1

'In proof by contradiction, we start by assuming the negation of what we're trying to prove.'

(a) Write down the negation of each statement:
(i) All rich people are happy.
(ii) There are no prime numbers between 10 million and 11 million.
(iii) If p and q are prime numbers then $(pq + 1)$ is prime number.
(iv) All numbers of the form $2^n - 1$ are either prime numbers of multiples of 3.
(v) At least one of the above four statements is true.

(b) Prove the following statements by contradiction:
(i) There is no greatest even integer.
(ii) If n^3 is even then n is even.
(iii) If pq is even then at le4ast one of p and q is even.
(iv) If $p + q$ is odd then at least one of p and q is odd.

TAP FOR ANSWERS

2

(a) Explain what you understand by the following terms:
(i) Critical value
(ii) Critical region
(iii) Acceptance region

(b) A test statistic has a distribution B(10, p). given that $H_0: p = 0.2, H_1: p > 0.2$, find the critical region for the test using a 5% significance level.

(c) A random variable has a distribution B(20, p). A single observation is used to test $H_0: p = 0.15$ against $H_1: p < 0.15$. Using a 5% level of significance, find the critical region of this test.

(d) A random variable has distribution B(20, p). A single observation is used to test $H_0: p = 0.4$ against $H_1: p \neq 0.4$
(i) Using the 5% level of significance, find the critical region of this test.
(ii) Write down the actual significance level of the test.

TAP FOR ANSWERS

A loom makes table cloths with an average thickness of 2.5mm. the thickness, T mm, can be modelled using a normal distribution. Given that 65% of table cloths are less than 2.55 mm thick, find:

(a) The standard deviation of the thickness
(b) The proportion of table cloths with thickness between 2.4 mm and 2.6 mm.

A table cloth can be sold if the thickness is between 2.4 mm and 2.6 mm. a sample of 20 table cloths is taken.

(c) Find the probability that at least 15 table cloths can be sold.

TAP FOR ANSWERS

Summarised below are the distances, to the nearest mile, travelled to work by a random sample of 120 commuters.

Distance to the nearest mile (x)	Number of commuters
0-9	10
10-19	19
20-29	43
30-39	25
40-49	8
50-59	6
60-69	5
70-79	3
80-89	1

TAP FOR ANSWERS

4

For this distribution,

(a) use linear interpolation to estimate its median

The midpoint of each class was represented by x and the corresponding frequency by f giving
$\Sigma fx = 3550$ and $\Sigma fx^2 = 138020$

(b) estimate the mean and standard deviation of this distribution

The data is coded using the code

$$y = \frac{x-5}{10}$$

(c) Evaluate the mean and variance of the coded data.

TAP FOR ANSWERS

5

For one of the activities at a gymnastics competition, 8 gymnasts were awarded marks out of 10 for each of the artistic performance and technical ability. The value of the PMCC for this data is 0.774. Stating your hypothesis clearly and using a 1% level of significance, test for evidence of a positive association between technical ability and artistic performance. Interpret this value.

TAP FOR ANSWERS

6

The recruitment director of a large accounting firm believes that maths graduates are more successful when applying for positions in his firm compared with graduates of other subjects.

One in five job applicants to this firm is successful.

The recruitment director selects a random sample of 25 maths graduate applicants

(a) State the hypothesis of the test clearly.

(b) Find the critical region to test at the 5% level of significance the directors belief

(c) State the actual significance level for a test

(d) Ten successful maths graduate applicants were found in the sample. Does this support this directors claim

TAP FOR ANSWERS

7

(a) Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution.

A company sells orchids of which 45% produce pink flowers.
A random sample of 20 orchids is taken and X produces pink flowers.

(b) Find $\mathrm{P}(X = 10)$.

A second random sample of 240 orchids is taken.

(c) Using a suitable approximation, find the probability that fewer than 110 orchids produce pink flowers.

(d) The probability that at least q orchids product pink flowers is 0.2. find q.

TAP FOR ANSWERS

8

Prove the identities

(a) $(\sec x - \operatorname{cosec} x)(\sec x + \operatorname{cosec} x) \equiv (\tan x - \cot x)(\tan x + \cot x)$

(b) $(\operatorname{cosec} x - \sin x)(\sec x - \cos x) \equiv \cos x \sin x$

TAP FOR ANSWERS

9

(a) Sketch, in the interval $-2\pi \leq x \leq 2\pi$, the graph of $y = 2 - 3\sec x$

(b) Hence deduce the range of values of k for which the equation $2 - 3\sec x = k$ has no solutions.

TAP FOR ANSWERS

10

(a) Express $12\sin x - 5\cos x$ in the form $R\sin(x - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
Hence solve the equation $12\sin x - 5\cos x = 7$ for $0 < x < 2\pi$ giving x correct to 3 decimal places.

(b) Given that $\sin x(\cos y + 2\sin y) = \cos x(2\cos y - \sin y)$, find the value of $\tan(x + y)$.

TAP FOR ANSWERS

11

In the following questions, use the trapezium rule with n intervals of equal width to find an estimate of the definite integral, giving your answer to 3 decimal places

(a) $\int_0^4 (16 - x^2)^{\frac{1}{2}}\, dx, \ n = 6$

(b) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin^{\frac{1}{2}} x\, dx, \quad n = 6$

(c) $\int_1^2 log_{10} x\, dx, \quad n = 5$

(d) $\int_0^{\frac{\pi}{2}} \cos^{\frac{1}{3}} x\, dx, \quad n = 5$

TAP FOR ANSWERS

12

Show that for small angles, the function $f(x) = sin^2x\ cosx$ can be approximated by a function of the form $h(x) = A + Bx + Cx^2 + Dx^3 + Ex^4$ and use this approximation to evaluate $sin^2\left(\frac{\pi}{24}\right) cos\left(\frac{\pi}{24}\right)$.

TAP FOR ANSWERS

13

$$g(x) = x^2 - 3x - 5$$

(a) Show that the equation $g(x) = 0$ can be written as $x = \sqrt{3x + 5}$.
(b) Sketch on the same axes the graphs of $y = x$ and $y = \sqrt{3x + 5}$.
(c) Use your diagram to explain why the iterative formula $x_{n+1} = \sqrt{3x_n + 5}$ converges to a root of $g(x)$ when $x_0 = 1$.

$g(x) = 0$ can also be rearranged to form the iterative formula $x_{n+1} = \frac{x_n^2 - 5}{3}$

(d) With reference to a diagram, explain why this iterative formula diverges when $x_0 = 7$.

TAP FOR ANSWERS

14

Prove by contradiction that $\sqrt{\frac{1}{2}}$ is an irrational number.

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 19B

15

Complete this old spec paper
https://www.madasmaths.com/archive/iygb_practice_papers/c2_practice_papers/c2_u.pdf

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 19B

1 - Answers

(a) (i) At least one rich person is not happy.
(ii) There is at least one prime number between 10 million and 11 million
(iii) If p and q are prime numbers three exists a number of the form $(pq + 1)$ that is not prime.
(iv) There is a number of the form $2^n - 1$ that is either not prime or not a multiple of 3.
(v) None of the above statements are true.

(b) (i) Assumption: there is a greatest even integer $2n$.
$2(n+1)$ is also an integer and $2(n+1) > 2n$ $\quad 2n + 2$ = even + even = even. So there exists an even integer greater than $2n$. this contradicts the assumption. Therefore there is no greatest even integer.

(ii) Assumption: there exists a number n such that n^3 is even by n is odd.
n is odd so write $n = 2k + 1$ $\quad n^3 = (2k+1)^3 = 8k^3 + 12k^2 + 6k + 1 = 2(4k^3 + 6k^2 + 3k) + 1 \Rightarrow$ n^3 is odd. This contradicts the assumption that n^3 is even. Therefore if n^3 is even then n must be even.

(iii) Assumption: if pq is even then neither p nor q is even.
p is odd, $p = 2k + 1$ $\quad q$ is odd, $q = 2m + 1$
$pq = (2k+1)(2m+1) = 2km + 2k + 2m + 1 = 2(km + k + m) + 1 \Rightarrow pq$ is odd. This contradicts the assumption that pq is even. Therefore if pq is even then at least one of p and q is even.

(iv) Assumption: if $p + q$ is odd then neither p nor q is odd
p is even, $p = 2k$ $\quad q$ is even, $q = 2m$
$p + q = 2k + 2m = 2(k + m) \Rightarrow$ so $p + q$ is even. This contradicts the assumption that pq is even.
Therefore, if $p + q$ is odd that at least one of p and q is odd.

TAP TO RETURN

2 - Answers

(a) (i) the critical value is the first value to fall inside of the critical region.
(ii) A critical region is a region of the probability distribution which, if the test statistic falls with it, would cause you to reject the null hypothesis.
(iii) The acceptance region is the area in which we accept the null hypothesis.

(b) The critical value is $x = 5$ and the critical region is $x \geq 5$ since $\mathrm{P}(X \geq 5) = 0.0328 < 0.05$

(c) The critical value is $x = 0$ and the critical region is $X = 0$.

(d) (i)The critical region is $X \geq 13$ and $X \leq 3$.
(ii) 0.037 = 3.7%

TAP TO RETURN

3 - Answers

(a) 0.1299 mm

(b) 0.5586

(c) 0.0644

TAP TO RETURN

4 - Answers

(a) $Q_2 \approx 26.7$

(b) $\bar{x} \approx 29.6, \sigma_x \approx 16.6$

(c) $\bar{y} \approx 2.46, \sigma_y \approx 0.166$

TAP TO RETURN

5 - Answers

Insufficient evidence

TAP TO RETURN

6 - Answers

(a) $H_0 : p = \frac{1}{5}, H_1 : p > \frac{1}{5}$,

(b) $x \geq 9$,

(c) 4.68%,

(d) Yes

TAP TO RETURN

7 - Answers

(a) n large, p close to 0.5

(b) 0.1593

(c) 0.5772

(d) 115

TAP TO RETURN

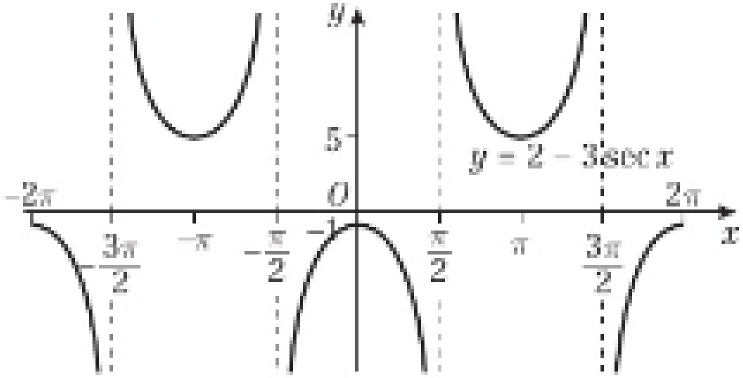
8 - Answers

Proof

TAP TO RETURN

9 - Answers

(a)



(b) $-1 < k < 5$

TAP TO RETURN

10 - Answers

(a) $R = 13$, $\alpha = 0.3948$. $x = 0.963$ or 2.968

(b) *2*

TAP TO RETURN

11 - Answers

(a) 12.248

(b) 0.946

(c) 0.167

(d) 1.234

TAP TO RETURN

12 - Answers

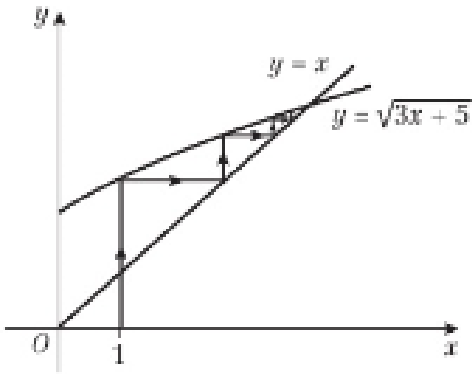
$x^2 - \frac{x^4}{2}$; 0.01699

TAP TO RETURN

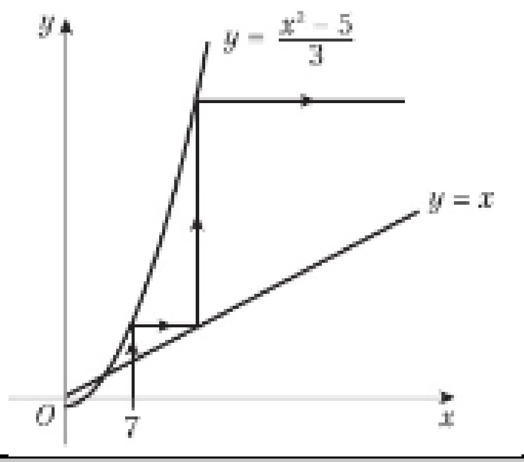
13 - Answers

(a) $g(x) = 0 \Rightarrow x^2 - 3x - 5 = 0$
$\Rightarrow x^2 = 3x + 5 \Rightarrow x = \sqrt{3x+5}$

(b, c)



(d)



TAP TO RETURN

14 - Answers

Assume $\sqrt{\frac{1}{2}}$ is a rational number.

Then $\sqrt{\frac{1}{2}} = \frac{a}{b}$ for some integers *a* and *b*.

Further assume that this fraction is in its simplest terms: there are no common factors between *a* and *b*.

So $0.5 = \frac{a^2}{b^2}$ or $2a^2 = b^2$.

Therefore b^2 must be a multiple of 2. We know that this means *b* must also be a multiple of 2.

Write $b = 2c$, which means $b^2 = (2c)^2 = 4c^2$.

Now $4c^2 = 2a^2$, or $2c^2 = a^2$.

Therefore a^2 must be a multiple of 2, which implies *a* is also a multiple of 2.

If *a* and *b* are both multiples of 2, this contradicts the statement that there are no common factors between *a* and *b*.

Therefore, $\sqrt{\frac{1}{2}}$ is an irrational number.

TAP TO RETURN

15 - Answers

https://www.madasmaths.com/archive/iygb_practice_papers/c2_practice_papers/c2_u_solutions.pdf

TAP TO RETURN