BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 20A

1

For each of the following,
(i) Find the binomial expansion up to and including the x^3 term.
(ii) State the range of values for x for which the expansion is valid

(a) $\sqrt{4+2x}$

(b) $\frac{1}{2+x}$

(c) $\frac{1}{(4-x)^2}$

(d) $\sqrt{9+x}$

(e) $\frac{1}{\sqrt{2+x}}$

(f) $\frac{5}{3+2x}$

(g) $\frac{1+x}{2+x}$

(h) $\sqrt{\frac{2+x}{1-x}}$

TAP FOR ANSWERS

$y = \mathrm{f}(x)$, where $\mathrm{f}(x) = x^2 \sin x - 2x + 1$. The points P, Q, and R are roots of the equation. The points A and B are stationary points, with x-coordinates a and b respectively.

(a) Show that the curve has a root in each of the following intervals:
(i) [0.6, 0.7]
(ii) [1.2, 1.3]
(iii) [2.4, 2.5]

(b) Explain why $x_0 = a$ is not suitable to use as a first approximation to α when applying the Newton-Raphson method to f(x).

(c) Using $x_0 = 2.4$ as a first approximation, apply the Newton-Raphson method to f(x) to obtain a second approximation. Give your answer to 3 decimal places.

TAP FOR ANSWERS

A small bus company provides a service for a small town and some neighbouring villages. IN a study of their service a random sample of 20 journeys was taken and the distances x, in kilometres, and journey times t, in minutes, were recorded. The average distance was 4.535 km and the average journey time was 15.15 minutes. Given that the PMCC is calculated to be 0.37, stating your hypotheses clearly test, at the 5% level, whether or not there is evidence of a positive correlation between journey time and distance.

TAP FOR ANSWERS

The probability of a telesales representative making a sale on a customer is 0.1.

(a) Find the probability that a telesales representative achieves
(i) No sales in 10 calls
(ii) More than 4 sales in 20 calls

Representatives are required to achieve a mean of at least 4 sales each day

(c) Find the least number of callas a representative should make each day, in order to achieve this requirement.

(d) Calculate the least number of calls that a representative meeds to make in a day for the probability of at least 1 sale, to exceed 0.98.

TAP FOR ANSWERS

A herbalist claims that a particular remedy is successful in curing a particular disease in 52% of cases.
A random sample of 25 people who took the remedy is taken.

(a) Find the probability that more than 12 people in the sample were cured.

A second random sample of 300 people was taken and 170 were cured.

(b) Assuming the herbalist's claim is true, use a suitable approximation to find the probability that at least 170 were cured.

(c) Using your answer to part (b), comment on the herbalist's claim.

TAP FOR ANSWERS

6

Solve the following equations on the interval $0 \leq \theta \leq 2\pi$. Give exact answers.

$$\sec^2 x + \tan x - 1 = 0$$

(b) Prove that for $0 \leq x \leq 1, \arccos x = \arcsin \sqrt{1 - x^2}$

(c) Give a reason why this result is not true for $-1 \leq x \leq 0$

TAP FOR ANSWERS

7

Two variables S and x satisfy the formula $S = 3 \times 7^x$

(a) Show that $\log S = \log 3 + x \log 7$

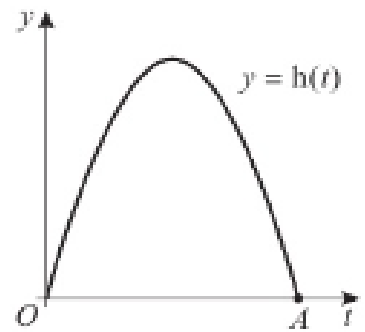
(b) The straight line graph of $\log S$ against x is plotted. Write down the gradient and the value of the intercept on the y axis.

TAP FOR ANSWERS

Ed throws a ball for his dog. The vertical height of the ball is modelled by the Function

$$h(t) = 40\sin\left(\frac{t}{10}\right) - 9\cos\left(\frac{t}{10}\right) - 0.5t^2 + 9, t \geq 0$$

$y = h(t)$ is shown in the diagram.



(a) Show that the t-coordinate of A is the solution to

$$t = \sqrt{18 + 80\sin\left(\frac{t}{10}\right) - 18\cos\left(\frac{t}{10}\right)}$$

To find an approximation for the t-coordinate of A, the iterative formula

$$t_{n+1} = \sqrt{18 + 80\sin\left(\frac{tn}{10}\right) - 18\cos\left(\frac{tn}{10}\right)}$$ is used

(b) Let $t_0 = 8$. Find the values of t_1, t_2, t_3 and t_4. Give your answers to 3 decimal places.
(c) Find h'(t).
(d) Taking 8 as a first approximation, apply the Newton-Raphson method once to h(t) to obtain a second approximation for the time when the height of the ball is zero. Give your answer to 3 decimal places.
(e) Hence suggest an improvement to the range of validity of the model.

TAP FOR ANSWERS

9

Use proof by contradiction to prove the statement ‘There are no integer solutions to the equation $x^2 - y^2 = 2$’

TAP FOR ANSWERS

10

(a) Express $\frac{8x+4}{(1-x)(2+x)}$ as partial fractions.

(b) Hence or otherwise expand $\frac{8x+4}{(1-x)(2+x)}$ in ascending powers of x as far as the term in x^2.

(c) State the set of values of x for which the expansion is valid.

TAP FOR ANSWERS

10b

(a) Express $-\frac{2x}{(2+x)^2}$ as partial fractions.

(b) Hence prove that $-\frac{2x}{(2+x)^2}$ can be expressed in the form $-\frac{1}{2}x + Bx^2 + Cx^3$ where constants B and C are to be determined.

(c) State the set of values for x for which the expansion is valid.

TAP FOR ANSWERS

Complete this old spec paper
https://www.madasmaths.com/archive/iygb_practice_papers/c3_practice_papers/c3_q.pdf

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 20A

Answers 1

(a) (i) $2+\frac{x}{2}-\frac{x^2}{16}+\frac{x^3}{64}$ (ii) $|x|<2$

(b) (i) $\frac{1}{2}-\frac{x}{4}+\frac{x^2}{8}-\frac{x^3}{16}$ (ii) $|x|<2$

(c) (i) $\frac{1}{16}+\frac{x}{32}-\frac{3x^2}{256}+\frac{x^3}{256}$ (ii) $|x|<4$

(d) (i) $3+\frac{x}{6}-\frac{x^2}{216}+\frac{x^3}{3888}$ (ii) $|x|<9$

(e) (i) $\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{8}x+\frac{3\sqrt{2}}{64}x^2-\frac{5\sqrt{2}}{256}x^3$ (ii) $|x|<2$

(f) (i) $\frac{5}{3}-\frac{10}{9}x+\frac{20}{27}x^2-\frac{40}{81}x^3$ (ii) $|x|<\frac{3}{2}$

(g) (i) $\frac{1}{2}+\frac{1}{4}x-\frac{1}{8}x^2+\frac{1}{16}x^3$ (ii) $|x|<2$

(h) (i) $\sqrt{2}+\frac{3\sqrt{2}}{4}x+\frac{15\sqrt{2}}{32}x^2+\frac{51\sqrt{2}}{128}x^3$ (ii) $|x|<1$

TAP TO RETURN

Answers 2

(a) $\mathrm{f}(0.6) = 0.0032 \ldots > 0,\ \mathrm{f}(0.7) = 0.0843 \ldots < 0$
Sign change implies root in the interval.
$\mathrm{f}(1.2) = -0.0578 \ldots < 0,\ \mathrm{f}(1.3) = 0.0284 \ldots > 0$
Sign change implies root in the interval.
$\mathrm{f}(2.4) = 0.0906 \ldots > 0, \mathrm{f}(2.5) = -0.2595 \ldots < 0$
Sign change implies root in the interval.

(b) It's a turning point, so $\mathrm{f}'(x) = 0$, and you cannot divide by zero in the Newton-Raphson formula.

(c) 2.430

3 - Answers

No evidence of correlation

TAP TO RETURN

4 - Answers

(a) 0.3487, 0.0432,

(b) $n = 40$,

(c) $n = 38$

TAP TO RETURN

5 - Answers

(a) 0.581

(b) 0.0594

(c) Assuming the claim is correct, there is a less than 6% chance that 170 or more people would be cured out of 300, so it is likely that the herbalist has understated the actual cure rate.

TAP TO RETURN

6 - Answers

$$0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi$$

b) Let $y = \arccos x$. $x \in [0, 1] \Rightarrow y \in \left[0, \frac{\pi}{2}\right]$
$\cos y = x$, so $\sin y = \sqrt{1 - \cos^2 y} = \sqrt{1 - x^2}$
(Note, $\sin y \neq -\sqrt{1 - x^2}$ since $y \in \left[0, \frac{\pi}{2}\right]$, so $\sin y \geq 0$)
$y = \arcsin \sqrt{1 - x^2}$
Therefore, $\arccos x = \arcsin \sqrt{1 - x^2}$ for $x \in [0, 1]$.

(c) For $x \in (-1, 0)$, $\arccos x \in \left(\frac{\pi}{2}, \pi\right)$, but arcsin only has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

TAP TO RETURN

7 - Answers

(a) use log rules

(b) Gradient log 7, intercept log 3

TAP TO RETURN

8 - Answers

(a) $h(t) = 0$

$$40\sin\left(\frac{t}{10}\right) - 9\cos\left(\frac{t}{10}\right) - 0.5t^2 + 9 = 0$$

$$40\sin\left(\frac{t}{10}\right) - 9\cos\left(\frac{t}{10}\right) + 9 = 0.5t^2$$

$$80\sin\left(\frac{t}{10}\right) - 18\cos\left(\frac{t}{10}\right) + 18 = t^2$$

$$\Rightarrow t = \sqrt{18 + 80\sin\left(\frac{t}{10}\right) - 18\cos\left(\frac{t}{10}\right)}$$

(b) $t_1 = 7.928, t_2 = 7.896, t_3 = 7.882, t_4 = 7.876$

(c) $h'(t) = 4\cos\left(\frac{t}{10}\right) + 0.9\sin\left(\frac{t}{10}\right) - t$

(d) 7.874 (3 d.p.)

(e) Restrict the range of validity to $0 \leq t \leq A$

TAP TO RETURN

BHASVIC MαTHS

A1 DOUBLES ASSIGNMENT 20A

9 - Answers

Assumption: there is an integer solution to the equation $x^2 - y^2 = 2$.
Remember that $x^2 - y^2 = (x - y)(x + y) = 2$
To make a product of 2 using integers, the possible pairs are (2, 1), (1, 2), (-2, 1), (-1, -2).
Consider each possibility in turn.

$x - y = 2$ and $x + y = 1 \Rightarrow x = \frac{3}{2}, y = -\frac{1}{2}$

$x - y = 1$ and $x + y = 2 \Rightarrow x = \frac{3}{2}, y = \frac{1}{2}$

$x - y = -2$ and $x + y = -1 \Rightarrow x = -\frac{3}{2}, y = \frac{1}{2}$

$x - y = -1$ and $x + y = -2 \Rightarrow x = -\frac{3}{2}, y = -\frac{1}{2}$

This contradicts the statement that there is an integer solution to the equation $x^2 - y^2 = 2$.
Therefore the original statement must be true: There are no integer solutions to the equation $x^2 - y^2 = 2$.

TAP TO RETURN

10 - Answers

(a) $\frac{4}{1-x} - \frac{4}{2+x}$

(b) $2 + 5x + \frac{7}{2}x^2$

(c) valid $|x| < 1$

TAP TO RETURN

10b - Answers

(a) $-\frac{2}{2+x}+\frac{4}{(2+x)^2}$

(b) $B=\frac{1}{2}, C=-\frac{3}{8}$

(c) $|x|<2$

TAP TO RETURN

11 - Answers

(a) $-\frac{2}{2+x}+\frac{4}{(2+x)^2}$

(b) $B=\frac{1}{2}, C=-\frac{3}{8}$

(c) $|x|<2$

TAP TO RETURN

12 - Answers

(a) https://www.madasmaths.com/archive/iygb_practice_papers/c1_practice_papers/c1_v_solutions.pdf

TAP TO RETURN

12 - Answers

(a) https://www.madasmaths.com/archive/iygb_practice_papers/c3_practice_papers/c3_q_solutions.pdf

TAP TO RETURN