BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 14B

Skills 1

Sketch and state the ranges of the following functions (defined on $\mathbb{R}$): show asymptotes clearly

(a) $f(x) = \frac{1}{x+2} + 1$

(b) $g(x) = 1 - e^{2x}$

(c) $h(x) = \ln(1 + x)$

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Skills 2

Differentiate each of the following functions

(a) $\ln 8x$

(b) $2e^x - 2\ln(x^2)$

(c) $\frac{3x}{1-\sin x}$

(d) $\frac{e^x}{\ln x}$

(e) $3\ln x - \ln 3x$

(f) $\ln\sqrt{x} - 2\ln(1/x)$

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Skills 1 - Answers

Check sketches on Desmos

(a) $f(x) \in \mathbb{R},\ f(x) \neq -2$

(b) $g(x) \in \mathbb{R},\ g(x) < 1$

(c) $h(x) \in \mathbb{R},$

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Skills 2 – Answers

(a) $\frac{1}{x}$

(b) $2e^x - \frac{4}{x}$

(c) $\frac{3-3\sin x+3x\cos x}{(1-\sin x)^2}$

(d) $\frac{e^x(x\ln x-1)}{x(\ln x)^2}$

(e) $\frac{2}{x}$

(f) $\frac{5}{2x}$

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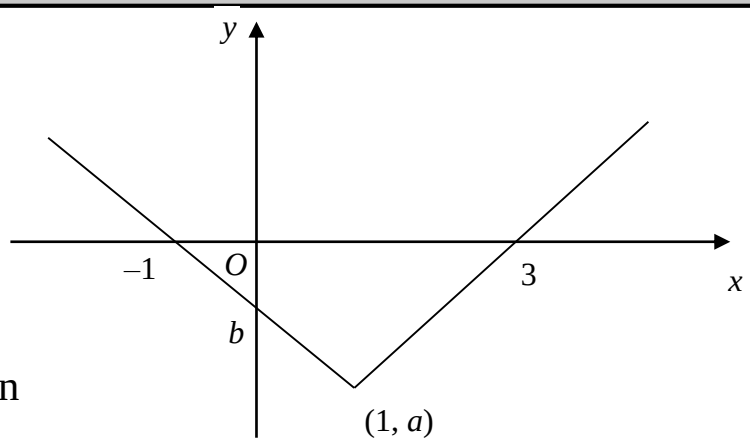
1

Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the x-axis at $(3, 0)$. The other line meets the x-axis at $(-1, 0)$ and the y-axis at $(0, b)$, $b < 0$.



In separate diagrams, sketch the graph with equation

(a) $y = f(x + 1)$,

(b) $y = f(|x|)$.

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $f(x) = |x - 1| - 2$, find

(c) the value of a and the value of b,

(d) the value of x for which $f(x) = 5x$.

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2

The points A and B have coordinates (4, 6) and (12, 2) respectively. The straight line l_1 passes through A and B.

(a) Find an equation for l_1 in the form $ax + by = c$, where a, b and c are integers.

The straight line l_2 passes through the origin and has gradient -4.

(b) Write down an equation for l_2.

(c) The lines l_1 and l_2 intersect at the point C. Find the exact coordinates of the mid-point of AC.

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3

The circle C has equation $x^2 + y^2 - 12x + 8y + 16 = 0$

(a) Find the centre and radius of C

(b) Given that C crosses the x axis at the points A and B, find the length AB giving your answer in the form $k\sqrt{5}$.

TAP FOR ANSWERS

4

A stone is dropped from the top of a tower. One second later another stone is thrown vertically downwards from the same point with a velocity of 14 m s^{-1}. If they hit the ground together, find the height of the tower.

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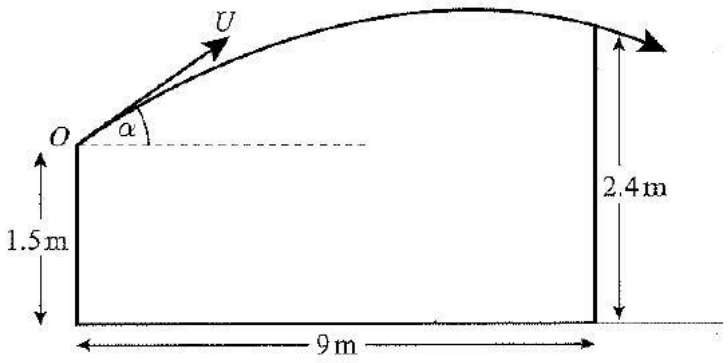
5

A girl playing volleyball on horizontal ground hits the ball towards the net 9 m away from a point 1.5 m above the ground. The ball moves in a vertical plane which is perpendicular to the net. The ball just passes over the top of the net, which is 2.4 m above the ground, as shown in the diagram.



The ball is modelled as a particle projected with initial speed U m s^{-1} from point O, 1.5 m above the ground at an angle α to the horizontal.

By writing down expressions for the horizontal and vertical distances from O to the ball, t seconds after it was hit, show that when the ball passes over the net

$$0.9 = 9\tan\alpha - \frac{81g}{2U^2\cos^2\alpha}$$

Given that $\alpha = 30°$

Find the speed of the ball as it passes over the net.

TAP FOR ANSWERS

6

A ship S is moving with constant velocity $(3\mathbf{i} + 3\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of S is $(-4\mathbf{i} + 2\mathbf{j})$ km.

(a) Find the position vector of S at time t hours.

A ship T is moving with constant velocity $(-2\mathbf{i} + n\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of T is $(6\mathbf{i} + \mathbf{j})$ km. The two ships meet at the point P.

(b) Find the value of n.

(c) Find the distance OP.

TAP FOR ANSWERS

7

(a) Prove that the derivative of sin (3x) is 3 cos(3x) from first principles

(b) Prove that the derivative of cos (3x) is -3 sin(3x) from first principles

TAP FOR ANSWERS

8

A curve C has equation $y = \frac{e^{2x}}{(x-2)^2}, x \neq 2$.

(a) Show that $\frac{dy}{dx} = \frac{Ae^{2x}(Bx-C)}{(x-2)^3}$ where A, B and C are integers to be found.

(b) Find the equation of the tangent of C at the point $x = 1$.

TAP FOR ANSWERS

9

Curve C has equation $x = (\arccos y)^2$. Show that

$$\frac{dy}{dx} = -\frac{\sqrt{1-\cos^2\sqrt{x}}}{2\sqrt{x}}$$

TAP FOR ANSWERS

10

The moment magnitude scale is used by seismologists to express the sizes of earthquakes. The scale is calculated using the formula

$$M = \frac{2}{3}\log_{10}(S) - 10.7$$

Where S is the seismic moment in dyne cm.

(a) Find the magnitude of an earthquake with a seismic moment of 2.24×10^{22} dyne cm.

(b) Find the seismic moment of an earthquake with
(i) Magnitude 6
(ii) Magnitude 7

(c) Using your answers to part b or otherwise, show that an earthquake of magnitude 7 is approximately 32 times as powerful as an earthquake of magnitude 6.

TAP FOR ANSWERS

11

$f(x) = 3x + 2$ and $g(x) = \frac{1}{4}$ with $x \neq 0$

(a) Find $f^{-1}(x), g^{-1}(x)$ and $gf(x)$

(b) Show that $(gf)^{-1}(x) = f^{-1}g^{-1}(x) = \frac{1}{3}\left(\frac{1}{x} - 2\right)$

Note: you will need to show *both* that $f^{-1}g^{-1}(x) = \frac{1}{3}\left(\frac{1}{x} - 2\right)$ and that $(gf)^{-1}(x) = \frac{1}{3}\left(\frac{1}{x} - 2\right)$.

TAP FOR ANSWERS

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12

Find the inverses of the following functions where each function is defined on its given domain, $x \in \mathbb{R}$

(a) $f(x) = (x-1)^2 + 4, x \geq 1$

(b) * $f(x) = x^2 + 4x - 1, x \geq -2$

(c) * $f(x) = x^2 + 4, x \geq -2$

* co*mplete the square first*

TAP FOR ANSWERS

1 - Answers

(a) and (b) use graph sketching app

(c) $a = -2, \quad b = -1$

(d) $x = -\frac{1}{6}$

TAP TO RETURN

2 - Answers

(a) $x + 2y - 16 = 0$

(b) $y = -4x$

(c) $\left(\frac{6}{7}, \frac{53}{7}\right)$

TAP TO RETURN

3 - Answers

(a) centre (6,–4), radius = 6

(b) $k = 4$

TAP TO RETURN

4 - Answers

23 m

TAP TO RETURN

5 - Answers

(a) R(→): $x = 9 = U\cos\alpha \times t$, so $t = \frac{9}{U\cos\alpha}$

R(↑): $y = U\sin\alpha \times t - \frac{1}{2}gt^2$

Substitute for $t \Rightarrow y = U\sin\alpha\left(\frac{9}{U\cos\alpha}\right) - \frac{1}{2}g\left(\frac{9}{U\cos\alpha}\right)^2$

Use $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ and $y = 0.9$. Rearrange to give $0.9 = 9\tan\alpha - \frac{81g}{2U^2\cos^2\alpha}$.

(b) 8.8 m s^{-1}

TAP TO RETURN

6 - Answers

(a) $(-4 + 3t)\mathrm{i} + (2 + 3t)\mathrm{j}$

(b) 3.5

(c) 8.25 km

TAP TO RETURN

7 - Answers

Proof

TAP TO RETURN

8 - Answers

(a) $\frac{(x-2)^2(2e^{2x})-e^{2x}[2(x-2)]}{(x-2)^4} = \frac{2(x-2)^2e^{2x}-2e^{2x}(x-2)}{(x-2)^4}$

$= \frac{2(x-2)e^{2x}-2e^{2x}}{(x-2)^3} = \frac{2e^{2x}(x-2-1)}{(x-2)^3} = \frac{2e^{2x}(x-3)}{(x-2)^3}$

$A = 2, B = 1, C = 3$

(b) $y = 4e^2x - 3e^2$

TAP TO RETURN

9 - Answers

$$\frac{dy}{dx} = 2\arccos y \times -\frac{1}{\sqrt{1-y^2}} = -\frac{2\arccos y}{\sqrt{1-y^2}}$$

$$\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{2\arccos y} = -\frac{\sqrt{1-\cos^2\sqrt{x}}}{2\sqrt{x}}$$

TAP TO RETURN

10 - Answers

(a) 4.2

(b) (i) 1.12×10^{25} dyne cm
(ii) 3.55×10^{26} dyne cm

(c) Divide (b)(ii) by (b)(i)

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11 - Answers

$f^{-1}(x) = {}^{1}/_{3}(x - 2)$, $g^{-1}(x) = {}^{1}/_{x}$, $x \neq 0$, $gf(x) = {}^{1}/_{(3x + 2)}$, $x \neq -{}^{2}/_{3}$

TAP TO RETURN

12 - Answers

(a) $f^{-1}(x) = 1 + \sqrt{x - 4}$

(b) $f^{-1}(x) = -2 + \sqrt{x + 5}$

(c) $f^{-1}(x) = \sqrt{x + 4} - 2$

TAP TO RETURN