given that the coefficient of friction is 0.2, and when the system is released from rest, the acceleration is $5ms^{-2}$. Find the Tension in the string , and m.

TAP FOR ANSWERS

6



A and B are particles connected by a light inextensible string which passes over a smooth fixed pulley attached to a corner of a smooth plane inclined at 37°. Particle B hangs freely. If A has mass 3kg, find the mass of B given that the system is in equilibrium.

TAP FOR ANSWERS

7

A curve C has parametric equations $x = \frac{1}{3}\sin t\,, \quad y = \sin 3t\,, \quad 0 < t < \frac{\pi}{2}$

(a) Show that the Cartesian equation of the curve is given by $y = ax(1 - bx^2)$ where a and b are integers to be found.

(b) State the domain and range of $y = \mathrm{f}(x)$ in the given domain of t.

TAP FOR ANSWERS

8

Show that the curve with parametric equations

$$x = 2\cos t, \quad y = \sin\left(t - \frac{\pi}{6}\right), \quad 0 < t < \pi$$

can be written in the form

$$y = \frac{1}{4}\left(\sqrt{12 - 3x^2} - x\right), -2 < x < 2$$

TAP FOR ANSWERS

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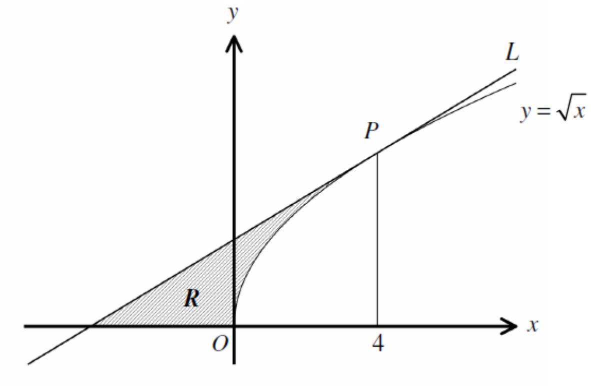

The figure above shows the graph of the curve C with equation

$$y = \sqrt{x}, \qquad x \geq 0$$

The point P lies on C where $x = 4$.
The straight line L is the tangent to C at P.

(a) Find an equation of L

The finite region R, shown shaded in the figure above, is bounded by the curve C, the tangent L and the x axis.

(b) Find the exact area of R

TAP FOR ANSWERS

10

NEW TECHNIIQUES!

You now know all of the differentiation table for the Single A Level spec. This means you can integrate everything on the RHS of the table!

This method is called 'Reverse Chain Rule'. It is a 3 step process.
1 Guess
2 Differentiate
3 Adjust

For example:
$\int cosx\,dx$
We know sinx differentiates to cosx, so I GUESS $\int cosx\,dx = sinx + c$
I DIFFERENTIATE to check i.e. $\frac{d}{dx}(sinx + c) = cosx$
And we can see there is no need to ADJUST.

TAP FOR ANSWERS

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A1 DOUBLES ASSIGNMENT 15B

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Another example:
$\int \sec^2 3x \, dx$
We know $tan3x$ differentiates to $3\sec^2 3x$ so I GUESS $\int \sec^2 3x \, dx = tan3x + c$
I DIFFERENTIATE to check i.e. $\frac{d}{dx}(tan3x + c) = 3\sec^2 3x$
So now I would need to ADJUST as we started with $\sec^2 3x$ *not* $3\sec^2 3x$. So the answer is:

$$\int \sec^2 3x \, dx = \frac{1}{3} tan3x + c$$

Differentiate to check and you'll see it works.

Try these out:
(a) $\int 8cosec^2 x \, dx$

(b) $\int 4\cos\left(\frac{1}{2}x\right) dx$

(c) $\int -2sec4xtan4x \, dx$

(d) $\int cosx \sin^3 x \, dx$ * hint – think about what $\sin^4 x$ differentiates to!

TAP FOR ANSWERS

BHASVIC MαTHS

A1 DOUBLES ASSIGNMENT 15B

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NEW TECHNIQUES!

We can only integrate something if it looks like something on the RHS of the differentiation table. Manipulating trig identities can help us to do this.

For example: $\int \cos^2 x \, dx$ We cannot integrate this as $\cos^2 x$ is not on the RHS of the table. But we know $cos2x \equiv 2\cos^2 x - 1$, therefore rearranging gives: $\cos^2 x \equiv \frac{1}{2} + \frac{1}{2}cos2x$ which we can integrate.

Find:

(a) $\int \sin^2 x \, dx$

(b) $\int \tan^2 x \, dx$

TAP FOR ANSWERS

Given that $f(x) = \dfrac{2}{x-1} - \dfrac{6}{(x-1)(2x+1)}, x > 1$,

(a) Prove that $f(x) = \dfrac{4}{2x+1}$

(b) Find the range of f

(c) Find $f^{-1}(x)$ and state its domain

(d) State the range of $f^{-1}(x)$

TAP FOR ANSWERS

1 - Answers

(a) (a) Let $y = \arccos x \Rightarrow \cos y = x \Rightarrow \frac{dy}{dx} = -\sin y$

$\frac{dy}{dx} = -\frac{1}{\sin y} = -\frac{1}{\sqrt{1-\cos^2 y}} = -\frac{1}{\sqrt{1-x^2}}$

(b) Let $y = \arctan x$

Then, $\tan y = x$

$\frac{dy}{dx} = \sec^2 y$

$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$

TAP TO RETURN

2 - Answers

(a) $\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x}$

(b) 2

(c) $24x - 9y + 12\sqrt{3} - 8\pi = 0$

TAP TO RETURN

3 - Answers

(a) Centre (-4,6) radius = 8

(b) $4\sqrt{3}$

TAP TO RETURN

4 - Answers

(b) 320 seconds

(c) 3840m

TAP TO RETURN

5 - Answers

(a) 250N (2sf), 0.58 (2sf)

(bi) 40N (2sf)

(bii) no

(c) 51N (2sf), 11kg (2sf)

TAP TO RETURN

6 - Answers

1.81 kg (3sf)

TAP TO RETURN

7 - Answers

(a) $y = 9x(1 - 12x^2) \Rightarrow a = 9, b = 12$

(b) Domain: $0 < x < \frac{1}{3}$, Range: $-1 < y < 1$

TAP TO RETURN

8 - Answers

$$y = \sin t \cos\left(\frac{\pi}{6}\right) - \cos t \sin\left(\frac{\pi}{6}\right)$$

$$= \frac{\sqrt{3}}{2}\sin t - \frac{1}{2}\cos t = \frac{\sqrt{3\left(1-\frac{x^2}{4}\right)}}{2} - \frac{1}{4}x$$

$$= \frac{1}{4}\left(2\sqrt{3-\frac{3}{4}x^2} - x\right) = \frac{1}{4}\left(\sqrt{12-3x^2} - x\right)$$

$$t = 0 \Rightarrow x = 2, t = \pi \Rightarrow x = -2, \text{so} -2 < x < 2$$

TAP TO RETURN

9 - Answers

(a) $\frac{1}{4}x + 1$

(b) $\frac{8}{3}$

TAP TO RETURN

10 - Answers

(a) $-8cot x + c$

(b) $8\sin\left(\frac{1}{2}x\right) + c$

(c) $-\frac{1}{2}sec4x + c$

(d) $\frac{1}{4}\sin^4 x + c$

TAP TO RETURN

11 - Answers

(a) $\frac{1}{2}x - \frac{1}{4}sin2x + c$

(b) $tanx - x + c$

TAP TO RETURN

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12 - Answers

(a) Proof

(b) $f \in \Re : f \neq 0$

(c) $f^{-1}(x) = \frac{4-x}{2x}, x \in \Re : x \neq 0$

(d) $f^{-1}(x) > 1$

TAP TO RETURN