BHASVIC MαTHS
A1 DOUBLES SKILLS 1 - PART B

1

Convert the below into the form $ax^m + bx^n$:

(a) $\frac{1+5x}{4x}$

(b) $\frac{3x-4\sqrt{x}}{x^3}$

(c) $\frac{\sqrt[4]{16x^3}-\sqrt[3]{27x^3}}{2x^2}$

(d) $\frac{4-5x}{3x^2}$

(e) $\frac{\sqrt{4x^3}-1-2x}{\sqrt[3]{x}}$

(f) $\frac{4x+\sqrt{81x^2}}{9\sqrt{x}}$

TAP FOR ANSWERS

NEW TECHNIQUES

Co-ordinate Geometry –line

The equation of straight line can be constructed using $m(x-a)=(y-b)$ where (a,b) is a point on the line and **m** is the gradient of the line. You can then re-arrange to get the equation in the form $ax+by+c=0$, where a, b and c are integers.

Example:

Find the equation of the line through $(1,-2)$ with gradient $\frac{1}{2}$ (giving your answer in the form $ax+by+c=0$, where a, b and c are integers):

(substituting point and gradient into the formula)	$\mathbf{\frac{1}{2}(x-1)=y+2}$
(multiplying both sides by 2)	$\mathbf{x-1=2y+4}$
(getting it all on one side for required form)	$\mathbf{x-2y-5=0}$

TAP FOR ANSWERS

2

Give your answers to these questions in the form $ax + by + c = 0$ where a, b and c are **integers**.

(a) Find the equation of the line through $(2, -3)$ with gradient $\frac{3}{2}$

(b) Find the equation of the line through $(-2, -1)$ with gradient $-\frac{7}{2}$

(c) Find the equation of the line through $(2, 3)$ with gradient $-\frac{1}{2}$

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES SKILLS 1 - PART B

1 - Answers

(a) $\frac{1}{4}x^{-1} + \frac{5}{4}$

(b) $3x^{-2} - 4x^{-\frac{5}{2}}$

(c) $x^{-\frac{5}{4}} - \frac{3}{2}x^{-1}$

(d) $\frac{4}{3}x^{-2} - \frac{5}{3}x^{-1}$

(e) $2x^{\frac{7}{6}} - x^{-\frac{1}{3}} - 2x^{\frac{2}{3}}$

(f) $\frac{13}{9}x^{\frac{1}{2}}$

TAP TO RETURN

2 - Answers

(a) $3x - 2y - 12 = 0$

(b) $7x + 2y + 16 = 0$

(c) $x + 2y - 8 = 0$

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES PROBLEM SHEET 1 - PART B

1

Find all of the roots of the function $r(x) = x^8 - 17x^4 + 16$

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES PROBLEM SHEET 1 - PART B

2

The function f is defined as $f(x) = 2^{2x} - 20(2^x) + 64, x \in \mathbb{R}$.

(a) Write f(x) in the form $(2^x - a)(2^x - b)$, where a and b are real constants.

(b) Hence find the two roots of f(x).

TAP FOR ANSWERS

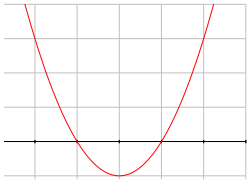
3

(a) Evaluate the discriminant, $b^2 - 4ac$, for each of the quadratics

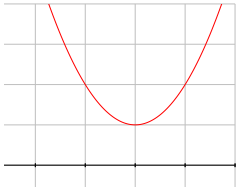
$$f(x) = x^2 - 2x + 1$$
$$g(x) = x^2 - 2x + 3$$
$$h(x) = x^2 - 2x - 5$$

(b) Hence state which of the following sketches could show the graph of $f(x)$, $g(x)$ and $h(x)$ and explain why.

Quadratic A



Quadratic B



Quadratic C

(c) For the quadratic $f(x) = x^2 - kx - k + 1$, the discriminant $= -7$, find the possible values of k. *Make sure you are correctly identifying a, b and c!*

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES PROBLEM SHEET 1 - PART B

4

For this question, $f(x) = 4kx^2 + (4k + 2)x + 1$, where k is a real constant.

(a) Find the discriminant of f(x) in terms of k.

(b) By simplifying your answer to part **a** or otherwise, prove that f(x) has two distinct real roots for all non-zero values of k.

(c) Explain why f(x) cannot have two distinct real roots when $k = 0$.

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES PROBLEM SHEET 1 - PART B

5

A fertiliser company uses a model to determine how the amount of fertiliser used, f kilograms per hectare, affects the grain yield g, measured in tonnes per hectare.

$$g = 6 + 0.03f - 0.00006f^2$$

(a) According to the model, how much grain would each field yield without any fertiliser?

(b) One farmer currently uses 20kg of fertiliser per hectare. How much more fertiliser would he need to use to increase his grain yield by 1 tonne per hectare?

TAP FOR ANSWERS

SWC REVIEW!

GRAPH TRANSFORMATIONS

$f(x) + b$ is $f(x)$ moved up b units. Moving the function down works the same way; $f(x) - b$ is $f(x)$ moved down b units.

$f(x + b)$ gives $f(x)$shifted b units to the left. Shifting to the right works the same way; $f(x - b)$ is $f(x)$ shifted b units to the right.

(a) If $\boldsymbol{f(x) = x^2}$, sketch the following transformations, stating the co-ordinates of the y and x axis intercepts.

(i) $y = f(x) + 1$
(ii) $y = f(x) - 3$
(iii) $y = f(x + 1)$
(iv) $y = f(x - 2)$

TAP FOR ANSWERS

(b) If $\boldsymbol{f(x) = x^3}$, sketch the following transformations, stating the co-ordinates of the *y* and *x* axis intercepts.

(i) $y = f(x) + 1$
(ii) $y = f(x) - 3$
(iii) $y = f(x + 1)$
(iv) $y = f(x - 2)$

(c) If $\boldsymbol{f(x) = \frac{1}{x}}$, sketch the following transformations. Show clearly the equations of any asymptotes and where the graph cuts the *y* and *x* axes.

(i) $y = f(x) + 1$
(ii) $y = f(x) - 3$
(iii) $y = f(x + 1)$
(iv) $y = f(x - 2)$

TAP FOR ANSWERS

6

(d) If $\boldsymbol{f(x) = \frac{1}{x^2}}$, sketch the following transformations. Show clearly the equations of any asymptotes and where the graph cuts the *y* and *x* axes.

(i) $y = f(x) + 1$
(ii) $y = f(x) - 3$
(iii) $y = f(x + 1)$
(iv) $y = f(x - 2)$

TAP FOR ANSWERS

7

The straight line l passes through $A\ (1, 3\sqrt{3})$ and $B\ (2 + \sqrt{3}, 3 + 4\sqrt{3})$.
Show that l meets the x-axis at the point C (-2, 0)

TAP FOR ANSWERS

8

NEW TECHNIQUES

A car starts from rest and accelerates uniformly for 15 seconds until it reaches a speed of $12ms^{-1}$. It then maintains this speed for 40 seconds before decelerating uniformly to rest in 30 seconds.

Draw a speed-time graph to model this motion and calculate the total distance travelled by the car (**you will need to use the fact that the distance travelled equals area under a speed-time graph**).

TAP FOR ANSWERS

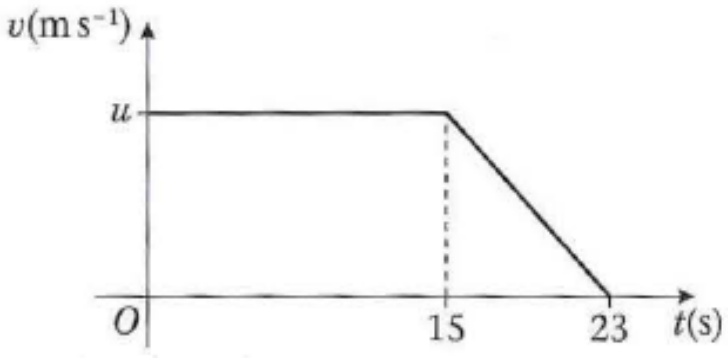
BHASVIC MαTHS
A1 DOUBLES PROBLEM SHEET 1 - PART B

9

NEW TECHNIQUES

The diagram is a speed-time graph representing the motion of a cyclist along a straight road. At time $t = 0$ s, the cyclist is moving with speed u m s^{-1}. The speed is maintained until time $t = 15$ s, when she slows down with constant deceleration, coming to rest when $t = 23$ s. The total distance she travels in 23 s is 152 m.

Find the value of u.



TAP FOR ANSWERS

10

NEW TECHNIQUES

This is a suvat table. Only model a situation with a suvat table if acceleration is **constant**

s	
u	
v	
a	
t	

s = displacement in metres, u = initial velocity in metres per second,
v = final velocity in metres per second, a = acceleration in metres per second2,
t = time in seconds

The *suvat* table can also be drawn horizontally:

s	u	v	a	t

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES PROBLEM SHEET 1 - PART B

The five *suvat* equations are given below. Each equation contains only 4 out of the 5 variables.

$$s = vt - \frac{1}{2}at^2 \qquad v = u + at \qquad s = \left(\frac{u+v}{2}\right)t$$

$$s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as$$

Write out (many times) and <u>memorise</u> the *suvat* equations.

For each of the situations on the next page, make and complete a *suvat* table. Put a question mark in the box for whichever piece of information is required by the question, and a cross for the variable you're not interested in. **Make sure you've used displacement not distance, and make sure you're consistently using the same direction (up or down, or left or right) as the 'positive' direction.**

TAP FOR ANSWERS

For each situation, select an equation from the 5 listed on the previous page. The easiest way to do this is to see what information is missing from your table (neither given nor required by the question) and choose the equation with this variable missing.

Write down the equation, substitute in the values from your *suvat* table and then rearrange to find the required information**. Don't forget units on your final answer!**

<u>In all mechanics questions, you need to round your final answer to 2 or 3 significant figures at the end.</u>

(a) A particle moves in a straight line. When $t = 0$ its velocity is 3 m s^{-1}. When $t = 4$ its velocity is 12 m s^{-1}. Find its acceleration, assumed to be constant.

TAP FOR ANSWERS

10

(b) A car is approaching traffic lights at 15 m s^{-1} when the driver applies the brake and comes to a stop in 45 m. Find the deceleration, assumed constant, and the time taken to stop.

(c) A particle has constant acceleration 6 m s^{-2} whilst travelling in a straight line between points *A* and *B*. It passes *A* at 2 m s^{-1} and *B* at 5 m s^{-1}. Calculate the distance *AB*.

(d) A person on the top of a tower of height 45m holds their arm over the side of the building and **drops** a stone vertically downwards. The stone takes 3.03s to reach the ground. Use this information to prove that the value of acceleration due to gravity is 9.8 to 2 significant figures.

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES PROBLEM SHEET 1 - PART B

11

A particle is projected vertically upwards with speed 24.5m s $^{-1}$. Find the total time for which it is 21 m or more above its point of projection.

TAP FOR ANSWERS

A1 DOUBLES PROBLEM SHEET 1 - PART B

NEW TECHNIQUES

FP1 Complex Numbers: Solving polynomials with real coefficients

Reminder: from Moving on Day / Starting with Confidence

Remember $i^2 = -1$

TASK: Solve the following equation to find x in the form $x = a + ib$

EXAMPLE: $x^2 + 16 = 0$

So, $x^2 = -16$

Then, $x = \pm\sqrt{-16}$

Then, $x = \pm i\sqrt{16}$

Therefore, $x = \pm 4i$

NOW YOU TRY:

TAP FOR ANSWERS

Part A

Find *x* such that:

(i) $x^2 + 81 = 0$

(i) $x^2 + 2x + 5 = 0$

(iii) $-4x = x^2 + 7$

(iv) $x^3 + 2x^2 + 17x = 0$

As you may have noticed, if $z = a + bi$ is a complex root of a polynomial with real coefficients (e.g. $ax^2 + bx + c = 0$, where a, b and c are real) then $z^* = a - ib$ (the complex conjugate, z^* or $\bar{z}$) is also a root.

We know that if 2 and 3 are the roots of an equation, then we can write it in the form

$(x - 2)(x - 3) = 0$.

TAP FOR ANSWERS

12

The same thing is true here: if we know that the two complex roots are 1 + 2i and 1 – 2i, then we can write the equation as $\big(x-(1+2i)\big)\big(x-(1-2i)\big)=0$.

To expand this and find our quadratic, we can use the grid as follows:

We want to expand $(x-1-2i)(x-1+2i)$

	x	-1	2i
x	x^2	$-x$	$2ix$
-1	$-x$	1	-2i
-2i	$-2ix$	2i	4

Adding the contents of our 9 boxes, we get $x^2-x-x-2ix+2ix+1+2i-2i+4=x^2-2x+5$

So our quadratic is $x^2-2x+5=0$.

TAP FOR ANSWERS

PART B

One root of the quadratic equation $x^2 + px + q = 0$, where p and q are real, is the complex number 2 – 3i.

(i) Write down the other root.

(ii) Find the values of p and q

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES PROBLEM SHEET 1 - PART B

1 - Answers

1, 1, 2 and -2

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES PROBLEM SHEET 1 - PART B

2 - Answers

(a) $f(x) = (2^x - 16)(2^x - 4)$

(b) 4 and 2

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES PROBLEM SHEET 1 - PART B

3 - Answers

(a) 0,-8,24

(b) $f(x)$ is quadratic C ($b^2 - 4ac = 0$ so 1 root - graph touches the *x* axis)
$g(x)$ is quadratic B ($b^2 - 4ac < 0$ so no roots - graph doesn't cut the *x* axis)
$h(x)$ is quadratic A ($b^2 - 4ac > 0$ so two distinct roots - graph cuts the *x* axis)

(c) k = –1 or k = –3

TAP TO RETURN

4 - Answers

(a) $16k^2 + 4$

(b) $k^2 \geq 0$ for all k so $16k^2 + 4 > 0$

(c) When $k = 0$, $f(x) = 2x + 1$; this is a linear function with only one root.

TAP TO RETURN

5 - Answers

(a) 6 tonnes

(b) 39.6 kilograms per hectare

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES PROBLEM SHEET 1 - PART B

6 - Answers

In the library computers you can plot the graphs on ‘autograph’. On your phone you could use the free app ‘desmos’. Or, use your graphical calculator to check. It is important you try these yourself first, don’t go straight to the answers!

TAP TO RETURN

7 - Answers

Proof

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES PROBLEM SHEET 1 - PART B

8 - Answers

750m

TAP TO RETURN

9 - Answers

$u = 8$

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES PROBLEM SHEET 1 - PART B

10 - Answers

(a) 2.25 ms^{-2}

(b) $a = 2.5$ ms^{-2}, $t = 6$

(c) 1.75m

TAP TO RETURN

11 - Answers

2.8 seconds

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES PROBLEM SHEET 1 - PART B

12 - Answers

Part A:

(i) $\pm 9i$

(ii) $-1 \pm 2i$

(iii) $-2 \pm \sqrt{3}i$

(iv) $0, -1 \pm 4i$

Part B:

(i) 2 + 3i

(ii) p=-4, q=13

TAP TO RETURN