BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 3A

1. Use the rule that if $y = ax^n$, then the gradient is given by $\frac{dy}{dx} = na\,x^{n-1}$ to find the gradient of the following graphs at the point where $x = 3$

(a) $y = x^2$
(b) $y = 3x^2$
(c) $y = 4x^3$
(d) $y = 8x^5$

2. In each of the following y is given as a function of x. Find the derived function $\frac{dy}{dx}$

(a) $y = (2x^2 + 3)(x + 1)$ *(Hint: Expand first!)*
(b) $y = \sqrt[5]{x}$
(c) $y = 2x^{-5}$
(d) $y = x^2 - x^{-2}$
(e) $y = x(x^2 - 3)$
(f) $y = \frac{x^3 - 1}{2x}$

TAP FOR ANSWERS

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Sketch the following, stating the x and y intercepts and the equations of any asymptotes:

(a) $y = (x-3)(x+3)(x-2)(x+2)$
(b) $y = x(x+3)^2(x-2)$
(c) $y = \frac{1}{x-1} + 2$
(d) $y = 64x - 9x^3$
(e) $y = \frac{2}{x^2}$
(f) $y = \sin 3x$
(g) $y = \cos\left(\frac{x}{2}\right)$
(h) $y = \tan\left(x + \frac{3\pi}{2}\right)$

TAP FOR ANSWERS

3

A quadratic graph $y = ax^2 + bx + c$ has a minimum point at $(4, -3)$ and passes through the point $(5,0)$. Find the values of a, b and c.

TAP FOR ANSWERS

4

One pair of solutions for the simultaneous equations
$y = kx + 3$
$2x^2 - xy = -7$

Is (1, m) where k and m are constants.

(a) Find the values of k and m .
(b) Find the second pair of solutions for the simultaneous equations

TAP FOR ANSWERS

5

(a) A line passes through points $(p, 3)$, $(p + 2, 5)$ and $(1, 2)$ where $p > 0$. Find the value of p.

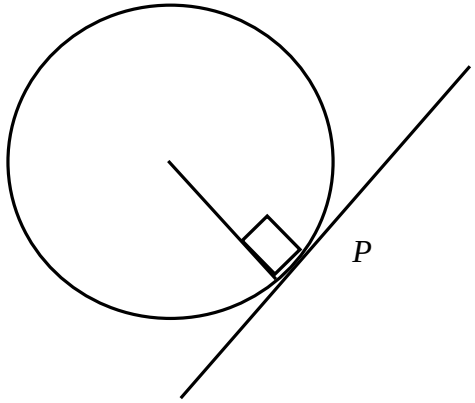
(b) The midpoint of $(5, p)$ and $(q, 10)$ is $(6, 6)$. Find the value of p and q.

TAP FOR ANSWERS

6

Remember that the tangent to a circle at P is always perpendicular to the radius joining P to the centre of the circle.

Use this information to find the equation of the tangent to the circle $(x + 2)^2 + (y - 2)^2 = 73$ at the point (1,-6), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.



TAP FOR ANSWERS

7

Prove, from first principles, that the derivative of $10x^2$ is $20x$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

TAP FOR ANSWERS

8

For the curve C with equation $y = x^4 - 8x^2 + 3$,

(a) find $\frac{dy}{dx}$

The point A, on the curve C, has x-coordinate 1.

(b) Find an equation for the normal to C at A, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

TAP FOR ANSWERS

9

$$\frac{(x^2-3)^2}{x^3}, x \neq 0.$$

(*a*) Show that $f(x) \equiv x - 6x^{-1} + 9x^{-3}$.

(*b*) Hence, or otherwise, differentiate $f(x)$ with respect to x.

TAP FOR ANSWERS

10

(a) Find the coordinates for which the tangent to the curve $y = x^2 - 9x + 4$ is perpendicular to the line $y - x - 1 = 0$

(b) Find the possible values of c for which the line $24x + 3y + c = 0$ is tangent to the curve $f(x) = \frac{1}{3}x^3 + \frac{9}{x}$

TAP FOR ANSWERS

11

(a) Find the equation of the tangent to $f(x) = \frac{1}{4x}$ at the point $x = 2$

(b) At which points is the tangent to $f(x) = \frac{1}{4x}$ parallel to the line $x + 4y + 12 = 0$.

TAP FOR ANSWERS

12

(*a*) Find the equations of the tangent and normal to the curve $y = \frac{4}{x}$ at the point where $x = 2$.

(*b*) Show that the tangent does not intersect the curve again.

(*c*) Show that the normal does intersect the curve again, and find the coordinates of the point of intersection.

TAP FOR ANSWERS

13

The normals to the curve $2y = 3x^3 - 7x^2 + 4x$ at the origin (0,0) and the point A(1,0) meet at N. Show the coordinates of N are $(\frac{4}{5}, -\frac{2}{5})$

TAP FOR ANSWERS

14

(a) Find the equation of the line l in the form ax + by + c = 0, which goes through the point P (5, 9) and has gradient 2.

(b) The circle C has equation $x^2 - 8x + y^2 - 4y + 15 = 0$. A line is a tangent to a circle if it touches it once only (rather than intersecting it twice or not touching it at all). Use this information to show that l is a tangent to C.

(c) Find, as a surd, the length from P to where l touches the circle.

TAP FOR ANSWERS

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1 - Answers

1.

(a) 6

(b) 18

(c) 108

(d) 3240

2.

(a) $6x^2 + 4x + 3$

(b) $\frac{1}{5}x^{-\frac{4}{5}}$

(c) $-10x^{-6}$

(d) $2x + 2x^{-3}$

(e) $3x^2 - 3$

(f) $x + \frac{1}{2}x^{-2}$

TAP TO RETURN

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2 - Answers

In the library computers you can plot the graphs on ‘autograph’. On your phone you could use the free app ‘desmos’. Or, use your graphical calculator to check. It is important you try these yourself first, don’t go straight to the answers! And **don’t forget to label the intercepts & asymptotes**.

TAP TO RETURN

3 - Answers

$a = 3, b = -24, c = 45$

TAP TO RETURN

4 - Answers

(a) $k = 6, m = 9$

(b) $\left(\frac{-7}{4}, \frac{-15}{2}\right)$

TAP TO RETURN

5 - Answers

(a) $p = 2$,

(b) $p = 2$, $q = 7$

TAP TO RETURN

6 - Answers

$3x - 8y - 51 = 0$

TAP TO RETURN

7 - Answers

Proof – use

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

TAP TO RETURN

8 - Answers

$(a)\frac{dy}{dx} = 4x^3 - 16x$

$(b)x - 12y - 49 = 0$

TAP TO RETURN

9 - Answers

(a)Show

$(b) f'(x) = 1 + 6x^{-2} - 27x^{-4}$

TAP TO RETURN

10 - Answers

(a) $(4, -16)$

(b) $c = -52, c = 52$

TAP TO RETURN

11 - Answers

(a) $x + 16y - 4 = 0$

(b) (1, ¼) (-1, -¼)

TAP TO RETURN

12 - Answers

(a) $x + y - 4 = 0$, $x - y = 0$ (c) $(-2, -2)$

TAP TO RETURN

13 - Answers

Show that

TAP TO RETURN

14 - Answers

(a) $2x - y - 1 = 0$ (b) discriminant = 0 (c) $3\sqrt{5}$

TAP TO RETURN