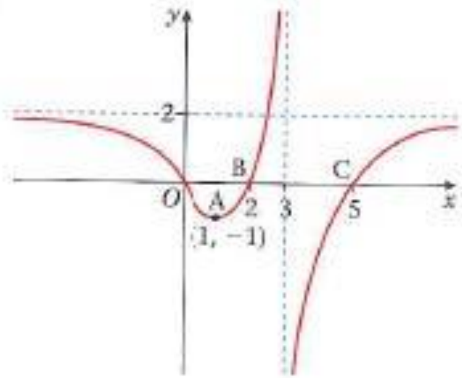
BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 13B

Skills 1



The diagram shows a sketch of the graph of $y = f(x)$.

The curve has a minimum at the point A (1, -1) passed through x-axis at the origin, and the points B (2, 0) and C (5, 0); the asymptotes have equations $x = 3$ and $y = 2$.

(a) Sketch on separate axes, the graph of

(i) $y = |f(x)|$ (ii) $y = -f(x+1)$ (iii) $y = f(-2x)$

(b) State the number of solutions to the equation

(i) $3|f(x)| = 2$ (ii) $2|f(x)| = 3$

TAP FOR ANSWERS

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Skills 2

Let
f(x) = $3x^2 - 2$.
g(x) = $x^3 - 1$

Find the **derivative** of the following:

(a) f(x)

(b) g(x)

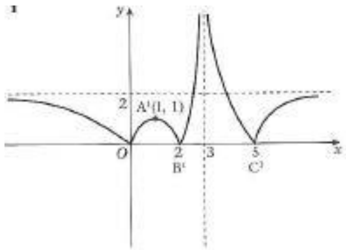
(c) f(x) g(x)

(d) $\frac{f(x)}{g(x)}$

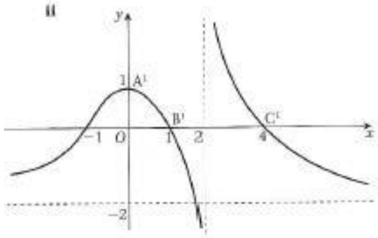
TAP FOR ANSWERS

Skills 1 - Answers

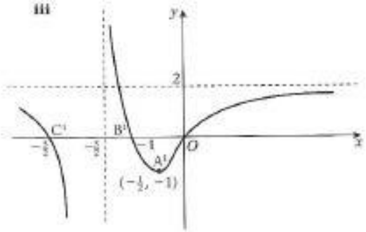
(a) (i)



(ii)



(iii)



(b) (i) 6

(ii) 4

TAP TO RETURN

Skills 2 – Answers

(a) 6x

(b) $3x^2$

(c) $15x^4 - 6x^2 - 6x$

(d) $\frac{-3x^4+6x^2-6x}{(x^3-1)^2}$

TAP TO RETURN

1

Find the value of k for which $y = 3x + 1$ is a tangent to the curve $x^2 + y^2 = k$

TAP FOR ANSWERS

2

The function k is defined by $\mathrm{k}(x) = \frac{a}{x^2}, a > 0, x \in \mathbb{R}, x \neq 0$

(a) Sketch the graph of $y = \mathrm{k}(x)$.

(b) Explain why it is not necessary to sketch $y = |\mathrm{k}(x)|$ and $y = \mathrm{k}(|x|)$.

The function m is defined by $\mathrm{m}(x) = \frac{a}{x^2}, a < 0, x \in \mathbb{R}, x \neq 0$.

(c) Sketch the graph of $y = \mathrm{m}(x)$

(d) State with a reason whether the following statements are true or false.

(i) $|\mathrm{k}(x)| = |\mathrm{m}(x)|$ (ii) $\mathrm{k}(|x|) = \mathrm{m}(|x|)$ (iii) $\mathrm{m}(x) = \mathrm{m}(|x|)$

TAP FOR ANSWERS

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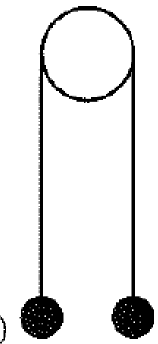
3

Two particles A and B have masses m kg and 3 kg respectively, where $m > 3$. The particles are connected by a light inextensible string which passes over a smooth, fixed pulley. Initially A is 2.5 m above horizontal ground. The particles are released from rest with the string taut and the hanging parts of the string vertical, as shown I the figure. After A has been descending for 1.25 s, it strikes the ground. Particle A reaches the ground before B has reached the pulley.



(a) Show that the acceleration of B as it ascends if 3.2 m s^{-2}.
(b) Find the tension in the string as A descends.
(c) Show that $m = \frac{65}{11}$.
(d) State how you have used the information that the string is inextensible.

When A strikes the ground it does not rebound and the string becomes slack. Particle B then moves freely under gravity, without reaching the pulley, until the string becomes taut again.

(e) Find the time between the instant when A strikes the ground and the instant when the string becomes taut again.

TAP FOR ANSWERS

4

A parcel of mass 8 kg rests on a smooth slope, and is connected by a light inextensible string which passes over a smooth pulley to a mass of 2kg, which hangs freely. The system is in equilibrium. Find the angle of the slope.

TAP FOR ANSWERS

5

A projectile is launched from a point on a horizontal plane with initial speed u m s $^{-1}$ at an angle of elevation α. The particle moves freely under gravity until it strikes the plane. The range of the projectile is R m.

(a) Show that the time of flight of the particle is $\frac{2u \sin \alpha}{g}$ seconds

(b) Show that $R = \frac{U^2 \sin 2\alpha}{g}$.

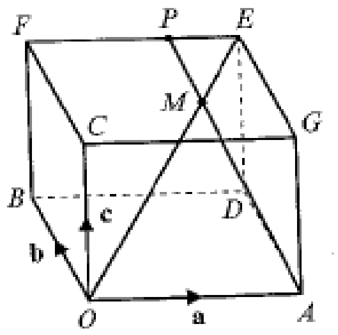
(c) Deduce that, for a fixed u, the greatest possible range is when $\alpha = 45°$

(d) Given that $R = \frac{2u^2}{5g}$, find the two possible values of the angle of elevation at which the projectile could have been launched.

TAP FOR ANSWERS

6

The diagram shows a cuboid whose vertices are *O*, *A*, *B*, *C*, *D*, *E*, *F* and *G*. **a**, **b**, and **c** are the position vectors of the vertices *A*, *B*, and *C* respectively. The point *M* lies on *OE* such that $OM : ME = 3 : 1$. the straight line *AP* passes through point *M*. Given that $AM : MP = 3 : 1$, prove that P lies on the line *EF* and find the ratio $FP : PE$.



TAP FOR ANSWERS

7

(a) Prove, from first principles, that the derivative of $2x^3$ is $6x^2$

(b) Prove, from first principles, that the derivative of sin 2x is 2cos2x

TAP FOR ANSWERS

8

Given that $\mathrm{f}(x) = \frac{2x}{x+5} + \frac{6x}{x^2+7x+10}, x > 10$

(a) Show that $f(x) = \frac{2x}{x+2}$

(b) Hence find $\mathrm{f}'(3)$

TAP FOR ANSWERS

9

The normal to the curve $y = \sec^2 x$ at the point $P\left(\frac{\pi}{4}, 2\right)$ meets the line $y = x$ at the point Q. Find the exact coordinates of Q.

TAP FOR ANSWERS

10

The gradient of a curve is given by $f'(x) = x^2 - 3x - \frac{2}{x^2}$. Given that the curve passes through the point $(1, 1)$, find the equation of the curve in the form $y = f(x)$.

TAP FOR ANSWERS

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(a) For each of these functions, find the inverse function, $f^{-1}(x)$ and state its domain:

(i) $f(x) = \frac{x+6}{5}, x \in \mathbb{R}$

(ii) $f(x) = \frac{5}{x}, \{x \in \mathbb{R}: x \neq 0\}$

(iii) $f: x \to \sqrt{x+4}, \{x \in \mathbb{R}: x \geq -4\}$

(iv) $f: x \to \frac{3x+2}{x-1}, \{x \in \mathbb{R}: x \neq 1\}$

(b) (i) State why the inverse $f^{-1}(x)$ does not exist for $f: x \to 2|(x-3)^2 - 5\{x \in \mathbb{R}\}$

(ii) Change the domain of the above function so that the inverse does exist.

TAP FOR ANSWERS

12

By solving the equation $z^4 + 6z^2 + 25 = 0$ for z^2, or otherwise, express each of the four roots of the equation in the form $x + iy$.

TAP FOR ANSWERS

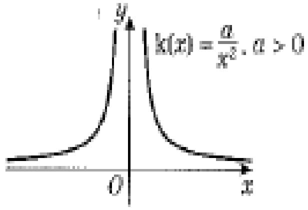
1 - Answers

$k = \frac{1}{10}$

TAP TO RETURN

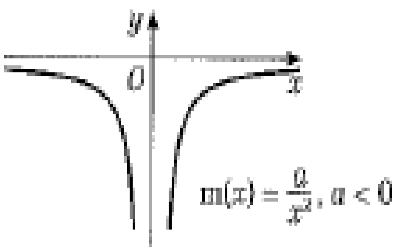
2 - Answers

(a)



(b) Both these graphs would match the original graph.

(c)



(d) (i) True, $|k(x)| = \left|\frac{a}{x^2}\right| = \left|\frac{-a}{x^2}\right| = |m(x)|$

(ii) False, $k(|x|) = \frac{a}{|x|^2} \neq \frac{-a}{|x|^2} = m(|x|)$

(iii) True, $m(|x|) = \frac{-a}{|x|^2} = \frac{-a}{x^2} = m(x)$

TAP TO RETURN

3 - Answers

(a) $s = ut + \frac{1}{2}at^2$ so $2.5 = 0 + \frac{1}{2} \times a \times 1.25^2, a = 3.2$ ms^{-2}

(b) 39 N

(c) For A, $R(\downarrow): mg - T = ma$
$T = m(9.8 - 3.2), T = 6.6m$
Substituting for $T: 39 = 6.6m$
$m = \frac{65}{11}$

(d) Same tension in string either side of the pulley.

(e) $\frac{40}{49}$ s

TAP TO RETURN

4 - Answers

14.5^o

TAP TO RETURN

5 - Answers

12° and 78° (nearest degree)

TAP TO RETURN

6 - Answers

Show that $\overrightarrow{FP} = \frac{2}{3}\mathbf{a}$ (multiple methods possible)

Show that $\overrightarrow{PE} = \frac{1}{3}\mathbf{a}$ (multiple methods possible)

Therefore FP and PE are parallel, so P lies on FE

$FP{:}PE = 2 : 1$

TAP TO RETURN

7 - Answers

Proof

TAP TO RETURN

8 - Answers

(a) $\frac{2x}{x+5}+\frac{6x}{(x+5)(x+2)}=\frac{2x(x+2)}{(x+5)(x+2)}+\frac{6x}{(x+5)(x+2)}$
$=\frac{2x(x+2+3)}{(x+5)(x+2)}=\frac{2x(x+5)}{(x+5)(x+2)}=\frac{2x}{(x+2)}$

(b) $\frac{4}{25}$

TAP TO RETURN

9 - Answers

$y = 4x - \pi + 2$

TAP TO RETURN

10 - Answers

$$y = \frac{x^3}{3} - \frac{3}{2}x^2 + 2x^{-1} + \frac{1}{6}$$

TAP TO RETURN

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11 - Answers

(a) (i) $f^{-1}(x) = 5x - 6$, $x \in \mathbb{R}$

(ii) $f^{-1}(x) = {}^{5}/_{x}$, $\{x \in \mathbb{R}: x \neq 0\}$

(iii) $f^{-1} : x \to x^2 - 4$, $\{x \in \mathbb{R}: x \geq 0\}$

(iv) $f^{-1} : x \to {}^{(x+2)}/_{(x-3)}$, $\{x \in \mathbb{R}: x \neq 3\}$

(b) (i) The inverse is a 1 to many function (3bii) $x \in \mathbb{R}$, $x \geq 3$

(ii) $X \geq 3$

TAP TO RETURN

12 - Answers

$\pm(1+2i);\ \pm(1-2i)$

TAP TO RETURN