BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 1A

1

Solve the following quadratics by:

i) Using **Completing the square** only, leaving your answers in surd form:

(a) $x^2 + 6x + 1 = 0$

(b) $x^2 + 12x + 3 = 0$

(c) $4x - 2 = -x^2$

(d) $x^2 - 10x = 5$

ii) Using the **Quadratic Formula** only, leaving your answers in surd form where appropriate:

(a) $x^2 + 3x + 1 = 0$

(b) $4x^2 - 4x - 1 = 0$

(c) $4x^2 - 7x = 2$

(d) $2x - 7 = -11x^2$

TAP FOR ANSWERS

The equation of straight line can be constructed using $m(x-a)=(y-b)$ where (a,b) is a point on the line and **m** is the gradient of the line. You can then re-arrange to get the equation in the form $ax+by+c=0$, where a, b and c are integers.

Example:

Find the equation of the line through $(1,-2)$ with gradient $\frac{1}{2}$ (giving your answer in the form $ax+by+c=0$, where a, b and c are integers):

(substituting point and gradient into the formula)	$\frac{1}{2}(x-1)=y+2$
(multiplying both sides by 2)	$x-1=2y+4$
(getting it all on one side for required form)	$x-2y-5=0$

TAP FOR ANSWERS

2

(i) Give your answers to these questions in the form $ax + by + c = 0$ where a, b and c are **integers**.

(a) Find the equation of the line through $(2, -3)$ with gradient $\frac{3}{2}$

(b) Find the equation of the line through $(-2, -1)$ with gradient $-\frac{7}{2}$

(c) Find the equation of the line through $(2, 3)$ with gradient $-\frac{1}{2}$

TAP FOR ANSWERS

(ii) Using the line equation $y - y_1 = m(x - x_1)$, find the line equations passing through the points (x_1, y_1) and (x_2, y_2) in the form $ax + by + c = 0$ where $a, b, and\ c$ are integers.

(a) $(2,4)\ and\ (3,8)$

(b) $(-2,0)\ and\ (2,8)$

(c) $(3,-1)\ and\ (7,3)$

(d) $(-1,-5)\ and\ (-3,3)$

(e) $\left(\frac{1}{3},\frac{2}{5}\right)\ and\ \left(\frac{2}{3},\frac{4}{5}\right)$

(f) $\left(-\frac{3}{4},\frac{1}{7}\right)\ and\ \left(\frac{1}{4},\frac{3}{7}\right)$

TAP FOR ANSWERS

3

$y=f(x)+b$ is $y=f(x)$ moved up b units. Moving the function down works the same way; $y=f(x)-b$ is $y=f(x)$ moved down b units.

$y=f(x+b)$ gives $y=f(x)$shifted b units to the left. Shifting to the right works the same way; $y=f(x-b)$ is $y=f(x)$ shifted b units to the right.

(a) If $\boldsymbol{f(x)=x^2}$, sketch the following transformations, stating the co-ordinates of the y and x axis intercepts.

(i) $y=f(x)+1$
(ii) $y=f(x)-3$
(iii) $y=f(x+1)$
(iv) $y=f(x-2)$

TAP FOR ANSWERS

(b) If $\boldsymbol{f(x) = x^3}$, sketch the following transformations, stating the co-ordinates of the y and x axis intercepts.

(i) $y = f(x) + 1$
(ii) $y = f(x) - 3$
(iii) $y = f(x + 1)$
(iv) $y = f(x - 2)$

(c) If $\boldsymbol{f(x) = \frac{1}{x}}$, sketch the following transformations. Show clearly the equations of any asymptotes and where the graph cuts the y and x axes.

(i) $y = f(x) + 1$
(ii) $y = f(x) - 3$
(iii) $y = f(x + 1)$
(iv) $y = f(x - 2)$

TAP FOR ANSWERS

(d) If $\boldsymbol{f(x) = \frac{1}{x^2}}$, sketch the following transformations. Show clearly the equations of any asymptotes and where the graph cuts the *y* and *x* axes.

(i) $y = f(x) + 1$
(ii) $y = f(x) - 3$
(iii) $y = f(x + 1)$
(iv) $y = f(x - 2)$

TAP FOR ANSWERS

4

The straight line l passes through $A\ (1, 3\sqrt{3})$ and $B\ (2 + \sqrt{3}, 3 + 4\sqrt{3})$.
Show that l meets the x-axis at the point C (-2, 0)

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 1A

5

Write the following in completed square form $(x + q)^2 + p$ where q and p are real constants and hence state the turning point:

(a) $y = x^2 + 6x + 2$

(b) $y = x^2 + \frac{3}{2}x - 1$

(c) $y = x^2 - 0.1 - 0.6x$

(d) A quadratic graph $y = x^2 + bx + c$ has a turning point of $(2, -5)$. Find the values of b and c.

TAP FOR ANSWERS

How to complete the square if a≠1:

Example

Express $y = 2x^2 - 4x - 7$ in the form $p(x+q)^2 + r$ where p,q and r are real constants.

Method

$\frac{y}{2} = x^2 - 2x - \frac{7}{2}$ *(Firstly divide all by 2 to get an x^2 at the start. Don't forget to divide **everything** by 2.)*

$\frac{y}{2} = (x-1)^2 - 12 - \frac{7}{2}$ *(Now complete the square as usual on the right hand side.)*

$\frac{y}{2} = (x-1)^2 - \frac{9}{2}$ *(Once you're done multiply everything by 2)*

$\boldsymbol{y = 2(x-1)^2 - 9}$

TAP FOR ANSWERS

6 - continued

Example continued

Hence solve $2x^2 - 4x - 7 = 0$

Method

$2(x-1)^2 - 9 = 0$

$2(x-1)^2 = 9$

$(x-1)^2 = \frac{9}{2}$

$(x-1) = \pm\sqrt{\frac{9}{2}}$

$x = -1 \pm \sqrt{\frac{9}{2}}$

Now you try… (P.T.O.)

TAP FOR ANSWERS

6 - continued

Express the following in the form $p(x+q)^2+r$ where p,q and r are real constants and hence solve $y=0$

(a) $y=2x^2+12x-15$

(b) $y=3x^2-12x-9$

(c) $y=-x^2+6x+2$

(d) $y=-x^2-5x+3$

TAP FOR ANSWERS

7

A car manufacturer uses a model to predict the fuel consumption, y miles per gallon (mpg), for a specific model of car travelling at a speed of x mph.

$$y = -0.01x^2 + 0.975x + 16, x > 0$$

(a) Use the model to find two speeds at which the car has a fuel consumption of 32.5mpg, giving your answers to 3 significant figures.

(b) Rewrite in the form $A - B(x - C)^2$, where A, B and C are constants to be found. Round answers to 2 decimal places where appropriate.

(c) Using your answer to part (b), find the speed at which the car has greatest fuel efficiency, giving your answer to 2 decimal places.

(d) Use the model to calculate the fuel consumption of a car travelling at 120mph. Comment on the validity of using this model for very high speeds.

TAP FOR ANSWERS

8

(a) Find the coordinates where the line $y = x + 1$ intersects the curve $x^2 - 2y + 3 = 0$? What can you conclude from the number of solutions that you have found?

(b) Find the coordinates where the line $y = 3x$ crosses the circle $x^2 + y^2 = 120$

TAP FOR ANSWERS

9

An open box with volume $48cm^3$ is to be made by cutting $4cm$ squares from each corner of a square piece of metal and folding up the sides. Find the exact length of the side of the square piece of metal. Let $x\ cm$ be the length of the side of the square.

The square of Barbara's age six years ago is equal to her age in six years' time. Find her present age.

TAP FOR ANSWERS

10

(a) Express 27^{2x+1} in the form 3^y, stating y in terms of x.

(b) A rectangle has a length of $(1 + \sqrt{3})$cm and area of $\sqrt{12}\text{cm}^2$.
Calculate the width of the rectangle in cm.
Express your answer in the form $a + b\sqrt{3}$, where a and b are integers to be found.

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 1A

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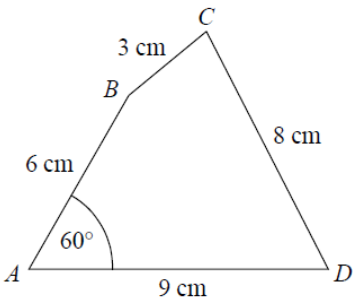
The diagram shows the quadrilateral $ABCD$ in which AB =6 cm, BC = 3 cm, CD =8 cm, AD =9cm and $\angle BAD = 60°$

(a) Using the cosine rule, show that $BD = 3\sqrt{7}\ cm$.

(b) Find the size of $\angle BCD$ in degrees, giving your answer to 2 decimal places.

(c) Find the area of quadrilateral $ABCD$, giving your answer to 2 decimal places.



TAP FOR ANSWERS

These are called hidden or disguised quadratics. Here's an example:

Find all the roots of the following function $f(x) = x^8 - 4x^4 + 3$

The roots are when y = 0 so solve $0 = x^8 - 4x^4 + 3$

Let $y = x^4$ (the middle term)

Then $y^2 = x^8$ (square it)

So $0 = y^2 - 4y + 3$

$0 = (y-3)(y-1)$

$y = 3 or 1$

$x^4 = 3$ or $x^4 = 1$ (don't forget to solve for your original value, x)

$x = 3^{\frac{1}{4}}$ or $x = 1$

Find all roots of the following functions:

(a) $f(x) = x^4 - 13x^2 + 36$

(b) $p(x) = x^6 + 7x^3 - 8$

(c) $m(x) = 6x^{\frac{2}{3}} + 5x^{\frac{1}{3}} - 4 = 0$

(d) $g(x) = x - 4\sqrt{x} + 3$

TAP FOR ANSWERS

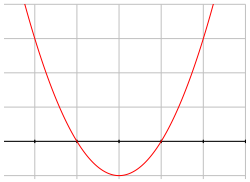
13

(a) Evaluate the discriminant, $b^2 - 4ac$, for each of the quadratics

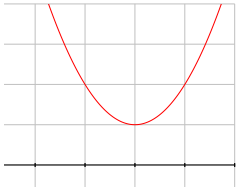
$$f(x) = x^2 - 2x + 1$$
$$g(x) = x^2 - 2x + 3$$
$$h(x) = x^2 - 2x - 5$$

(b) Hence state which of the following sketches could show the graph of $f(x)$, $g(x)$ and $h(x)$ and explain why.

Quadratic A



Quadratic B



Quadratic C

(c) For the quadratic $f(x) = x^2 - kx - k + 1$, the discriminant $= -7$, find the possible values of k. *Make sure you are correctly identifying a, b and c!*

TAP FOR ANSWERS

14

For this question, $f(x) = 4kx^2 + (4k + 2)x + 1$, where k is a real constant.

(a) Find the discriminant of f(x) in terms of k.

(b) By simplifying your answer to part **a** or otherwise, prove that f(x) has two distinct real roots for all non-zero values of k.

(c) Explain why f(x) cannot have two distinct real roots when $k = 0$.

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 1A

1 - Answers

(i)

(a) $-3 \pm 2\sqrt{2}$

(b) $-6 \pm \sqrt{33}$

(c) $-2 \pm \sqrt{6}$

(d) $5 \pm \sqrt{30}$

(ii)

(a) $-\frac{3}{2} \pm \frac{\sqrt{5}}{2}$

(b) $\frac{1}{2} \pm \frac{\sqrt{2}}{2}$

(c) $2, -\frac{1}{4}$

(d) $-\frac{1}{11} \pm \frac{\sqrt{78}}{11}$

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 1A

2 - Answers

i)

(a) $3x - 2y - 12 = 0$

(b) $7x + 2y + 16 = 0$

(c) $x + 2y - 8 = 0$

ii)

a) 4x-y-4=0
b) 2x-y+4=0
c) x-y-4=0
d) 4x+y+9=0
e) 6x-5y=0
f) 4x-14y+5=0

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 1A

3 - Answers

In the library computers you can plot the graphs on ‘autograph’. On your phone you could use the free app ‘desmos’. Or, use your graphical calculator to check. It is important you try these yourself first, don’t go straight to the answers!

TAP TO RETURN

4 - Answers

Proof

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 1A

5 - Answers

(a) $y = (x+3)^2 - 7$ TP (-3,-7)

(b) $y = \left(x + \frac{3}{4}\right)^2 - \frac{25}{16}$ TP $\left(\frac{-3}{4}, \frac{-25}{16}\right)$

(c) $y = (x - 0.3)^2 - 0.19$ TP (0.3, -0.19)

(d) $b = -4, c = -1$

TAP TO RETURN

6 - Answers

(a) $y = 2(x+3)^2 - 33$ $\quad x = -3 \pm \sqrt{\frac{33}{2}}$

(b) $y = 3(x-2)^2 - 21$ $\quad x = 2 \pm \sqrt{7}$

(c) $y = -(x-3)^2 + 11$ $\quad x = 3 \pm \sqrt{11}$

(d) $y = -(x+\frac{5}{2})^2 + \frac{37}{4}$ $\quad x = \frac{-5}{2} \pm \frac{1}{2}\sqrt{37}$

TAP TO RETURN

7 - Answers

(a) 21.8mph and 75.7mph

(b) A=39.77, B=0.01, C=48.75

(c) 48.75mph

(d) -11mpg; a negative answer is impossible so this model is not valid for very high speeds

TAP TO RETURN

8 - Answers

(a) (1,2) One repeated coordínate, therefore the line is a tangent to the curve

(b) $\left(2\sqrt{3}, 6\sqrt{3}\right)$ and $\left(-2\sqrt{3}, -6\sqrt{3}\right)$

TAP TO RETURN

9 - Answers

(a) $x = 8 + 2\sqrt{3}$ (note that you reject $x = 8 - 2\sqrt{3}$ as this solution is not practically possible if you have a side of $(x - 8)$)

(b) *10 years old*

TAP TO RETURN

10 - Answers

(a) $y = 6x + 3$

(b) $3 - \sqrt{3}$ cm

TAP TO RETURN

11 - Answers

(b) 77.98^{o} (2dp)

(c) 35.12 cm^2 (2dp)

TAP TO RETURN

12 - Answers

(a) $\pm 2, \pm 3$

(b) $-2, 1$

(c) $\left(\frac{1}{8}, \frac{-64}{27}\right)$

(d) $1, 9$

TAP TO RETURN

13 - Answers

(a) 0,-8,24

(b) $f(x)$ is quadratic C ($b^2 - 4ac = 0$ so 1 root - graph touches the x axis)
$g(x)$ is quadratic B ($b^2 - 4ac < 0$ so no roots - graph doesn't cut the x axis)
$h(x)$ is quadratic A ($b^2 - 4ac > 0$ so two distinct roots - graph cuts the x axis)

(c) k = –1 or k = –3

TAP TO RETURN

14 - Answers

(a) $16k^2 + 4$

(b) $k^2 \geq 0$ for all k so $16k^2 + 4 > 0$

(c) When $k = 0$, $f(x) = 2x + 1$; this is a linear function with only one root.

TAP TO RETURN