1

By completing the square, sketch the graphs of the following equations. For each graph, show the coordinates of the points where the graph crosses the coordinate axes, and write down the coordinate of the turning point and the equation of the line of symmetry.

(a) $y = 4x^2 - 20x + 16$ (b) $y = 0.5x^2 + 0.1x - 0.04$

(c) $y = -x^2 + 10x + 1$

2

i) Sketch the following curves of y = f(x), stating the equations of the asymptotes and the coordinates of any axis intercepts:

(a)
$$f(x) = 2 + \frac{1}{x}$$

(b)
$$f(x) = \frac{1}{x-3}$$

(c)
$$f(x) = \frac{2}{x}$$

ii) Sketch the following curves, showing any relevant features such as axis intercepts

(a)
$$y = (x - 1)^2 + 2$$

(b) $y = -\frac{1}{x+2}$
(c) $y = (x + 2)^2(x - 3)$
(d) $y = \frac{1}{3x}$

3

(a) Simplify
$$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$$
.

(b) Hence show that
$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \dots + \frac{1}{\sqrt{24}+\sqrt{25}} = 4$$

TAP FOR ANSWERS

4

Given the two points A(4, 7) and B(-2, 5):

- (a) Find the mid-point of *A* and *B*.
- (b) Find the exact distance AB. Leave your answer in the form $k\sqrt{10}$.
- (c) Find the equation of the line through *A* and *B*, giving your answer in the form ax + by + c = 0 where *a*, *b* and *c* are integers.
- (d) Find the area of the triangle with vertices at (0, -2), (0, 6) and (-2, -3)

5

The functions f and g are defined as for $x \in \mathbb{R}$ $f(x) = x^2 - 4x + 1$ g(x) = x + 1

- (a) The function h(x) is defined as f(x) + g(x) for $x \in \mathbb{R}$. Show that $h(x) = x^2 3x + 2$
- (b) Write h(x) in the form $(x p)^2 q$ where p and q are constants to be found.
- (c) Using the completed square form sketch the graph of y = h(x) showing all the coordinates where the graph crosses the axes.
- (d) Write down the coordinates of the turning point and the equation of the line of symmetry.

6

TAP FOR ANSWERS

In $\triangle PQR$, PQ = (x + 2)cm, PR = (5 - x)cm and $\angle QPR = 30^{\circ}$. The area of the triangle is Acm².

(a) Show that
$$A = \frac{1}{4}(10 + 3x - x^2)$$
.

(b) Use the method of completing the square, or otherwise, to find the maximum value of A, and give the corresponding value of *x*.

7

f(x) = (x - 1)(x - 2)(x + 1).

- (a) State the coordinates of the point at which the graph of y = f(x) intersects the y-axis.
- (b) The graph of y = af(x) intersects the y-axis at (0,-4). Find the value of a.
- (c) The graph of y = f(x + b) passes through the origin. Find three possible values of *b*.

8

Solve the simultaneous equations:

$$3^x = 9^{1-y}$$

 $x^2 + 4y^2 = 4$

TAP FOR ANSWERS



Solve the following simultaneous equations:

$$x^{2} - y^{2} + 5x = 41$$

5y - 4x = 1

TAP FOR ANSWERS

10

Use the discriminant to find the set of values of *k* for which:

- (a) $x^2 + 2kx k = 0$ has a repeated root
- (b) $x^2 + 2k = 0$ has no real roots
- (c) $(x-k)^2 + 3k = 0$ has distinct real roots

11

The points A and C lie on the y-axis and the point B lies on the x-axis as shown in the diagram below. The line through points A and B is perpendicular to the line through points B and C. Find the value of c.



12

TAP FOR ANSWERS

These sketches are graphs of quadratic functions in the form $y = ax^2 + bx + c$. Find the values of *a*, *b*, and *c* for each function:



13

A circle with centre (a,b) and radius *r* has equation $(x - a)^2 + (y - b)^2 = r^2$. Use this to answer the following questions:

(i) Write down the equation of each circle:
(a) Centre (3,2), radius 4
(b) Centre (-4,5), radius 6
(c) Centre (5, -6), radius 2√3
(d) Centre (2a, 7a), radius 5a
(e) Centre (-2√2, -3√2), radius 1

(ii) By completing the square in the x terms and the y terms, write the following circle equations in the form $(x - a)^2 + (y - b)^2 = r^2$, and hence state the centre and radius:

(a)
$$x^{2} + y^{2} - 2x + 8y - 8 = 0$$

(b) $x^{2} + y^{2} + 12x - 4y = 9$
(c) $x^{2} + y^{2} - 6y = 22x - 40$
(d) $x^{2} + y^{2} + 5x - y + 4 = 2y + 8$
(e) $2x^{2} + 2y^{2} - 6x + 5y = 2x - 3y - 3$

14

Sketch the graphs of the following equations. For each graph, show the coordinates of the points where the graph crosses the coordinate axes, and write down the coordinate of the turning point and the equation of the line of symmetry.

- (a) $y = 4x^2 20x + 16$
- (b) $y = 0.5x^2 + 0.1x 0.04$
- (c) $y = -x^2 + 10x + 1$

- (a) U shaped parabola. Cuts axes at (1,0), (4,0), (0,16). Turning point $(\frac{5}{2}, -9)$. Line of symmetry $x = \frac{5}{2}$
- (b) U shaped parabola. Cuts axes at $\left(\frac{1}{5}, 0\right), \left(-\frac{2}{5}, 0\right), (0, -0.04)$. Turning point $\left(-\frac{1}{10}, -\frac{9}{200}\right)$. Line of symmetry $x = -\frac{1}{10}$
- (c) \cap shaped parabola. Cuts axes at $(5 \pm \sqrt{26}, 0)$, (0,1). Turning point (5,26). Line of symmetry x = 5

2 - Answers

In the library computers you can plot the graphs on 'autograph'. On your phone you could use the free app 'desmos'. Or, use your graphical calculator to check. It is important you try these yourself first, don't go straight to the answers!

3 - Answers

Proof

4 - Answers

(a)	(1,	6)
` '	`		

(b) $2\sqrt{10}$

(c) x - 3y + 17 = 0

(d) 8

TAP TO RETURN

5 - Answers

b) $p = \frac{3}{2}, q = -\frac{1}{4}$

(c) U shaped parabola, cuts x axis at (1,0) and (2,0), cuts y axis at (0,2)

(d) TP $\left(\frac{3}{2}, \frac{-1}{4}\right)$ line of symmetry is $x = \frac{3}{2}$

$$A = \frac{49}{16}$$
, when $x = 1.5$

(a) (0,2)	
(b) -2	
(c) -1,1,2	TAP TO RETURN





a) $k = 0, -1$ (b) $k > 0$ (c) $k < 0$	
	TAP TO RETUR
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(a)
$$a = 1, b = -8, c = 15$$
 (b) $a = -1, b = 3, c = 10$

13 - Answers

TAP TO RETURN

i) (a)
$$(x - 3)^2 + (y - 2)^2 = 16$$

(b) $(x + 4)^2 + (y - 5)^2 = 36$
(c) $(x - 5)^2 + (y + 6)^2 = 12$
(d) $(x - 2a)^2 + (y - 7a)^2 = 25a^2$
(e) $(x + 2\sqrt{2})^2 + (y + 3\sqrt{2})^2 = 1$

ii) (a) Centre (1, -4), radius 5 (b) Centre (-6, 2), radius 7 (c) Centre (11, 3), radius $3\sqrt{10}$ (d) Centre $\left(-\frac{5}{2}, \frac{3}{2}\right)$, radius $\frac{5\sqrt{2}}{2}$ (e) Centre (2, -2), radius $\sqrt{\frac{13}{2}}$

- (a) U shaped parabola. Cuts x axis at (1,0) and (4,0). Cuts y axis at (0,16), TP $(\frac{5}{2}, -9)$, Line of symmetry $x = \frac{5}{2}$
- (b) U shaped parabola. Cuts x axis at $(\frac{1}{5}, 0)$ and $(\frac{-2}{5}, 0)$. Cuts y axis at (0, 0.04), TP $(\frac{-1}{10}, \frac{-9}{200})$ Line of symmetry $x = \frac{-1}{10}$
- (c) (c) \cap shaped parabola. Cuts x axis at $(5 \pm \sqrt{26})$. Cuts y axis at (0,1), TP (5,26) Line of symmetry x = 5