BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 6A

1

Use the discriminant to find the range of values of k (sketch the quadratic graph of k and show clearly where the graph is consistent with the inequality) if the equation:

(a) $x^2 + 2kx - k = 0$ has no real roots

(b) $x^2 + 2k = 0$ has a repeated real root

(c) $(x - k)^2 + 3k = 0$ has distinct real roots

TAP FOR ANSWERS

Sketch the following curves of $y = f(x)$, stating the equations of the asymptotes and the coordinates of any axis intercept:

(a) $f(x) = 2 + \frac{1}{x}$

(b) $f(x) = \frac{1}{x-3}$

(c) $f(x) = \frac{2}{x}$

TAP FOR ANSWERS

3

(a) By rationalising the denominator, write these in the form $A + B\sqrt{C}$:

(i) $\frac{2}{\sqrt{3}}$ (ii) $\frac{2}{1+\sqrt{3}}$ (iii) $\frac{3+\sqrt{5}}{2-\sqrt{5}}$

(b) Convert to the following in the form $ax^n + \beta x^m$

(i) $y = \frac{3x+2}{\sqrt{x}}$ (ii) $y = \frac{2\sqrt{x}-1}{x}$ (iii) $y = \frac{6x\sqrt{x}-3}{2x^2}$

TAP FOR ANSWERS

4

(a) The quadratic $0 = px^2 + px - x - 3$ has a discriminant equal to – 24. Find the value of p

(b) Calculate the possible values of k if $(k+1)x^2 + kx + k + 1 = 0$ has a repeated root

TAP FOR ANSWERS

5

(a) Use the factor theorem to show that $(x - 1)$ is a factor of $x^3 + x^2 - 9x + 7$.

(b) Use the fact that $(x + 2)$ is a factor of $x^3 + 3x^2 + kx + 8$ to find the value of k.

TAP FOR ANSWERS

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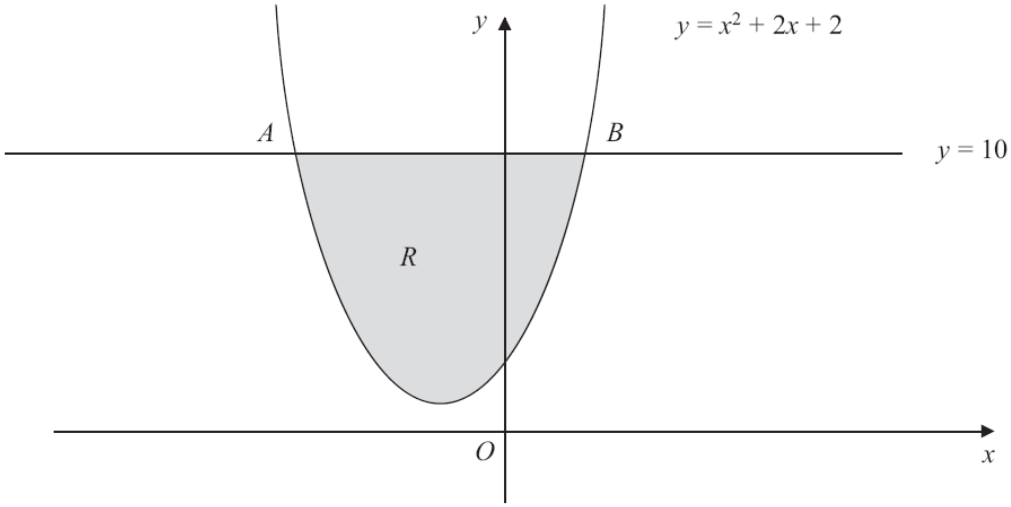
6

The line with equation $y = 10$ cuts the curve with equation $y = x^2 + 2x + 2$ at the points A and B as shown in Figure 1. The figure is not drawn to scale.

(*a*) Find by calculation the x-coordinate of A and the x-coordinate of B.

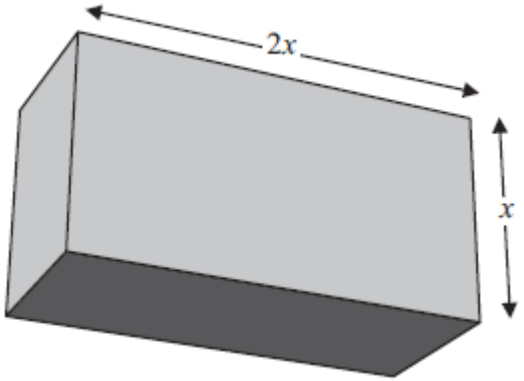
The shaded region R is bounded by the line with equation $y = 10$ and the curve as shown in Figure 1.

(b) Use calculus to find the exact area of R.



TAP FOR ANSWERS

7



A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.

The volume of the cuboid is 81 cubic centimetres.

(*a*) Show that the total length, L cm, of the twelve edges of the cuboid is given by $L = 12x + \frac{162}{x^2}$.

(*b*) Use calculus to find the minimum value of L.

(*c*) Justify, by further differentiation, that the value of L that you have found is a minimum.

TAP FOR ANSWERS

8

The circle C has equation $x^2 + 3x + y^2 + 6y = 3x - 2y - 7$

(a) Find the centre and radius of the circle.

(b) Find the points of intersection of the circle and the y-axis

(c) Show that the circle does not intersect the x-axis

TAP FOR ANSWERS

9

Solve the following equations on the interval $0 \leq x \leq 360°$

(a) $\sin(x - 90°) = \frac{1}{\sqrt{2}}$

(b) $\cos(x - 180°) = \frac{1}{\sqrt{2}}$

(c) $\tan(3x) = -1$

TAP FOR ANSWERS

Prove the following identities:

(a) $\tan x + \cot x \equiv \sec x \operatorname{cosec} x$

(b) $\sec x - \cos x \equiv \tan x \sin x$

TAP FOR ANSWERS

11

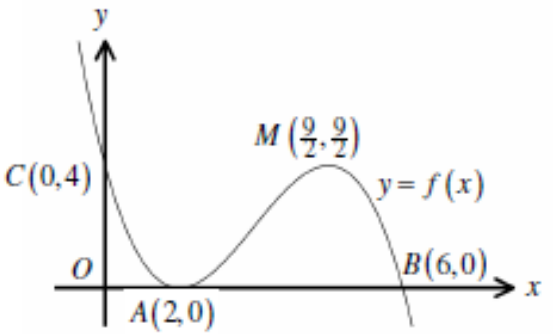
Solve the following equation on the interval $0 \leq \theta \leq 2\pi$. Where possible, give exact answers. Otherwise give your answers to 3sf:

$$\tan^2\theta + 2\sec\theta = 7$$

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 6A

12



The figure above shows the graph of the curve with equation $y = f(x)$. The curve meets the x axis at $A(2,0)$ crosses the x axis a the point $B(6,0)$, and crosses the y axis at the point $C(0,4)$. The curve has a maximum at $M\left(\frac{9}{2},\frac{9}{2}\right)$

Sketch on separate diagrams the graphs of

(a) $y = 2f(x)$

(b) $y = f(x+2)$

(c) $y = f\left(\frac{1}{2}x\right)$

TAP FOR ANSWERS

13

(a) Given that $P = 2x + \frac{100}{x}$ ($x > 0$), find the value of x for which P is minimum. Use the second derivative to justify your answer.

(b) $f(x) = x^3 + 4x^2 - 3x + 7$

Find the set of values of x for which $f(x)$ is increasing

TAP FOR ANSWERS

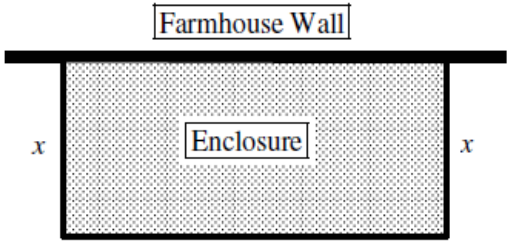
14

The figure above shows the plan of a rectangular enclosure to be built next to a farmhouse. One of the farmhouse's walls will form one of the sides of the enclosure and 25 metres of fencing will form the other three sides.

The width of the enclosure is x metres, as shown in the figure.

The area of the enclosure must be at most 75 m^2.

Given further that the width of the enclosure must be at least 3 metres but no more than 9 metres, determine the range of possible values of x.



TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 6A

1 - Answers

(a) $-1 < k < 0$

(b) $k = 0$

(c) $k < 0$

TAP TO RETURN

2 - Answers

(a) asymptotes $y = 2$, $x = 0$. Intercept $(-\frac{1}{2}, 0)$

(b) asymptotes $y = 0$, $x = 3$. Intercept $(0, -1/3)$

(c) asymptotes $y = 0$, $x = 0$. No intercepts.

TAP TO RETURN

3 - Answers

(a) Compare the original expression & your answer on a calculator.

(b)

(i) $3x^{\frac{1}{2}} + 2x^{\frac{-1}{2}}$

(ii) $2x^{\frac{-1}{2}} - x^{-1}$

(iii) $3x^{\frac{-1}{2}} - \frac{3}{2}x^{-2}$

TAP TO RETURN

4 - Answers

(a) $p = -5$

(b) $k = -2$ or $\frac{-2}{3}$

TAP TO RETURN

5 - Answers

(a) Proof

(b) $k = 6$

TAP TO RETURN

6 - Answers

(a) x=-4, 2 (b) R=36

TAP TO RETURN

7 - Answers

54cm

TAP TO RETURN

8 - Answers

(a) Centre (0, -4) and radius = 3

(b) (0, -1) and (0, -7)

(c) Students' own work. Equation $x^2 = -7$ has no real solutions.

TAP TO RETURN

9 - Answers

(a) 135 ° , 225 °

(b) 135°, 225 °

(c) 45°, 105°, 165°, 225°, 285°, 345°

TAP TO RETURN

10 - Answers

Proof

TAP TO RETURN

11 - Answers

$\frac{\pi}{3}, \frac{5\pi}{5}, 1.82, 4.46$

TAP TO RETURN

12 - Answers

Sketches – discuss in class

TAP TO RETURN

13 - Answers

(a) $5\sqrt{2}$

(b) $x < -3, OR\ x > \frac{1}{3}$

TAP TO RETURN

14 - Answers

$3 < x < 5$ or $\frac{15}{2} < x < 9$

TAP TO RETURN