

AB, *CD* and *EF* are arcs of concentric circles, centre *O*, such that *OACE* and *OBDF* are straight lines as shown in the diagram The area of the shaded region *CEFD* is denoted by A_1 and the area of the shaded sector *OAB* by A_2 .

TAP FOR ANSWERS

Given that OA = r cm, AC = 2 cm, OE = 8 cm and $\angle AOB = \theta$ radians, (a) find an expression for A_1 in terms of r and θ .

Given also that $A_1 = 7A_2$, (b) show that r = 2.5

2

A histogram was drawn to show the distribution of age in completed years of the participants on an outward-bound course.

There were 32 people aged 30-34 years on the course. The height of the rectangle representing this group was 19.2 cm and it was 1 cm in width.

Given that there were 28 people aged 35-39 years.

(a) find the height of the rectangle representing this group.

Given that the height of the rectangle representing people aged 40-59 years was 2.7 cm.

(b) find the number of people on the course in this age group.

3

The table shows information about the time, t minutes correct to the nearest minute, taken by 50 people to complete a race.

Time (minutes)	$t \leq 27$	$28 \leqslant t \leqslant 30$	$31 \leq t \leq 35$	$36 \leq t \leq 45$	$46 \leqslant t \leqslant 60$	$t \ge 61$	
Number of people	0	4	28	14	4	0	

(a) In a histogram illustrating the data, the height of the block for the $31 \le t \le 35$ class is 5.6 cm. Find the height of the block for the $28 \le t \le 30$ class. (There is no need to draw the histogram.)

(b) Calculate an estimate for the median of the data

(c) Calculate estimates of the mean and standard deviation of the data.

4

A particular species of orchid is being studied. The population p at time t years after the study started is assumed to be

 $p = \frac{2800ae^{0.2t}}{1+ae^{0.2t}}$, where *a* is a constant.

Given that there were 300 orchids when the study started,

(a) Show that a = 0.12

(b) Use the equation with a = 0.12 to predict the number of years before the population of orchids reaches 1850.

(c) Show that $p = \frac{336}{0.12 + e^{-0.2t}}$

(d) Hence show that the population cannot exceed 2800.

5

A particle of weight W is attached to the end B of a light string AB which is fixed at A. The string is inclined at 30° to the vertical by a force of magnitude P as shown. Find the value of P when W is: (a) $2\sqrt{3}$ N (b) $\sqrt{48}$ N (c) $\sqrt{300}$ N



TAP FOR ANSWERS

6

(i) Solve the quadratic $3^{2x} - 28(3^x) + 27 = 0$ by any method

(ii) Solve the following equations, giving answers correct to 2 d.p.

(a) $3e^{2x+5} = 4$

(b) $3^x = 5^{1-x}$

(c) $2\ln(2x-1) = 1 + \ln 7$

7

A scalene triangle has the coordinates (2, 0, 0), (5, 0, 0) and (4, 2, 3). Work out the area of the triangle.

8

For each of these propositions, decide whether it is true. If the proposition is true, prove it. If the proposition is not true, find a counter-example.

(a) The sum of two primes is prime.

(b) The sum of two primes is never prime.

TAP FOR ANSWERS

9

There are many different flu viruses. The numbers of flu viruses detected in the first few weeks of the 2012–2013 flu epidemic in the UK were as follows.

Week	1	2	3	4	5	6	7	8	9	10
Number of flu viruses	7	10	24	32	40	38	63	96	234	480

These data may be modelled by an equation of the form $y = a \times 10^{bt}$, where y is the number of flu viruses detected in week t of the epidemic, and a and b are constants to be determined.

- (i) Explain why this model leads to a straight-line graph of $\log_{10} y$ against *t*. State the gradient and intercept of this graph in terms of *a* and *b*.
- (ii) Complete the values of $\log_{10} y$ in the table, draw the graph of $\log_{10} y$ against *t*, and draw by eye a line of best fit for the data.

Hence determine the values of a and b and the equation for y in terms of t for this model.

During the decline of the epidemic, an appropriate model was

 $y = 921 \times 10^{-0.137w}$

where y is the number of flu viruses detected in week w of the decline.

(iii) Use this to find the number of viruses detected in week 4 of the decline.

10

A disease is known to be present in 2% of a population. A test is developed to help determine whether or not someone has the disease.

Given that a person has the disease, the test is positive with probability 0.95 Given that a person does not have the disease, the test is positive with probability 0.03

(a) Draw a tree diagram to represent this information.

A person is selected at random from the population and tested for this disease. (b) Find the probability that the test is positive.

A doctor randomly selects a person from the population and tests him for the disease.

Given that the test is positive,

(c) find the probability that he does not have the disease.

(d) Comment on the usefulness of this test.

11

- The points *A*, *B* and *C* have position vectors $\begin{pmatrix} 8 \\ -7 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 8 \\ -3 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 12 \\ -6 \\ 3 \end{pmatrix}$ respectively.
- (a) Find the vectors \overrightarrow{AB} , \overrightarrow{AC} and \overrightarrow{BC} .

(b) Find $|\overrightarrow{AB}|$, $|\overrightarrow{AC}|$ and $|\overrightarrow{BC}|$ giving your answers in exact form.

(c) Describe triangle *ABC*.

12

For each of these propositions decide whether it is true.

If the proposition is true, prove it.

If the proposition is not true find a counter-example.

a)
$$\sqrt{x^2 + y^2} = x + y$$

b) For any real numbers, x and y, $x^2 + y^2 \ge 2xy$

13

(a)(i) Write down the resolved part of the force F in the direction Ox.

(ii) Write down the resolved part of the force F in the direction Oy. $\frac{Y}{Y}$

A toboggan of mass 20 kg is pulled, with a rope, up a slope inclined at 15° to the horizontal. The rope is inclined at an angle of 15° to the slope, and the tension in the rope is 70 N.

Given that the toboggan is moving at constant speed:

(b)(i) Find the frictional force F.

(ii) Find the normal reaction *R*.

(iii) Find the coefficient of friction.



Х



Fig. 2

Figure 2 shows a cable car *C* of mass 1 tonne which has broken down. The cable car is suspended in equilibrium by two cables *A*C and *BC perpendicular to each other* and attached to fixed points *A* and *B*, at the same horizontal level on either side of a valley. The cable *AC* is inclined at an angle α to the horizontal where $\tan \alpha = \frac{3}{4}$.

Show that the tension in the cable AC is 5900 N (2sf) and find the tension in the cable BC.

15

Lauren wants to find the average daily mean windspeed in Hurn in 1987. She only has access to the large data set. She uses it to obtain a random sample of the daily mean windspeeds, t knots, on n days in Hurn in 1987. The data collected by Lauren are summarised as follows

$$\sum (t-5) = 55, \qquad \overline{t} = 10$$

(a) Find *n*.

Lauren uses the same sampling method to estimate that the average daily mean windspeed in Hurn in 2015 was 11 mph.

(b) Convert 11 mph into knots.

(c) Hence, compare the average daily mean windspeed in Hurn in 1987 and 2015.

(d) With reference to the large data set, state **one** limitation of your conclusion in part (c).

(e) Explain how Lauren can

(i) improve her data collection method

(ii) improve her data processing

to allow for a more reliable comparison in part (c).

(a)
$$A_1 = \frac{1}{2}\theta(60 - 4r - r^2) \text{ cm}^2$$

2 - Answers

(a) 16.8 cm

(b) 18

TAP TO RETURN

3 - Answers

(a) $h = \frac{4}{3} cm$

(b) $Q_2 = 34.25$

(c) $\bar{x} = 36.6$, $\sigma = 6.26(3 \text{sf})$

4 - Answers

(a) (c) (d) Proofs

(b) t = 13.9... = 14 years (nearest year)

TAP TO RETURN

(a) 2N	
(b) 4N	
(c) 10N	TAP TO RETURN





8 - Answers

(a) Not true

(b) Not true

TAP TO RETURN

9 - Answers



 $2.5 \leq a \leq 6.3$

iii) 260 or 261



(d) High probability of not having the disease for a person with a positive test

11 - Answers

(a)
$$\overrightarrow{AB} = 4\mathbf{j} - \mathbf{k}, \overrightarrow{AC} = 4\mathbf{i} + \mathbf{j} - \mathbf{k}, \overrightarrow{BC} = 4\mathbf{i} - 3\mathbf{j}$$

(b)
$$\left| \overrightarrow{AB} \right| = \sqrt{17}, \left| \overrightarrow{AC} \right| = 3\sqrt{2}, \left| \overrightarrow{BC} \right| = 5$$

(c) scalene

12 - Answers

(a) Not true

(b) True

TAP TO RETURN

13 - Answers

(a) (i) $Fcos\theta$

(ii) $Fsin\theta$

(b) (i) 16.9 N

(ii) 171N (3sf)

(iii) μ =0.099

TAP TO RETURN

14 - Answers

800g = 7800N (2sf)

15 - Answers

a) 11

b) 9.56 mph

c) Hurn had a lower average daily mean windspeed in 1987 than in 2015

d) The Large Data Set only covers the months May to October or

The Large Data Set does not cover the whole year

- e) i) use a larger data set so it is more representative
- ii) consider standard deviation/variation, so she can compare/take into account the spread of the data **or**

consider another average/the median/the mode, so she can compare with respect to other averages **or**

consider/exclude outliers, to avoid the average being influenced by extreme values