1

(a) $\int \sec^2 y \tan^5 y \, dy$

(b) $\int \operatorname{cosec} 3u \cot 3u \, du$

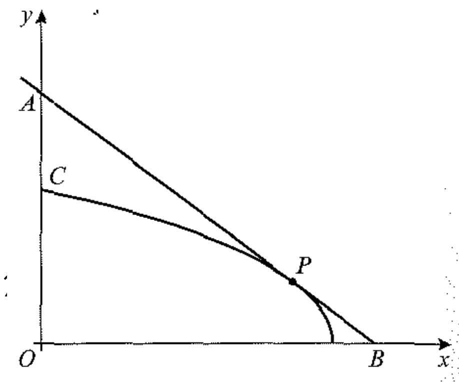
(c) $\int 4x(3x^2+1)^6 \, dx$

(d) $\int \frac{\sec^2 3x}{2+\tan 3x} \, dx$

(e) $\int \frac{4-x}{(x-2)(x-3)} \, dx$

TAP FOR ANSWERS

2



The diagram shows the curve C with parametric equations

$$x = a\sin^2 t, \qquad y = a\cos t, \qquad 0 \leq t \leq \frac{1}{2}\pi$$

where a is a positive constant. The point P lies on C and has coordinates $\left(\frac{3}{4}a, \frac{1}{2}a\right)$.

(a) Find $\frac{dy}{dx}$, giving your answer in terms of t.
Find an equation of the tangent to C at P.

(b) The tangent to C at P cuts the coordinate axes at points A and B.

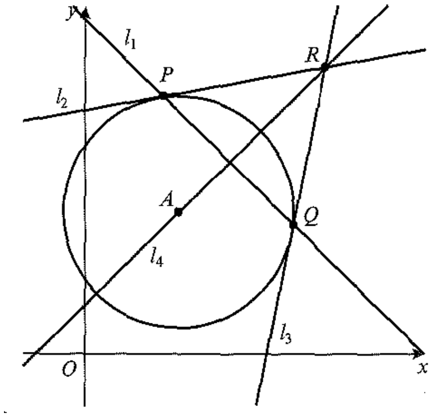
(c) Show that the triangle AOB has area ka^2 where k is a constant to be found.

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 18A

3

The circle C has a centre at (6, 9) and a radius of $\sqrt{50}$.



The line l_1 with equation $x + y - 21 = 0$ intersects the circle at the points P and Q.

(a) Find the coordinates of the point P and the point Q.

(b) Find the equations of l_2 and l_3, the tangents at the points P and Q respectively.

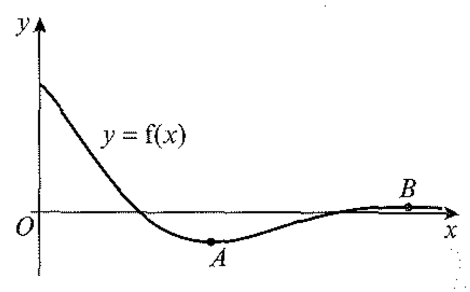
(c) Find the equation of l_4, the perpendicular bisector of the chord PQ.

(d) Show that the two tangents and the perpendicular bisector intersect and find the coordinates of R, the point of intersection.

(e) Calculate the area of the kite $APRQ$.

TAP FOR ANSWERS

4



The curve C with equation $y = \mathrm{f}(x)$ is shown in the diagram, where

$$\mathrm{f}(x) = \frac{\cos 2x}{e^x}, 0 \leq x \leq \pi$$

The curve has a local minimum at A and a local maximum at B.

(a) Show that the x-coordinates of A and B satisfy the equation $\tan 2x = -0.5$ and hence find the coordinates of A and B.
(b) Using your answer to part (a), find the coordinates of the maximum and minimum turning points on the curve with equation $y = 2 + 4f(x - 4)$
(c) Determine the range of values for which $\mathrm{f}(x)$ is concave.

TAP FOR ANSWERS

Express the following in the form $R\sin(\theta \pm \alpha)$ or $R\cos(\theta \pm \alpha)$ as appropriate (with α in radians) and hence find the **minimum** value of the function, and the first positive value of θ for which it occurs: check using your graphic calculator

(a) $\cos\theta + \sin\theta$ [use $R\cos(\theta - \alpha)$]

(b) $5\cos\theta - 12\sin\theta$ [use $R\cos(\theta + \alpha)$]

(c) $\sqrt{3}\sin\theta + 3\cos\theta$ [use $R\sin(\theta + \alpha)$]

(d) $3\sin\theta - 7\cos\theta$ [use $R\sin(\theta - \alpha)$]

TAP FOR ANSWERS

(a) Express $65\cos\theta - 20\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the value of α correct to 4 decimal places.

A city wants to build a large circular wheel as a tourist attraction. The height of a tourist on the circular wheel is modelled by the equation

$$H = 70 - 65\cos 0.2t + 20\sin 0.2t$$

where H is the height of the tourist above the ground in metres, t is the number of minutes after boarding and the angles are given in radians. Find:

(b) The maximum height of the wheel
(c) The time for one complete revolution
(d) The number of minutes the tourist will be over 100 m above the ground in each revolution.

TAP FOR ANSWERS

7

Given that

$y = \arccos x,$ $\quad -1 \leq x \leq 1$ and $0 \leq y \leq \pi$

(a) express $\arcsin x$ in terms of y

Hint: Use $\cos\theta = \sin(\frac{\pi}{2} - \theta)$

(b) Hence evaluate $\arccos x + \arcsin x$. Give your answer in terms of π.

TAP FOR ANSWERS

8

The side of a cube of length x cm, is increasing at the constant rate of $1.5\ cm\ s^{-1}$

Find the rate at which the volume of the cube is increasing when its side is 6 cm

TAP FOR ANSWERS

9

A curve has implicit equation $x^3 + y^3 + 3y^2 + 3y - 6x = 50 + 2xy$
Find an equation of the normal to the curve at the point P(4,2)

TAP FOR ANSWERS

10

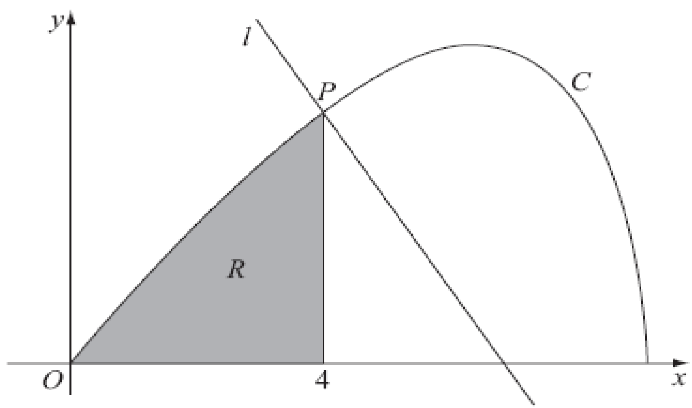


Figure 3 shows the curve C with parametric equations

$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leq t \leq \frac{\pi}{2}$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P.

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

TAP FOR ANSWERS

$$f(x) = \frac{9x^2+4}{9x^2-4}, x \neq \pm\frac{2}{3}$$

(a) Given that $f(x) = A + \frac{B}{3x-2} + \frac{C}{3x+2}$, find the values of the constants A, B and C.

(b) Hence find the exact value of $\int_{-\frac{1}{3}}^{\frac{1}{3}} \frac{9x^2+4}{9x^2-4}\, dx$, writing your answer in the form $a + b\ln c$, where a. b and c are rational numbers to be found.

TAP FOR ANSWERS

(a) Show that $\sin^2 x + 3\cos^2 x \equiv 2 + \cos 2x$.

(b) Hence evaluate $\int_{\pi/12}^{\pi/4} (\sin^2 x + 3\cos^2 x)\,dx$
check using your calculator to see if you're right

(c) Show that $\frac{4\cos 2x}{\sin^2 2x} \equiv \mathrm{cosec}^2 x - \sec^2 x$

(d) Hence evaluate $\int_{\pi/6}^{\pi/3} \frac{4\cos 2x}{\sin^2 2x}\,dx$
check using your calculator to see if you're right

TAP FOR ANSWERS

Find the first three terms in the expansion of

$$\frac{1}{(1+x)^2}$$

Hence deduce the expansions, stating the values of x for which each expansion is valid.

a) $\frac{1}{(1-3x)^2}$

b) $\frac{1}{\left(1+\frac{2x}{3}\right)^2}$

c) $\frac{1}{(4-3x)^2}$

TAP FOR ANSWERS

14

$$\frac{2x^2 + 5x - 10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}$$

(a) Find the values of A, B and C.

(b) Hence, or otherwise, expand $\frac{2x^2+5x-10}{(x-1)(x+2)}$ in ascending powers of x, as far as the term in x^2. Give each coefficient as a simplified fraction.

TAP FOR ANSWERS

(a) Complete this old spec paper

(b) https://www.madasmaths.com/archive/iygb_practice_papers/c3_practice_papers/c3_o.pdf

TAP FOR ANSWERS

1 - Answers

(a) $\frac{1}{6}\tan^6 y + c$

(b) $-\frac{1}{3}\text{cosec}3u + c$

(c) $\frac{2}{21}(3x^2 + 1)^7 + c$

(d) $\frac{1}{3}ln(2 + tan3x) + c$

(e) $\ln|x - 3| - 2\ln|x - 2| + c$

TAP TO RETURN

2 - Answers

(a) $-\frac{1}{2}\sec t$

(b) $4y + 4x = 5a$

(c) Tangent crosses the x-axis at $x = \frac{5}{4}a$, and crosses the y-axis at $y = \frac{5}{4}a$. So area $AOB = \frac{1}{2}\left(\frac{4}{5}a\right)^2 = \frac{25}{32}a^2, k = \frac{25}{32}$

TAP TO RETURN

3 - Answers

(a) $P(5, 16)$ and $Q(13, 8)$

(b) $l_2: y = \frac{1}{7}x + \frac{107}{7}$ and $l_3: y = 7x - 83$

(c) $l_4: y = x + 3$

(d) All 3 equations have solution $x = \frac{43}{3}, y = \frac{52}{3}$ so $R(15, 18)$

(e) $\frac{200}{3}$

TAP TO RETURN

4 - Answers

(a) $f'(x) = -\frac{2\sin 2x + \cos 2x}{e^x}$

$f'(x) = 0 \Leftrightarrow 2\sin 2x + \cos 2x = 0 \Leftrightarrow \tan 2x = -0.5$

A (1.34, -0.234), *B* (2.91, 0.0487)

(b) Maximum (6.91, 2.20); minimum (5.34, 1.06) to 3 s.f.

(c) $0 < x \leq 0.322, 1.89 \leq x < \pi$

TAP TO RETURN

5 - Answers

(a) $R = \sqrt{2}, \alpha = \frac{\pi}{4}$ min

$-\sqrt{2}, \theta = \frac{5\pi}{4}$

(b) $R = 13, \alpha = 1.18$ min

$-13, \theta = 1.96$

(c) $R = 2\sqrt{3}, \alpha = \frac{\pi}{3}$ min

$-2\sqrt{3}, \theta = \frac{7\pi}{6}$

(d) $R = \sqrt{58}, \alpha = \arctan\frac{7}{3}$ min

$-\sqrt{58}, \theta = 5.88$

6 - Answers

(a) R = 68.0074, $\alpha = 0.2985$

(b) 138.0 m

(c) 31.4 minutes

(d) 11.1 minutes

TAP TO RETURN

7 - Answers

(a) $\frac{\pi}{2} - y$

(b) $\frac{\pi}{2}$

TAP TO RETURN

8 - Answers

(a) $162\ cm^3\ s^{-1}$

TAP TO RETURN

9 - Answers

$y = \frac{1}{2}x$

TAP TO RETURN

10 - Answers

(a) $t = \frac{\pi}{3}$

(b) proof

TAP TO RETURN

11 - Answers

(a) $A = 1$, $B = 2$, $C = -2$

(b) $a = \frac{2}{3}, b = -\frac{4}{3}, c = 3$

TAP TO RETURN

12 - Answers

(b) $\frac{\pi}{3}+\frac{1}{4}$

(d) 0

TAP TO RETURN

13 - Answers

a) $1 + 6x + 27x^2, \ |x| < \frac{1}{3}$

b) $1 - \frac{4}{3}x + \frac{4}{3}x^2, \ |x| < \frac{3}{2}$

c) $\frac{1}{2} + \frac{3x}{4} + \frac{27x^2}{16}, \ |x| < \frac{4}{3}$

TAP TO RETURN

14 - Answers

$A = 2$, $B = -1$, $C = 4$

$$5 + \frac{3}{2}x^2 + \cdots$$

TAP TO RETURN

15 - Answers

https://www.madasmaths.com/archive/iygb_practice_papers/c3_practice_papers/c3_o_solutions.pdf

TAP TO RETURN