### Skills 1

Sketch the following functions on the given domain and hence find their ranges:

- (a)  $f(x) = x^2 + 4x + 3$  Domain f:  $x \in \mathbb{R}$
- (b)  $g(t) = 2t^2 4t 1$  Domain g:  $t \in \mathbb{R}$

# Skills 2

The functions *f* and *g* are defined on the whole of R by  $f(x) = x^2 + 1$ , g(x) = x + 3

Find:

(a) fg(0) (b) fg(1) (c)  $f^2(2)$  (d) fg(x)

(e) gf(x) (f) ff(x)

#### Skills 1 - Answers

- (a) Range f:  $f(x) \ge -1$
- (b) Range g:  $g(x) \ge -3$

#### Skills 2 – Answers

| (a) 10                |       |
|-----------------------|-------|
| (b) 17                |       |
| (c) 26                |       |
| (d) $(x+3)^2 + 1$     | TAP . |
| (e) $x^2 + 4$         | TO R  |
| (f) $(x^2 + 1)^2 + 1$ | ETUR  |
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A person throws a ball in a sports hall. The height of the ball, *h* m, can be modelled in relation to the horizontal distance from the point it was thrown from by the quadratic equation:  $h = -\frac{3}{10}x^2 + \frac{5}{2}x + \frac{3}{2}$ 

The hall has a sloping ceiling which can be modelled with equation  $h = \frac{15}{2} - \frac{1}{5}x$ .

Determine whether the model predicts that the ball will hit the ceiling.

#### 2

The circle C has equation  $x^2 + y^2 - 12x + 8y + 16 = 0$ 

- (a) Find the centre and radius of *C*
- (b) Given that *C* crosses the *x* axis at the points *A* and *B*, find the length *AB* giving your answer in the form  $k\sqrt{5}$

3

A car accelerates at a constant rate, starting from rest at a point A and reaching a speed of 65 km s<sup>-1</sup> 26 s. This speed is then maintained and the car passes a point B 3 minutes after leaving A.

(a) Sketch a speed-time graph to illustrate the motion of the car.

(b) Find the distance from *A* to *B*.

4

Draw a force diagram and resolve forces horizontally and vertically. N.B. In the case o limiting friction,  $F = \mu R$ , where *R* is the normal reaction.

An airline passenger pushes a 15kg suitcase along the floor with his foot. A force (P) of 60N is needed to move the suitcase. Find:-

(a) the co-efficient of friction.

(b) the force needed to give the suitcase an acceleration of  $0.2 \text{ms}^{-2}$ .

#### 5

TAP FOR ANSWERS

Two masses of 3kg and 5kg are suspended either end of a light inextensible string which passes over a smooth fixed peg. The particles are held in the positions shown, with the string taut; they are then released from rest. Construct separate equations for each of the masses. Find the tension in the string and the acceleration of the particles.



6

[In this question, the horizontal unit vectors **i** and **j** are directed due East and North respectively.]

A coastguard station O monitors the movements of ships in a channel. At noon, the station's radar records two ships moving with constant speed. Ship A is at the point with position vector  $(-5\mathbf{i} + 10\mathbf{j})$  km relative to O and has velocity  $(2\mathbf{i} + 2\mathbf{j})$  km h<sup>-1</sup>. Ship B is at the point with position vector  $(3\mathbf{i} + 4\mathbf{j})$  km and has velocity  $(-2\mathbf{i} + 5\mathbf{j})$  km h<sup>-1</sup>.

(a) Given that the two ships maintain these velocities, show that they collide. The coast guard radios ship A and orders it to reduce its speed to move with velocity  $(\mathbf{i} + \mathbf{j}) \text{ km h}^{-1}$ .

Given that A obeys this order and maintains this new constant velocity,

- (b) find an expression for the vector  $\overrightarrow{AB}$  at time t hours after noon.
- (c) find, to 3 significant figures, the distance between A and B at 1400 hours,
- (d) find the time at which *B* will be due north of *A*.

7

A large tank is in the shape of a cuboid with a rectangular base and no top. Two of the vertical opposite faces of the cuboid are square and the height of the cuboid is x metres.

(a) given that the surface area of the tank is 54  $m^2$ , show that the capacity, V, of the tank is given by  $V = 18x - \frac{2}{3}x^3$ .

(b) Find the maximum value for V, fully justifying the fact that it is the maximum value.

#### 8

NEW TECHS!  $f(x) = \frac{5x^2 + 7x}{2x^4}$ 

(a) Split f(x) into two fractions and find an expression for f'(x)

(b) Now consider the functions  $g(x) = 5x^2 + 7x$  and  $h(x) = 2x^4$ . Find expressions for g'(x) and h'(x)

(c) Now, 
$$f(x) = \frac{g(x)}{h(x)}$$
 Which one of the following rules is true?  
A  $f(x) = \frac{g'(x)}{h'(x)}$   
B  $f(x) = \frac{g'(x)h(x)+g(x)h'(x)}{h(x)}$   
C  $f(x) = \frac{g'(x)h(x)-g(x)h'(x)}{h(x)^2}$   
D  $f(x) = \frac{g'(x)h(x)+g(x)h'(x)}{h(x)^2}$   
E  $f(x) = \frac{g(x)h'(x)-g'(x)h(x)}{h(x)}$ 





The figure above shows the curve with equation

$$y = x^3 - 12x^2 + 45x - 34$$

TAP FOR ANSWERS

The points *A* and *B* lie on the curve, where x = 1 and x = 4 respectively. The finite region *R* is bounded by the curve and the straight line segment *AB*. Show that the area of *R*, shown shaded in the figure is  $\frac{81}{4}$ 

#### 10

Evaluate the following

$$\lim_{\delta x \to 0} \sum_{x=0}^{36} (2 + \sqrt{x})^2 \, dx$$

TAP FOR ANSWERS

#### 11

The function t is defined by t:  $x \mapsto 5 - 2x$ 

Solve the equation  $t^2(x) - (t(x))^2 = 0$ 

TAP FOR ANSWERS

#### 12

The function h is defined by  $h(x) = x^2 - 6x + 20$  and has domain  $x \ge a$ . Given that h(x) is a one-to-one function find the smallest possible value of the constant *a*.

You have a tracking test the week after you get back from Christmas. We have given you 4 less questions than normal here – use this time to revise anything you feel is weak to prepare for the test.

#### 1 - Answers

Yes, the ball will hit the ceiling.

#### 2 - Answers

(a) Centre (6, -4) radius 6

(b) k = 4

TAP TO RETURN

#### 3 - Answers

(b) 10855 km

#### 4 - Answers

(a)  $\mu = 0.408$ 

(b) P=63N

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#### 5 - Answers

(a) 1.96 ms<sup>-2</sup>

(b) T=23.5N

#### 6 - Answers

(b)  $AB = (8 - 3t)\mathbf{i} + (-6 + 4t)\mathbf{j}$ 

(c) 2.83 km

(d) 1440 hours



#### 8 - Answers

| (a) $-\frac{5}{x^3} - \frac{21}{2x^4}$ |
|--|
| (b) 10x + 7, 8x <sup>3</sup>           |
| (c) C                                  |
|  |
|  |

TAP TO RETURN

#### 9 - Answers

Proof

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