BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 5A

1

Sketch the following curves of $y = f(x)$, showing the coordinates of the turning point and any points of intersection with the coordinate axes:

(a) $f(x) = 4x - x^2$

(b) $f(x) = 16 - x^2$

(c) $f(x) = 2x^2 + 4x$

TAP FOR ANSWERS

Find $\frac{dy}{dx}$ when:

(a) $y = \frac{3x+2}{2x^2}$

(b) $y = \frac{5-2\sqrt{x}}{x^3}$

(c) $y = \frac{1-2x}{x\sqrt{x}}$

TAP FOR ANSWERS

3

(a) Given $f(x) = (x) = 2x^2 - 3x + 4$ and $g(x) = 4x + 1$
Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same axes

(b) Find the coordinates of any points of intersection.

(c) Write down the set of values for which $f(x) > g(x)$

(d) Write down the set of values for which $f(x) < g(x)$

TAP FOR ANSWERS

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4

The number of bacteria in a refrigerated food is given by

$$N = 20T^2 + 120 - 20T, \qquad T > 0$$

where T is the temperature of food in ^{0}C

(a) Express N in the form $p(T-q)^2+r$ where p, q, r are to be found

(b) What is the minimal number of bacteria and what is the temperature when this occurs?

(c) Find the temperature to 3 sf when the number of bacteria is 140

(d) Explain why $T > 0$.

TAP FOR ANSWERS

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The curve C_1 has equation $y = -\frac{a}{x^2}$ where a is a positive constant. The curve C_2 has the equation $y = x^2(3x + b)$ where b is a positive constant.

(a) Sketch $C_1$and C_2 on the same set of axes, showing clearly the coordinates of any point where the curves touch of cross the axes.

(b) Using your sketch state, giving reasons, the number of solutions to the equation $x^4(3x + b) + a = 0$.

TAP FOR ANSWERS

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(i) Solve the following equations on the interval $0 \leq x \leq 2\pi$

(a) $\sin 3x = -1$

(b) $\cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{2}}$

(c) $\tan\left(x + \frac{3\pi}{2}\right) = -1.4$

(ii) Prove the following identity:

$$\sec^2 x - \operatorname{cosec}^2 x \equiv \tan^2 x - \cot^2 x$$

TAP FOR ANSWERS

7

(a) Find an equation of the tangent and the normal at the point where $x = 2$ on the curve with equation $y = \frac{8}{x} - x + 3x^2, x > 0$.

(b) The normals to the curve $2y = 3x^3 - 7x^2 + 4x$, at the points $O(0, 0)$ and $A(1,0)$, meet at the point N.

(i) Find the coordinates of N.

(ii) Calculate the area of triangle OAN.

TAP FOR ANSWERS

8

Solve, for $0 \leq \theta < 180°$, the equation

$2 \cot^2 \theta - 9 \operatorname{cosec} \theta = 3$,

giving your answers to 1 decimal place.

TAP FOR ANSWERS

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If $\frac{dy}{dx} = x^2, y = \frac{x^3}{3} + c$ and if $\frac{dy}{dx} = 4x^5$ $y = \frac{2x^3}{3} + c.$

In general when $\frac{dy}{dx} = ax^n, y = \frac{ax^{n+1}}{n+1} + c$

We set out an integration like this: $\int x^2 dx = \frac{x}{3} + 6.$

Integrate the following:

(a) $\int x^3 dx =$

(b) $\int 3x^2 dx =$

(c) $\int \frac{2}{5} x^4 \, dx =$

(d) $\int 5x^{\frac{3}{2}} \, dx =$

(e) $\int 2x^{-2} dx =$

You can differentiate your answers to check they are correct.

TAP FOR ANSWERS

10

(a) Evaluate the following

$$\lim_{\delta x \to 0} \sum_{x=\frac{1}{2}}^{1} \frac{4-x}{2x^3} \delta x$$

(b) The curve with equation $y = f(x)$ passes through the point $(8, 7)$. Given that $f'(x) = 4x^{\frac{1}{3}} - 5$, find $f(x)$.

TAP FOR ANSWERS

11

(i) Use differentiation from first principles to prove that $\frac{d}{dx}(1-2x^3) = -6x^2$

(ii) A cuboid has base of width x cm, length $2x$ cm and height h cm. Its volume is 72 cm^2.

(a) Show that is surface area is given by $SA = 4x^2 + \frac{216}{x}$.

b) Find the value of x for which the surface area is a minimum.

c) Prove that the answer to part (b) gives a minimum surface area.

TAP FOR ANSWERS

12

(i) Use set notation to describe the set of values of x for which:

(a) $x^2 - 7x + 10 < 0$ and $3x + 5 < 17$
(b) $x^2 - x - 6 > 0$ and $10 - 2x < 5$
(c) $4x^2 - 3x - 1 < 0$ and $4(x + 2) < 15(x + 7)$
(d) $2x^2 - x - 1 < 0$ and $14 < 3x - 2$
(e) $x^2 - x - 12 > 0$ and $3x + 17 > 2$
(f) $x^2 - 2x - 3 < 0$ and $x^2 - 3x + 2 > 0$

(ii) Find the possible values of k for the quadratic equation $2kx^2 + 5kx + 5k - 3 = 0$ to have real roots.

(iii) A straight line has equation $y = 2x - k$ and a parabola has equation $y = 3x^2 + 2kx + 5$ where k is a constant. Find the range of values of k for which the line and parabola do not intersect.

TAP FOR ANSWERS

13

The line l_1 has equation $x + 2y - 1 = 0$. The line l_2 is perpendicular to l_1 and passes through the point $A(1, 5)$.

(a) Show that l_1 and l_2 cross at the point $(-1, 1)$

The points $B(-3, 2)$ and $C(3, -1)$ lie on l_1.

(b) Find the area of the triangle with vertices A, B, C.

TAP FOR ANSWERS

14

$f(x) = x^2 - 2x - 8$

(a) Sketch the graph of $y = f(x)$

(b) On the same set of axes, sketch the graph of $y = f'(x)$

(c) Explain why the x-coordinate of the turning point of $y = f(x)$ is the same as the x-coordinate of the point where the graph of $y = f'(x)$ crosses the x-axis

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 5A

1 - Answers

(a) intercepts (0, 0), (4, 0), turning point (2, 4)

(b) intercepts (–4, 0), (4, 0), (0, 16) turning point (0, 16)

(c) intercepts (–2, 0), (0, 0) turning point (–1, –2)

TAP TO RETURN

2 - Answers

(a) $-\frac{3}{2}x^{-2} - 2x^{-3}$

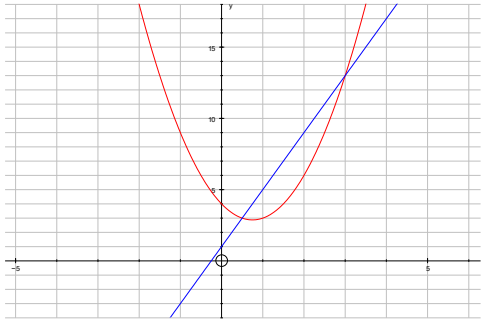
(b) $-15x^{-4} + 5x^{-\frac{7}{2}}$

(c) $-\frac{3}{2}x^{-\frac{5}{2}} + x^{-\frac{3}{2}}$

TAP TO RETURN

3 - Answers

(a)



(b) $\left(\frac{1}{2}, 3\right)$ (3,13)

(c) $x < \frac{1}{2}$ or $x > 3$

(d) $\frac{1}{2} < x < 3$

TAP TO RETURN

4 - Answers

(a) $p = 20, q = 0.5, r = 115$

(b) Min = 115 when T = 0.5

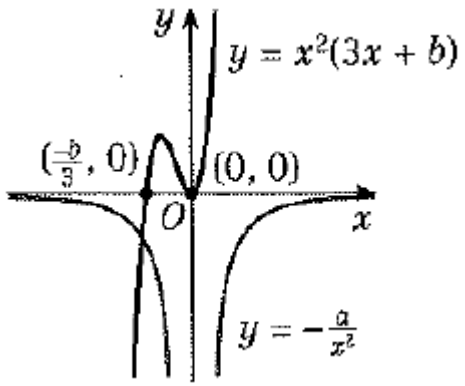
(c) 1.62°C

(d) The amount of bacteria doesn't increase if the temperature goes down

TAP TO RETURN

5 - Answers

(a)



(b) 1; only one intersection of the two curves.

TAP TO RETURN

6 - Answers

(ii) (a) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(b) $\frac{\pi}{2}$

(c) 0.62, 3.76

TAP TO RETURN

7 - Answers

(a) $9x - y - 4 = 0$ and $9y + x - 128 = 0$

(b)

(i) $\left(\frac{4}{5}, -\frac{2}{5}\right)$

(ii) $\frac{1}{5}$

TAP TO RETURN

8 - Answers

$\theta = 11.5°, 168.5°$

TAP TO RETURN

9 - Answers

(a) $\frac{x^4}{4} + c$

(b) $x^3 + c$

(c) $\frac{2}{25}x^5 + c$

(d) $2x^{\frac{5}{2}} + c$

(e) $-2x^{-1} + c$

TAP TO RETURN

10 - Answers

(a) $\frac{5}{2}$

(b) $f(x) = 3x^{\frac{4}{3}} - 5x - 1$

TAP TO RETURN

11 - Answers

(b) $x = 3$

TAP TO RETURN

12 - Answers

(i) (a) $\{x: 2 < x < 4\}$
(b) $\{x: x > 3\}$
(c) $\left\{x: -\frac{1}{4} < x < 1\right\}$
(d) No values
(e) $\{x: (-5 < x < -3) \cup (x > 4)\}$
(f) $\{x: (-1 < x < 1) \cup (2 < x < 3)\}$

(ii) $0 \leq k \leq \frac{8}{5}$

(iii) $-2 < k < 7$

TAP TO RETURN

13 - Answers

(a) (-1,1)

(b) 15

TAP TO RETURN

14 - Answers

(a) Sketches

(b) Discuss in class

(c) Discuss in class

TAP TO RETURN