BHASVIC MαTHS

A1 DOUBLES ASSIGNMENT 4A

1

The points A and B have coordinates (–2, –7) and (3, 8) respectively.

(a) Find the coordinates of the point at which the line through AB crosses the x-axis.

The mid-point of AB lies on the line with equation $y = kx$, where k is a constant.

(b) Find the value of k.

TAP FOR ANSWERS

Differentiate the following (remember to convert to the form $ax^n + \beta x^m$ first)

(a) $\frac{3x+2}{\sqrt{x}}$

(b) $\frac{2\sqrt{x}-1}{x}$

(c) $\frac{x^2-1}{4\sqrt{x}}$

TAP FOR ANSWERS

3

Given

$$f(x) = (x-3)(x-2)(x-1)$$

The factors of $f(x)$ are $(x-3)$ and $(x-2)$ and $(x-1)$

It is also clear that $f(3) = 0, f(2) = 0$ and $f(1) = 0$.

From this example we can infer that for any polynomial $f(x)$, if an α can be found such that $f(\alpha) = 0$, then $(x-\alpha)$ is a factor of $f(x)$

Try subbing in factors of -6 to find the three factors of the function below

$$f(x) = x^3 - 7x - 6$$

Hence write $f(x)$ in the form $(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)$

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 4A

Find the equation of the normal to the curve at the point where x=1

(a) $y = x^2 - 3x$

(b) $y = \frac{7}{x^3}$

(c) $y = \frac{4-3x^2}{x}$

(d) Find the equation of the normal to $y = 3x^2 - x + 1$ at $x = 0$

(e) Find the equation of the normal to $y = 2x + \frac{1}{x}$ at $x = \frac{1}{2}$

(f) Find the equation of the normal to $y = x^3 + x^2$ at $x = 1$

TAP FOR ANSWERS

BHASVIC MαTHS

A1 DOUBLES ASSIGNMENT 4A

A curve has the equation $y = x + \frac{3}{x}, x \neq 0.$

The point P on the curve has x coordinate 1.

(a) Show that the gradient of the curve at P is – 2.

(b) Find an equation for the normal to the curve at P, giving your answer in the form $y = mx + c$.

(c) Find the coordinates of the point where the normal to the curve at P intersects the curve again

TAP FOR ANSWERS

Find the values of x from 0 to 2π inclusive of the following equations. Give the answers in terms of π where possible, otherwise to 2d.p.

(a) $\tan x = \frac{1}{\sqrt{3}}$

(b) $\sin x = 0.7$

(c) $\cos\left(x + \frac{\pi}{3}\right) = \frac{1}{2}$

(d) $\sin\left(x - \frac{\pi}{6}\right) = 1$

(e) $\cos x = -\frac{1}{\sqrt{3}}$

(f) $\tan x = 0.2$

TAP FOR ANSWERS

Find the values of x from 0 to 2π inclusive of the following equations, giving the answers in terms of π.

(a) $\sin^2 x = \frac{1}{4}$

(b) $\tan^2 x = \frac{1}{3}$

(c) $\sin 2x = \frac{1}{2}$

(d) $\tan 2x = -1$

(e) $\cos 3x = \frac{\sqrt{3}}{2}$

(f) $\sin 3x = -1$

TAP FOR ANSWERS

8

Solve the following equations on the interval $0 \leq x \leq 360$

(a) $\sin(x - 45) = \frac{\sqrt{3}}{2}$

(b) $\cos(-x) = 0.2$

(c) $\tan(x - 180) = -\sqrt{3}$

TAP FOR ANSWERS

Prove the following identities:

(a) $\sec x + \tan x \equiv \frac{1}{\sec x - \tan x}$

(b) $\frac{\tan x \sec x}{1+\tan^2 x} \equiv \sin x$

(c) $\cot x + \tan x = \sec x \, cosec \, x$

TAP FOR ANSWERS

Prove the following identities: set out your proof correctly

(a) $\cos\theta + \sin\theta\tan\theta \equiv \sec\theta$

(b) $\sin^2 x(1 + \sec^2 x) \equiv \sec^2 x - \cos^2 x$

(c) $\frac{\sin\theta}{1+\cos\theta} + \frac{1-\cos\theta}{\sin\theta} \equiv \frac{2\sin\theta}{1+\cos\theta}$

TAP FOR ANSWERS

Prove that the equation $\frac{4x+3}{2x-1}+\frac{6x+1}{2x+3}=3$ has no real roots

TAP FOR ANSWERS

12

The normals to the curve $2y = 3x^3 - 7x^2 + 4x$, at the points $O(0,0)$ and $A(1,0)$, meet at the point N.
Find the coordinates of N.
Calculate the area of triangle OAN.

TAP FOR ANSWERS

(a) Solve the equation $1 + \tan^2 x = 3\tan x - 1$ on the interval $\boldsymbol{-\pi \leq x \leq \pi}$

(b) Use the identity $\frac{\sin x}{\cos x} \equiv \tan x$ to solve the equation $\sqrt{3}\cos x = \sin x$ on the interval $0 \leq x \leq 2\pi$

(c) Use the identity $\sin^2 x \equiv 1 - \cos^2 x$ to solve the equation $3 - 3\cos x = 2\sin^2 x$ on the interval $0 \leq x \leq 2\pi$

(d) Solve the following equation on the interval $0 \leq \theta \leq 2\pi$. Give exact answers.

$$\sec^2 x + \tan x - 1 = 0$$

TAP FOR ANSWERS

$$f(x) = \frac{1}{x}$$

(a) Given that $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, show that $f'(x) = \lim_{h\to 0} \frac{-1}{x^2+xh}$

(b) Deduce that $f'(x) = -\frac{1}{x^2}$

TAP FOR ANSWERS

1 - Answers

(a) line AB is $3x - y - 1 = 0$ so coordinate is $\left(\frac{1}{3}, 0\right)$

(b) $k = 1$

TAP TO RETURN

2 - Answers

(a) $\frac{3}{2}x^{-\frac{1}{2}} - x^{-\frac{3}{2}}$

(b) $-x^{-\frac{3}{2}} + x^{-2}$

(c) $\frac{3}{8}x^{\frac{1}{2}} + \frac{1}{8}x^{-\frac{3}{2}}$

TAP TO RETURN

3 - Answers

$f(x) = (x+1)(x+2)(x-3)$

TAP TO RETURN

4 - Answers

(a) $x - 3 - y = 0$

(b) $x - 21y + 146 = 0$

(c) $x - 7y + 6 = 0$

(d) $x - y + 1 = 0$

(e) $2x - 4y + 11 = 0$

(f) $x + 5y - 11 = 0$

TAP TO RETURN

5 - Answers

(b) $y = \frac{1}{2}x + \frac{7}{2}$

(c) $\left(6, \frac{13}{2}\right)$

TAP TO RETURN

6 - Answers

(a) $\frac{\pi}{6}, \frac{7\pi}{6}$

(b) 0.78, 2.37

(c) $0, \frac{4\pi}{3}, 2\pi$

(d) $\frac{2\pi}{3}$

(e) 2.19, 4.10

(f) 0.20, 3.34

TAP TO RETURN

7 - Answers

a) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

b) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

c) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

d) $\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

e) $\frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}, \frac{35\pi}{18}$

f) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

TAP TO RETURN

8 - Answers

(a) 105°, 165 °

(b) 78.5 °, 281.5 °

(c) 120 °, 300 °

TAP TO RETURN

9 - Answers

Proof

TAP TO RETURN

10 - Answers

Proof

TAP TO RETURN

11 - Answers

Proof

TAP TO RETURN

12 - Answers

a) $\left(\frac{4}{5}, -\frac{2}{5}\right)$

b) $\frac{1}{5}$

TAP TO RETURN

13 - Answers

(a) – 2.03, 1.11, $-\frac{3\pi}{4}, \frac{\pi}{4}$

(b) $\frac{\pi}{3}, \frac{4\pi}{3}$

(c) $0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$

(d) $0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi$

TAP TO RETURN

14 - Answers

Proof

TAP TO RETURN