Skills 1

TAP FOR ANSWERS

In each of the following $\frac{dy}{dx}$ is given, and y = 1 at x = 0. Find y in terms of x. a) $\frac{dy}{dx} = x^4 + 2x$ b) $\frac{dy}{dx} = -2x^{\frac{3}{4}}$ c) $\frac{dy}{dx} = (3x - 4)^2$ d) $\frac{dy}{dx} = (x - 1)(3x - 2)$

Skills 2

Sketch the curve and hence state the range for the given domain for the function $f(x) = x^2 - 2$

- (a) Domain $x \in \mathbb{R}: x = \{-2, -1, 0, 1, 2, \}$
- (b) Domain $x \in \mathbb{R}: -2 < x \le 7$

(c) Domain $x \in \mathbb{R}$: (x is a member of the set of all real numbers)

Skills 1 - Answers

(a)
$$y = \frac{x^5}{5} + x^2 + 1$$

(b) $y = -\frac{8}{7}x^{\frac{7}{4}} + 1$
(c) $y = 3x^3 - 12x^2 + 16x + 1$
(d) $y = x^3 - \frac{5}{2}x^2 + 2x + 1$

TAP TO RETURN

Skills 2 – Answers

(a) Range $f \in \mathbb{R}: f(x) = \{-2, -1, 2\}$
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(b) Range $f \in \mathbb{R}: -3 < f(x) \le 79$

remember minimum point

(c) Range $f \in \mathbb{R}$: $f(x) \ge -2$

The straight line l_1 passes through the points *P* and *Q* which have coordinates (7, 4) and (9, 7) respectively. The straight line l_2 has gradient 8 and passes through the origin, *O*. The lines l_1 and l_2 intersect at the point *R*.

Show that OP = OR

2

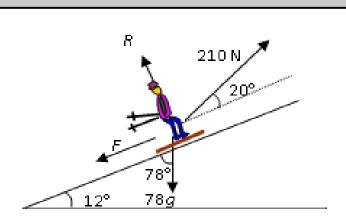
The circle C has centre (5, 2) and passes through the point (7, 3).

(a) Find an equation for *C*

(b) Show that the line y = 2x - 3 is tangent to *C* and find the coordinates of the point of contact

HINT: Show equal roots

3

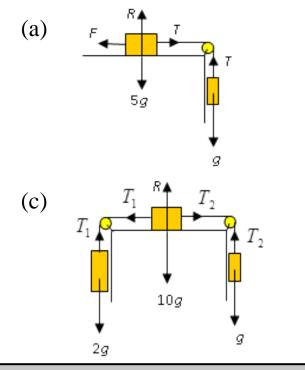


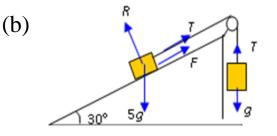
A skier of mass 78kg is pulled at constant speed up a rough slope of inclination 12°, by a force of magnitude 210N acting upwards at an angle of 20° to the slope. Find the magnitudes of the frictional force and the normal contact force acting on the skier.to 3 significant figures.

Why is the normal contact force so much larger than friction? Give a reason.

4

In the cases illustrated below, strings pass over small light pulleys. Contact between the blocks and the surfaces is rough. The blocks are at rest and the strings taut. By resolving forces in appropriate directions form equations to find the tension in the string and the frictional force exerted by each surface on the block with which it is in contact.





Add F (friction) to the diagram in the correct direction.

5

NEW TECHS!

The velocity vector $\underline{\mathbf{v}}$ is the change in displacement per unit of time.

The direction of travel is always the direction of the velocity vector $\underline{\mathbf{v}}$.

A bearing is the clockwise angle from North.

If a particle moves with constant velocity $\underline{\mathbf{v}}$ then its displacement from its initial position after time t is $\underline{\mathbf{v}}t$.

If a particle starts from a point with positon vector $\underline{\mathbf{r}}_{o}$ and moves constant velocity $\underline{\mathbf{v}}$ then by the triangle of vectors its position vector $\underline{\mathbf{r}}$ is given by $\underline{\mathbf{r}} = \underline{\mathbf{r}}_{o} + \underline{\mathbf{v}}t$

You may also be expected to use the vector form $\underline{\mathbf{v}} = \underline{\mathbf{u}} + \underline{\mathbf{a}}t$, for constant acceleration $\underline{\mathbf{a}}$ only.

Speed is the modulus of the velocity vector, ie speed = $\sqrt{v_x^2 + v_y^2}$

Also, distance is the modulus of the displacement vector.

Always draw a schematic diagram to clearly show the triangles that you may use to add or subtract vectors.

[In this question, **i** and **j** are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.]

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NEW TECHS!

A ship S is moving along a straight line with constant velocity. At time t hours the position vector of S is s km. When t = 0, s = 9i - 6j. When t = 4, s = 21i + 10j. Find

(a) the speed of S,

(b) the direction in which S is moving, giving your answer as a bearing.

(c) Show that at time $x \mathbf{s} = (3t+9) \mathbf{i} + (4t-6) \mathbf{j}$.

A lighthouse *L* is located at the point with position vector $(18\mathbf{i} + 6\mathbf{j})$ km. When *t* = *T*, the ship *S* is 10 km from *L*.

(d) Find the possible values of *T*.

6

Four points have coordinates A (7, 12 -1), B (11, 2, -9), C (14, -14, 3) and D (8, 1, 15) respectively.

(a) Show that *AB* and *CD* are parallel, and find the ratio *AB*:*CD* in its simplest form.

(b) Hence describe the quadrilateral *ABCD*.

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Given that $f(x) = \frac{2}{5}x^3 + 1$

(a) Find the gradient of the tangent to y = f(x) at the point where x = 1.

(b) Find the coordinates of the points where the gradient of the tangent to f(x) is equal to 30

(c) Find the coordinates of the point where the gradient of the tangent to f (x) is parallel to the line 24x - 5y + 15 = 0

(d) The coordinates of the stationary point on the curve

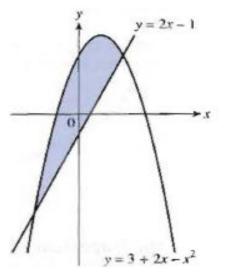
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Given that $\frac{6x+5x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $6x^p + 5x^q$, (a) write down the value of p and the value of q. Hence given that $\frac{dy}{dx} = \frac{6x+5x^{\frac{3}{2}}}{\sqrt{x}}$ and that y = 100 when x = 9, (b) find y in terms of x, simplifying the coefficient of each term.

TAP FOR ANSWERS



Find the area shaded between the line y = 2x - 1 and the curve



TAP FOR ANSWERS

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Evaluate the following

$$\lim_{\delta x \to 0} \sum_{x=1}^{5} 3\sqrt{x} - \frac{1}{\sqrt{x}} dx$$

TAP FOR ANSWERS

11

The functions of f and g are defined by:

f: $x \mapsto |9 - 4x|$ g: $x \mapsto \frac{3x-2}{2}$

(a) find fg(6)

(b) Solve fg(x) = x

12

The factor theorem, states that if (x - 1) is a factor of the polynomial f(x) then f(a) = 0.

Show that z = -1 is a root of the equation $z^3 + 9z^2 + 33z + 25 = 0$. Hence solve the equation completely.

1 - Answers

 $OP = OR = \sqrt{65}$

2 - Answers

(a) $(x-5)^2 + (y-2)^2 = 5$

(b) (3, 3)

TAP TO RETURN

3 - Answers

F=38.4N, R=676N

- (a) T=9.8N, F=9.8N
- (b) T=9.8N, F= 14.7N
- (c) $T_2=9.8N$, $T_1=19.6N$, F=9.8N

5 - Answers

(a) 5 km h-1

(b) 36.9°

(d) 1, 5 hours

6 - Answers

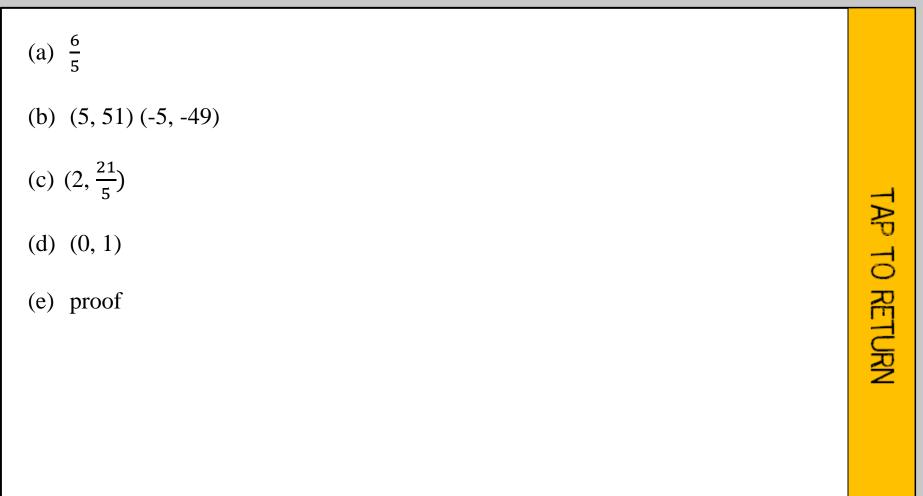
(a)

$$\overrightarrow{AB} = 4\mathbf{i} - 10\mathbf{j} - 8\mathbf{k} = 2(2\mathbf{i} - 5\mathbf{j} - 4\mathbf{k})$$

 $\overrightarrow{CD} = 6\mathbf{i} + 15\mathbf{j} + 12\mathbf{k} = -3(2\mathbf{i} - 5\mathbf{j} - 4\mathbf{k})$
 $\overrightarrow{CD} = -\frac{3}{2}\overrightarrow{AB}$, so *AB* is parallel to *CD*
AB:*CD* = 2 : 3

(b) *ABCD* is a trapezium

TAP TO RETURN



(a)
$$p = \frac{1}{2}, q = \frac{1}{6}$$

(b) $y = 4x^{\frac{2}{3}} + \frac{5x^2}{2} - \frac{421}{2}$





11 - Answers

(a) 23

(b)
$$x = \frac{13}{7}$$
 and $x = \frac{13}{5}$

TAP TO RETURN

12 - Answers

 $z = -1, -4 \pm 3i$