BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 18B

1

In each part, a random sample of size n is taken from a population having a normal distribution with mean μ and variance σ^2. Test the hypotheses at the stated levels of significance.

(a) $H_0: \mu = 21$, $H_1: \mu \neq 21$, $n = 20$, $\bar{x} = 21.2$, $\sigma = 1.5$, at the 5% level

(b) $H_0: \mu = 100$, $H_1: \mu < 100$, $n = 36$, $\bar{x} = 98.5$, $\sigma = 5.0$, at the 5% level

(c) $H_0: \mu = 5$, $H_1: \mu \neq 5$, $n = 25$, $\bar{x} = 6.1$, $\sigma = 3.0$, at the 5% level

(d) $H_0: \mu = 15$, $H_1: \mu > 15$, $n = 40$, $\bar{x} = 16.5$, $\sigma = 3.5$, at the 1% level

(b) $H_0: \mu = 50$, $H_1: \mu \neq 50$, $n = 60$, $\bar{x} = 48.9$, $\sigma = 4.0$, at the 1% level

TAP FOR ANSWERS

2

(a) A biased coin is thrown twice. The probability that it shows heads both times is 0.04. Find the probability that it shows tails both times.

(b) Another coin is biased so that the probability that it shows heads on any throw is p. The probability that the coin shows heads exactly once in two throws is 0.42. Find the two possible values of p.

TAP FOR ANSWERS

3

Daily mean windspeed is modelled as being normally distributed with a standard deviation of 3.1 knots.

A random sample of 25 recorded daily mean windspeeds is taken at Heathrow in 2015.

Given that the mean of the sample is 12.2 knots, test at the 2.5% level of significance whether the mean of the daily mean windspeeds is greater than 9.5 knots.

State your hypotheses clearly.

TAP FOR ANSWERS

BHASVIC MαTHS
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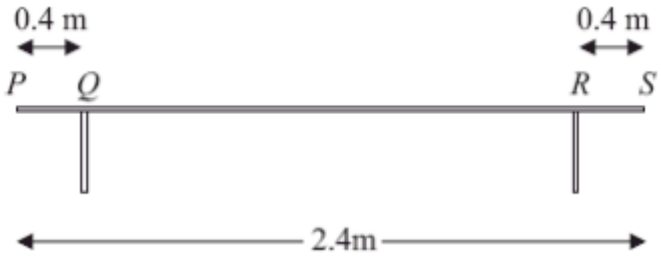


A bench consists of a plank which is resting in a horizontal position on two thin vertical legs. The plank is modelled as a uniform rod PS of length 2.4 m and mass 20 kg. The legs at Q and R are 0.4 m from each end of the plank, as shown in the diagram above. Two pupils, Arthur and Beatrice, sit on the plank. Arthur has mass 60 kg and sits at the middle of the plank and Beatrice has mass 40 kg and sits at the end P. The plank remains horizontal and in equilibrium. By modelling the pupils as particles, find the magnitude of the normal reaction between the plank and the leg at Q and the magnitude of the normal reaction between the plank and the leg at R.

TAP FOR ANSWERS

5

(a) Define the critical region of a test statistic.

A discrete random variable x has a Binomial distribution B(30, p). A single observation is used to test H0 : $p = 0.3$ against H1 : $p \neq 0.3$

(b) Using a 1% level of significance find the critical region of this test. You should state the probability of rejection in each tail which should be as close as possible to 0.005

(c) Write down the actual significance level of the test.

The value of the observation was found to be 15.

(d) Comment on this finding in light of your critical region

TAP FOR ANSWERS

6

A particle P moves on the *x*-axis. The acceleration of P at time t seconds, $t \geq 0$, is $(3t + 5)$ ms^{-2} in the positive *x*-direction. When $t = 0$, the velocity of P is 2 ms^{-1} in the positive *x*-direction. When $t = T$, the velocity of P is 6 ms^{-1} in the positive *x*-direction. Find the value of T.

TAP FOR ANSWERS

a Prove that for any positive numbers p and q:

$$p + q > \sqrt{4pq}$$

b Show, by means of a counter-example, that this inequality does not hold when p and q are both negative.

TAP FOR ANSWERS

8

At time $t = 0$ a particle P leaves the origin O and moves along the x-axis. At time t seconds the velocity of P is v ms^{-1}, where $v = 8t - t^2$.

(a) Find the maximum value of v.

(b) Find the time taken for P to return to O.

TAP FOR ANSWERS

Sketch the graph of each of the following. In each case, write down the coordinates of any points at which the graph meets the coordinate axes.

(a) $y = |x - 1|$

(b) $y = |2x + 3|$

(c) $y = |4x - 7|$

(d) $y = \left|\frac{1}{2}x - 5\right|$

(e) $y = |7 - x|$

(f) $y = |6 - 4x|$

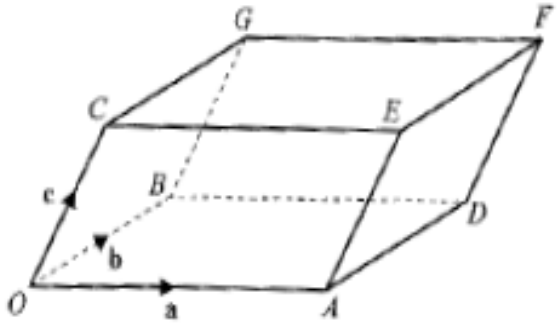
(g) $y = -|x|$

(h) $y = -|3x - 1|$

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 18B

10

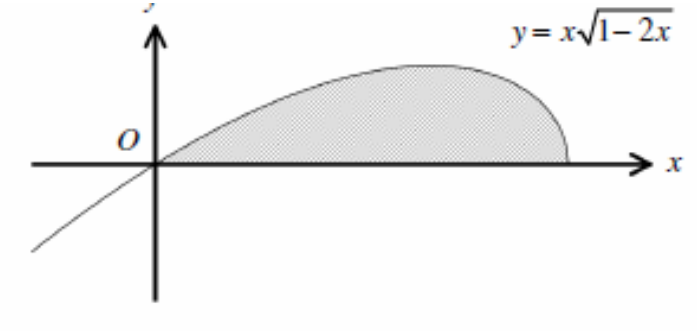


A parallelepiped is a three-dimensional figure formed by six parallelograms. The diagram shows a parallelepiped with vertices *o*, *A*, *B*, *C*, *D*, *E*, *F* and *G*.

a, **b** and **c** are the vectors $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively. Prove that the diagonals *OF* and *AG* bisect each other.

TAP FOR ANSWERS

11



The figure above shows the graph of the curve with equation

$$y = x\sqrt{1-2x}, \qquad x \leq \frac{1}{2}$$

Use integration by parts and reverse chain rule to find the area of the finite region bounded by the curve and the x axis.

TAP FOR ANSWERS

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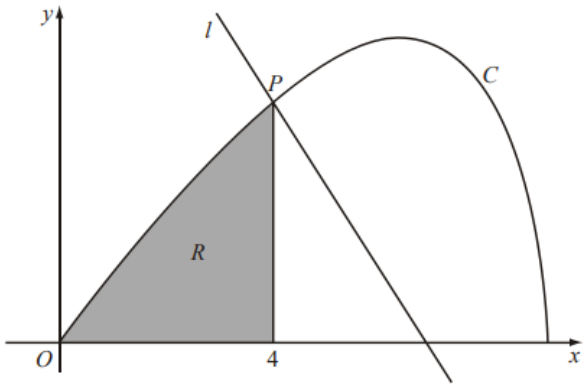
The diagram shows the curve C with parametric equations

$x = 8\cos t$, $\quad 0 \leq t \leq \frac{\pi}{2}$.

$y = 4\sin 2t$,

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.



(a) Find the value of t at the point P.

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in the diagram above.

(c) Show that the area of R is given by the integral $\int_{\pi/3}^{\pi/2} 64\sin^2 t \cos t \, dt$

(d) Use this integral to find the area of R, giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

TAP FOR ANSWERS

13

a When θ is small, show that the expression $\dfrac{4\cos 3\theta - 2 + 5\sin\theta}{1 - \sin 2\theta}$ can be written as $9\theta + 2$.

b Hence write down the value of $\dfrac{4\cos 3\theta - 2 + 5\sin\theta}{1 - \sin 2\theta}$ when θ is small.

TAP FOR ANSWERS

14

A curve C has equation

$$2^x + y^2 = 2xy$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates $(3, 2)$.

TAP FOR ANSWERS

15

The random variable X is normally distributed with mean μ and variance σ^2.

a Write down the distribution of the sample mean $\overline{X}$ of a random sample of size n.

A construction company wishes to determine the mean time taken to drill a fixed number of holes in a metal sheet.

b Determine how large a random sample is needed so that the expert can be 95% certain that the sample mean time will differ from the true mean time by less than 15 seconds.
Assume that it is known from previous studies that $\sigma = 40$ seconds.

TAP FOR ANSWERS

15

https://www.madasmaths.com/archive/iygb_practice_papers/c1_practice_papers/c1_u.pdf

TAP FOR ANSWERS

1 - Answers

(a) Not significant. Accept H_0.

(b) Significant. Reject H_0

(c) Not significant. Accept H_0

(d) Significant. Reject H_0

(e) Not significant. Accept H_0

TAP TO RETURN

2 - Answers

(a) 0.64

(b) 0.7, 0.3

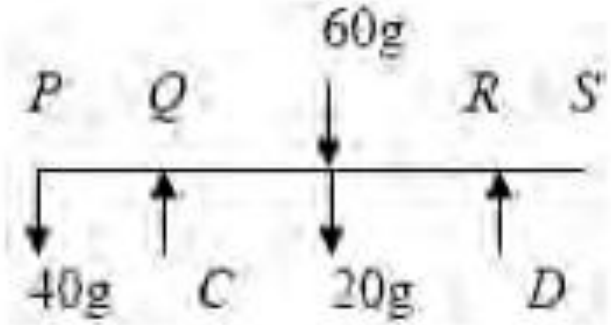
TAP TO RETURN

3 - Answers

$H_0: \mu = 9.5, H_1 > 9.5$. Critical region is $\bar{X} \geq 10.715$. $\bar{x} = 12.2 > 10.715$, so reject H_0 and conclude that the mean daily windspeed is greater than 9.5 knots.

TAP TO RETURN

4 - Answers



$C + D = 120g$

$M(Q)$, $80g.0.8 - 40g.0.4 = D.1.6$

solving

$C = 90g$; $D = 30g$

TAP TO RETURN

5 - Answers

(a) The set of values of the test statistic for which the null hypothesis is rejected in a hypothesis test.

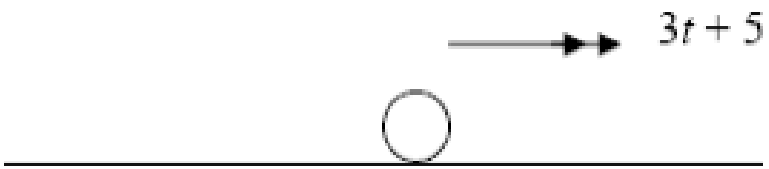
(b) $X \sim B(30,0.3)$
$P(X \leq 3) = 0.0093$
$P(X \leq 2) = 0.0021$
$P(X \geq 16) = 1 - 0.9936 = 0.0064$
$P(X \geq 17) = 1 - 0.9979 = 0.0021$
Critical region is $0 \leq x \leq 2$ or $16 \leq x \leq 30$

(c) Actual significance level 0.0021+0.0064=0.0085 or 0.85%

(d) 15 (it) is not in the critical region
not significant
No significant evidence of a change in P = 0.3
accept H0, (reject H1)
$P(x \geq 15) = 0.0169$

TAP TO RETURN

6 - Answers



$$\frac{dv}{dt} = 3t + 5$$

$$v = \int (3t + 5)\,dt$$

$$v = \tfrac{3}{2}t^2 + 5t \quad (+c)$$

$$t = 0 \quad v = 2 \quad \Rightarrow \quad c = 2$$

$$v = \tfrac{3}{2}t^2 + 5t + 2$$

$$t = T \qquad 6 = \tfrac{3}{2}T^2 + 5T + 2$$

$$12 = 3T^2 + 10T + 4$$

$$3T^2 + 10T - 8 = 0$$

$$(3T - 2)(T + 4) = 0$$

$$T = \tfrac{2}{3} \quad (T = -4)$$

$$\therefore T = \tfrac{2}{3} \quad \text{(or 0.67)}$$

TAP TO RETURN

7 - Answers

a $(p+q)^2 = p^2 + 2pq + q^2 = (p-q)^2 + 4pq$
$(p-q)^2 \geqslant 0$ since it is a square, so $(p+q)^2 \geqslant 4pq$
$p > 0, q > 0 \Rightarrow p + q > 0 \Rightarrow p + q \geqslant \sqrt{4pq}$

b e.g. $p = q = -1$: $p + q = -2$, $\sqrt{4pq} = 2$

TAP TO RETURN

8 - Answers

(a) $\frac{dv}{dt} = 8 - 2t$

$8 - 2t = 0$

Max $v = 8 \times 4 - 4^2 = 16\ (\text{ms}^{-1})$

(b) $\int 8t - t^2 dt = 4t^2 - \frac{1}{3}t^3 (+C)$

($t = 0$, displacement $= 0 \Rightarrow c = 0$)

$4T^2 - \frac{1}{3}T^3 = 0$

$T^2\left(4 - \frac{T}{3}\right) = 0 \Rightarrow T = 0,12$

$T = 12$ (seconds)

TAP TO RETURN

9 - Answers

USE DESMOS TO CHECK

TAP TO RETURN

10 - Answers

Let H = point of intersection of OF and AG.

$\overrightarrow{OH} = r\overrightarrow{OA} + s\overrightarrow{AG}$

$\overrightarrow{OF} = \mathbf{a} + \mathbf{b} + \mathbf{c}, \overrightarrow{AG} = -\mathbf{a} + \mathbf{b} + \mathbf{c}$

So $r(\mathbf{a} + \mathbf{b} + \mathbf{c}) = \mathbf{a} + s(-\mathbf{a} + \mathbf{b} + \mathbf{c})$

$r = 1 - s = s \Rightarrow r = s = \frac{1}{2}$, so $\overrightarrow{OH} = \frac{1}{2}\overrightarrow{OF}$ and $\overrightarrow{AH} = \frac{1}{2}\overrightarrow{AG}$

TAP TO RETURN

11 - Answers

1/15

TAP TO RETURN

12 - Answers

$t = \frac{\pi}{3}$

$\frac{64}{3} - 8\sqrt{3}$

TAP TO RETURN

13 - Answers

a $$\frac{4\cos 3\theta - 2 + 5\sin\theta}{1 - \sin 2\theta} \approx \frac{4\left(1 - \frac{(3\theta)^2}{2}\right) - 2 + 5\theta}{1 - 2\theta}$$

$$= \frac{4\left(1 - \frac{9\theta^2}{2}\right) - 2 + 5\theta}{1 - 2\theta} = \frac{4 - 18\theta^2 - 2 + 5\theta}{1 - 2\theta}$$

$$= \frac{(1 - 2\theta)(9\theta + 2)}{1 - 2\theta} = 9\theta + 2$$

b 2

TAP TO RETURN

14 - Answers

$4\ln 2 - 2$

TAP TO RETURN

15 - Answers

a $X \sim \mathrm{N}\left(\mu_1 \frac{\sigma^2}{n}\right)$ **b** Need $n = 28$ or more

TAP TO RETURN

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16 - Answers

https://www.madasmaths.com/archive/iygb_practice_papers/c1_practice_papers/c1_solutions.pdf

TAP TO RETURN