1

(i) Fully factorise the right-hand side of each equation.

(ii) Sketch the graph of each equation.

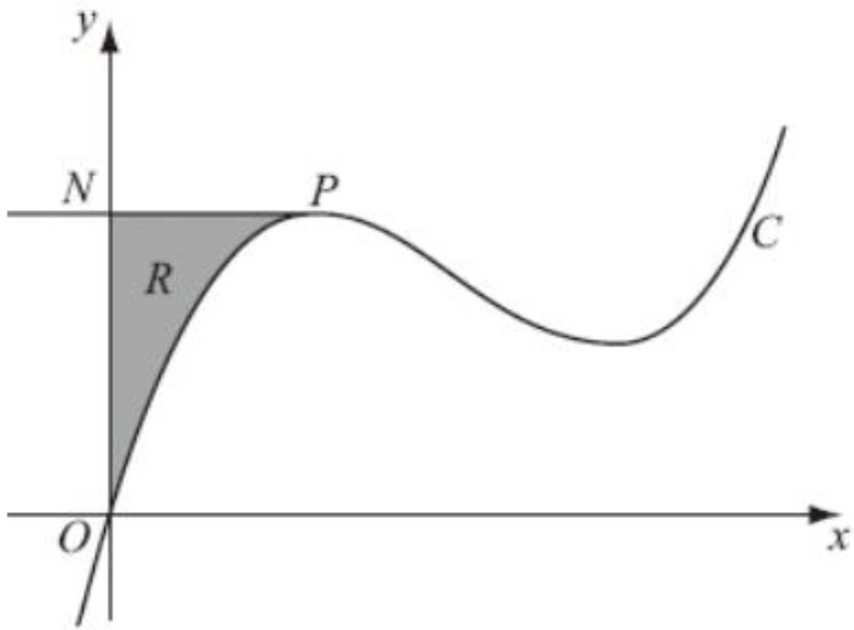
(a) $y = 2x^3 + 5x^2 - 4x - 3$

(b) $y = 2x^3 - 17x^2 + 38x - 15$

(c) $y = 3x^3 + 8x^2 + 3x - 2$

TAP FOR ANSWERS

2



The diagram above shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Q2 continues on next slide

TAP FOR ANSWERS

2

Given that the x-coordinate of P is 2,

(a) show that $k = 28$.

The line through P parallel to the x-axis cuts the y-axis at the point N. The region R is bounded by C, the y-axis and PN, as shown shaded in the diagram above.

(b) Use calculus to find the exact area of R.

TAP FOR ANSWERS

3

Given $f(x) = x^3 - 2x^2 - 7x - 4$

(a) Show that $(x + 1)$ is a factor of $f(x)$.

(b) Factorise $f(x)$ completely, and hence sketch the graph of $y = f(x)$, giving the intercepts with the coordinate axes.

TAP FOR ANSWERS

4

Evaluate $\int_1^8 \frac{1}{\sqrt{x}}\,dx$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

TAP FOR ANSWERS

5

a) Given that $(x - 1)$ is a factor of $5x^3 - 9x^2 + 2x + a$, find the value of a.

b)

Given that $$\frac{x^2 - 36}{x^2 - 11x + 30} \times \frac{25 - x^2}{Ax^2 + Bx + C} \times \frac{6x^2 + 7x - 3}{3x^2 + 17x - 6} \equiv \frac{x + 5}{6 - x}$$

find the values of the constants A, B and C, where A, B and C are integers.

TAP FOR ANSWERS

6

Prove that:

a $(\tan\theta + \cot\theta)(\sin\theta + \cos\theta) \equiv \sec\theta + \operatorname{cosec}\theta$

b $\dfrac{\operatorname{cosec} x}{\operatorname{cosec} x - \sin x} \equiv \sec^2 x$

c $(1 - \sin x)(1 + \operatorname{cosec} x) \equiv \cos x \cot x$

d $\dfrac{\cot x}{\operatorname{cosec} x - 1} - \dfrac{\cos x}{1 + \sin x} \equiv 2\tan x$

e $\dfrac{1}{\operatorname{cosec}\theta - 1} + \dfrac{1}{\operatorname{cosec}\theta + 1} \equiv 2\sec\theta\tan\theta$

f $\dfrac{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}{1 + \tan^2\theta} \equiv \cos^2\theta$

TAP FOR ANSWERS

7

The specification for a new rectangular car park states that the length is to be 5m more than the breadth. The perimeter of the car park is to be greater than 32m and the area of the car park is to be less than 104cm^2

(a) Form a linear inequality for the perimeter and solve it to find the range of values of x.

(b) Form a quadratic inequality for the area and solve it to find the range of values for x.

(c) Determine the set of possible values for x.

TAP FOR ANSWERS

8

$f(x) = x^3 + (p + 1)x^2 - 18x + q$, where p and q are integers.

Given that $(x - 4)$ is a factor of $f(x)$,

(a) show that $16p + q + 8 = 0$.

Given that $(x + p)$ is also a factor of $f(x)$, and that $p > 0$,

(b) show that $p^2 + 18p + q = 0$.

(c) Hence find the value of p and the corresponding value of q.

(d) Factorise $f(x)$ completely.

TAP FOR ANSWERS

9

Split the following into partial fractions:

(a) $\frac{5x^2-8x+1}{(2x)(x-1)^2}$

(b) $\frac{x^2}{x^2-4}$

(c) $\frac{1}{(x+1)(x+2)(x+3)}$

TAP FOR ANSWERS

10

A circular pipe has outer diameter 4 cm and thickness t cm.

(a) Show that the area of the cross-section, A cm^2, is given by $A = \pi(4t - t^2)$.
(b) Find the rate of increase of A with respect to t when $t = \frac{1}{4}$ and when $t = \frac{1}{2}$.

A piece of wire 16 cm long is cut into two pieces. Once piece is $8x$ cm long and is bent to form a rectangle measuring $3x$ by x cm. The other piece is bent to form a square.

Find in terms of x:
c) the length of a side of the square;
d) the area of the square.
e) show that the combined area of the rectangle and the square is A cm^2 where $A = 7x^2 - 16x + 16$.

Find:
f) The value of x for which A has its minimum value;
g) the minimum value of A.

TAP FOR ANSWERS

11

$$f(x) = x^4 + 3x^3 - 5x^2 - 3x + 1$$

(a) Find the coordinates of the stationary points of *f(x)*, and determine the nature of each.

(b) Sketch the graph of $y = f(x)$.

TAP FOR ANSWERS

12

Sketch the curve of $y = x^3 - 6x^2 + 9x$ showing clearly the coordinates of any point where the curve touches or crosses the axes.
The point with coordinates (-4,0) lies on the curve with equation
$y = (x-k)^3 - 6(x-k)^2 + 9(x-k)$

where k is a constant. Find the two possible values of k.

TAP FOR ANSWERS

13

(a) Sketch, on the same axes, in the interval $0 \leq x \leq 180$, the graphs of

$$y = \tan x° \text{ and } y = 2 \cos x°,$$

showing clearly the coordinates of the points at which the graphs meet the axes.

(b) Show that $\tan x° = 2 \cos x°$ can be written as

$$2 \sin^2 x° + \sin x° - 2 = 0.$$

(c) Hence find the values of x, in the interval $0 \leq x \leq 180$, for which $\tan x° = 2 \cos x°$.

TAP FOR ANSWERS

14

Two circles C_1 and C_2 have equations

$$(x - 2)^2 + y^2 = 9 \text{ and } (x - 5)^2 + y^2 = 9$$

respectively.

(a) For each of these circles state the radius and the coordinates of the centre.

(b) Sketch the circles C_1 and C_2 on the same diagram.

(c) Find the exact distance between the points of intersection of C_1 and C_2.

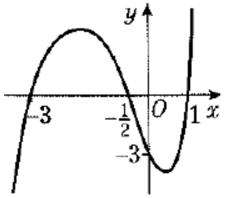
TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 9A

1 - Answers

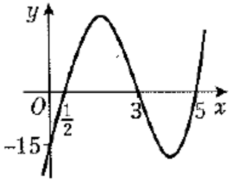
(a) (i) $(x-1)(x+3)(2x+1)$

(ii)



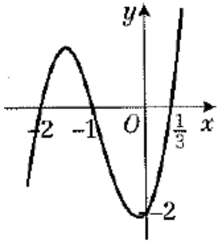
(b) (i) $(x-3)(x-5)(2x-1)$

(ii)



(c) (i) $(x+1)(x+2)(3x-1)$

(iii)



TAP TO RETURN

2 - Answers

(b) Area $= \frac{44}{3} \left(14\frac{2}{3} \text{ or } 14.\dot{6}\right)$

TAP TO RETURN

3 - Answers

(a) Proof

(b) $(x+1)^2(x-4)$

(c) Graph

TAP TO RETURN

4 - Answers

$$-2+4\sqrt{2}$$

TAP TO RETURN

5 - Answers

$a=2$

$A = 2$, $B = -9$ and $C = -18$

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 9A

6 - Answers

a L.H.S. $\equiv \left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)(\sin\theta+\cos\theta)$

$\equiv \frac{(\sin^2\theta+\cos^2\theta)}{\cos\theta\sin\theta}(\sin\theta+\cos\theta)$

$\equiv \frac{\sin\theta}{\sin\theta\cos\theta}+\frac{\cos\theta}{\cos\theta\sin\theta}$

$\equiv \sec\theta+\operatorname{cosec}\theta \equiv$ R.H.S.

b L.H.S. $\equiv \frac{\frac{1}{\sin x}}{\frac{1}{\sin x}-\sin x}$

$\equiv \frac{\frac{1}{\sin x}}{\frac{1-\sin^2 x}{\sin x}} \equiv \frac{1}{\sin x}\times\frac{\sin x}{\cos^2 x} \equiv \frac{1}{\cos^2 x} \equiv \sec^2 x \equiv$ R.H.S.

c L.H.S. $\equiv 1-\sin x+\operatorname{cosec} x-1 \equiv \frac{1}{\sin x}-\sin x$

$\equiv \frac{1-\sin^2 x}{\sin x} \equiv \frac{\cos^2 x}{\sin x} \equiv \cos x\frac{\cos x}{\sin x} \equiv \cos x\cot x$

$\equiv$ R.H.S.

d L.H.S. $\equiv \frac{\cot x(1+\sin x)-\cos x(\operatorname{cosec} x-1)}{(\operatorname{cosec} x-1)(1+\sin x)}$

$\equiv \frac{\cot x+\cos x-\cot x+\cos x}{\operatorname{cosec} x-1+1-\sin x} \equiv \frac{2\cos x}{\operatorname{cosec} x-\sin x}$

$\equiv \frac{2\cos x}{\frac{1}{\sin x}-\sin x} \equiv \frac{2\cos x}{\left(\frac{1-\sin^2 x}{\sin x}\right)} \equiv \frac{2\cos x\sin x}{\cos^2 x}$

$\equiv 2\tan x \equiv$ R.H.S.

e L.H.S. $\equiv \frac{\operatorname{cosec}\theta+1+\operatorname{cosec}\theta-1}{(\operatorname{cosec}^2\theta-1)} \equiv \frac{2\operatorname{cosec}\theta}{\cot^2\theta}$

$\equiv \frac{2}{\sin\theta}\cdot\frac{\sin^2\theta}{\cos^2\theta} \equiv \frac{2\sin\theta}{\cos^2\theta} \equiv \frac{2}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta}$

$\equiv 2\sec\theta\tan\theta \equiv$ R.H.S.

f L.H.S. $\equiv \frac{\sec^2\theta-\tan^2\theta}{\sec^2\theta} \equiv \frac{1}{\sec^2\theta} \equiv \cos^2\theta \equiv$ R.H.S.

TAP TO RETURN

7 - Answers

(a) $4x - 10 > 32, x > 10.5$
(b) $x(x - 5) < 104, -8 < x < 13$
(c) $10.5 < x < 13$

TAP TO RETURN

8 - Answers

c) p=2, q=-40

d) $x^3 + 3x - 18x - 40 = (x - 4)(x + 2)(x + 5)$

TAP TO RETURN

9 - Answers

a) $\frac{1}{2x} + \frac{2}{(x-1)} - \frac{1}{(x-1)^2}$

b) $1 + \frac{4}{x^2-4}$

c) $\frac{2}{2(x+1)} - \frac{1}{x+2} + \frac{1}{2(x+3)}$

TAP TO RETURN

10 - Answers

(b) $\frac{7}{2}\pi$; 3π

(c) $4 - 2x$

(d) $16 - 16x + 4x^2$

(f) $x = 1.143$

(g) A = 6.857

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 9A

11 - Answers

(a)(1,-3) local minimum; (-3,-35) local minimum; $\left(-\frac{1}{4}, \frac{357}{256}\right)$ local maximum

(b) Check Desmos

TAP TO RETURN

12 - Answers

(a) Check on desmos
(b) -4 and -7

TAP TO RETURN

13 - Answers

a) Use Desmos to check

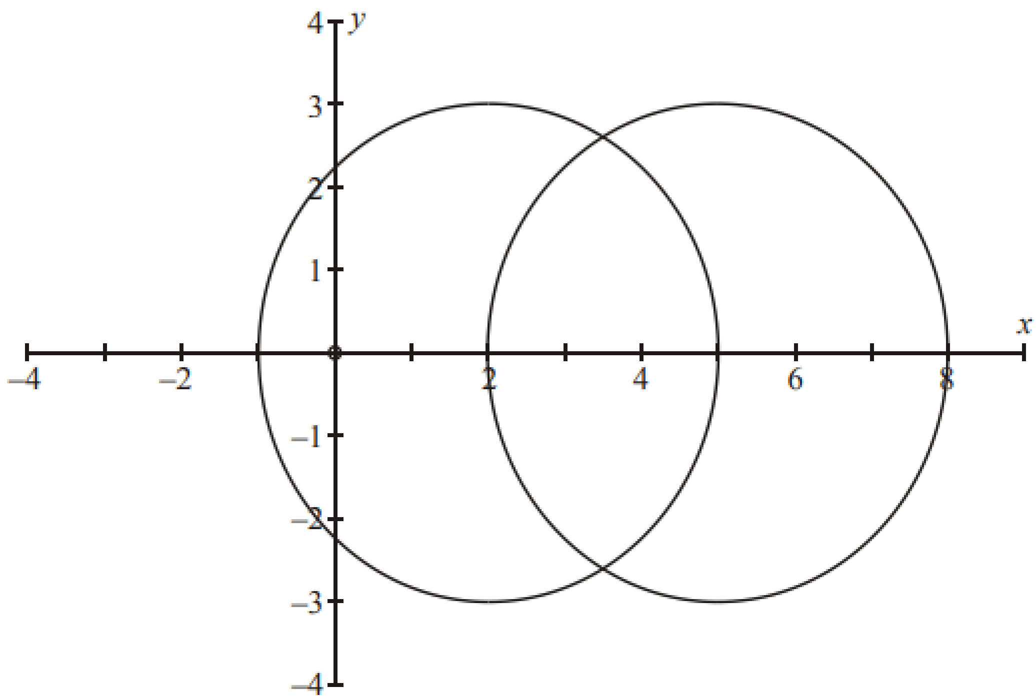
c) $x = 51.3$
$x = 128.7$ (accept 129)

TAP TO RETURN

14- Answers

(a) $r = 3$ (both circles)
Centres are at (2, 0) and (5, 0)

(b)



c) $3\sqrt{3}$

TAP TO RETURN