Data was collected about height above sea level, x metres, and the temperature,  $y^{o}C$ , at 7.00 am., on the same day in summer at nine places in Europe.

The PMCC was calculated to be -0.975.

Test this for negative correlation at the 5% significance level . Interpret your result in context.

#### 2

Find any point(s) of inflection of the following functions.

(a) 
$$f(x) = \cos^2 x - 2\sin x, 0 < x < 2\pi$$

(b) 
$$f(x) = -\frac{x^3 - 2x^2 + x - 1}{x - 2}, x \neq 2$$

(c) 
$$f(x) = -\frac{x^3}{x^2 - 4}, x \neq \pm 2$$

### 3

- a Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution.
- A company sells orchids of which 45% produce pink flowers.
- A random sample of 20 orchids is taken and X produce pink flowers.
- **b** Find P(X = 10).
- A second random sample of 240 orchids is taken.
- c Using a suitable approximation, find the probability that fewer than 110 orchids produce pink flowers.
- d The probability that at least q orchids produce pink flowers is 0.2. Find q.

4

A train engine of mass 6400kg is pulling a carriage of mass 1600kg along a straight horizontal railway track. The engine is connected to the carriage by a shunt which is parallel to the direction of motion of the coupling. The shunt is modelled as a light rod.

The engine provides a constant driving force of 12000N. The resistances to the motion of the engine and the carriage are modelled as constant forces of magnitude R N and 2000N respectively.

Given that the acceleration of the engine and the carriage is 0.5 ms<sup>-2</sup>.

(a) find the value of R

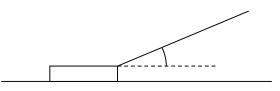
(b) show that the tension in the shunt is 2800N

5

(a) A particle is projected upwards with a speed of  $14 \text{ m s}^{-1}$ . Find for how long it is above 2 m.

(b) A ball is thrown vertically upwards from a height 1.6m above the ground, with a speed of 7 m s<sup>-1</sup>. Find the speed when it hits the ground.





A sledge has mass 30 kg. The sledge is pulled in a straight line along horizontal ground by means of a rope. The rope makes an angle  $20^{\circ}$  with the horizontal, as shown in Figure 3. The coefficient of friction between the sledge and the ground is 0.2. The sledge is modelled as a particle and the rope as a light inextensible string. The tension in the rope is 150 N. Find, to 3 significant figures,

(a) the normal reaction of the ground on the sledge,

(b) the acceleration of the sledge

When the sledge is moving at  $12 \text{ m s}^{-1}$ , the rope is released from the sledge.

(c) Find, to 3 significant figures, the distance travelled by the sledge from the moment when the rope is released to the moment when the sledge comes to rest.

7

The random variable X is normally distributed with mean  $\mu$  and variance  $\sigma^2$ .

- a Write down the distribution of the sample mean  $\overline{X}$  of a random sample of size *n*. (1 mark) A construction company wishes to determine the mean time taken to drill a fixed number of holes in a metal sheet.
- **b** Determine how large a random sample is needed so that the expert can be 95% certain that the sample mean time will differ from the true mean time by less than 15 seconds. Assume that it is known from previous studies that  $\sigma = 40$  seconds. (4 marks)

#### 8

Two helicopters *P* and *Q* are moving in the same horizontal plane. They are modelled as particles moving in straight lines with constant speeds. At noon *P* is at the point with position vector  $(20\mathbf{i} + 35\mathbf{j})$  km with respect to a fixed origin *O*. At time *t* hours after noon the position vector of *P* is  $\mathbf{p}$  km. When  $t = \frac{1}{2}$  the position vector of *P* is  $(50\mathbf{i} - 25\mathbf{j})$  km. Find

(a) the velocity of P in the form  $(a\mathbf{i} + b\mathbf{j}) \text{ km h}^{-1}$ ,

(b) an expression for  $\mathbf{p}$  in terms of t.

At noon Q is at O and at time t hours after noon the position vector of Q is q km. The velocity of Q has magnitude 120 km h<sup>-1</sup> in the direction of  $4\mathbf{i} - 3\mathbf{j}$ . Find

(c) an expression for  $\mathbf{q}$  in terms of t,

(d) the distance, to the nearest km, between P and Q when t = 2.

#### 9

Now, Integrate the following functions using an appropriate method when required:

- (a)  $\int -\sin(3x+1) dx$  (b)  $\int 4\cos\left(\frac{x}{2}\right) dx$
- (c)  $\int \tan x \, dx$
- (e)  $\int \sec^5 2x \tan 2x \, dx$
- (g)  $\int \sin^2 6x \, dx$

(i)  $\int 3\tan^2 4x \, dx$ 

(b)  $\int 4 \cos\left(\frac{x}{2}\right) d$ (d)  $\int \cot 4x dx$ 

(f)  $\int \tan 5x \, dx$ 

(h)  $\int 3\cos^2 2x \, dx$ 

#### 10

A curve C has equation  $y = \frac{1}{2}e^{2x} - 4x + 1$ ,  $x \in R$ The point P lies on C where  $x = \ln 4$ .

(a) Show that the equation of the tangent to the curve is  $y = 12x + 9 - 32 \ln 2$ 

The point Q lies on C where  $x = \ln 2$ .

The normal to the curve at the point Q meets the tangent to the curve at the point P, at the point R.

(b) Show that the co-ordinates of R are  $(\ln 2, 9 - 20\ln 2)$ 

#### 11

A particle *P* of mass 0.5 kg moves under the action of a single force **F** Newtons. At time *t* seconds, the velocity **v** m s<sup>-1</sup> of *P* is given by;

 $v = 3t^2i + (1 - 4t)j.$ 

Find:

(a) the acceleration of P at time t seconds,

(b) the magnitude of **F** when t = 2.

#### 12

A circle has parametric equations  $x = 4 \sin t - 3$ ,  $y = 4 \cos t + 5$ .

(a) Find the Cartesian equation of the circle

(b) Draw a sketch of the circle

(c) Find the exact coordinates of the points of intersection of the circle with the y axis

### 13

(a) Show that  $\cos^4 x \equiv \frac{1}{8}\cos 4x + \frac{1}{2}\cos 2x + \frac{3}{8}$ 

(b) Hence find  $\int \cos^4 x \, dx$ 

#### 14

2a C a 60°

A ladder *AB*, of mass *m* and length 3*a*, has one end *A* resting on rough horizontal ground. The other end *B* rests against a smooth vertical wall. A load of mass 2*m* is fixed on the ladder at the point *C*, where AC = a. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium at an angle of 60° with the ground.

Find the coefficient of friction between the ladder and the ground.

For the above problem, draw a complete and clear force diagram, including F<sub>max</sub> and R at A, perpendicular reaction S at B, the weight of the ladder, and the weight of the load at C.

Friction is limiting so you can use  $F_{max} = \mu R$ 

You can take moments about any point you like. Point A is easy then the equation will not have  $F_{max}$  or R. However, you can take moments about a point not even on the ladder – if you choose the point where R and S intersect, these forces will not then be in the equation, and you can find  $F_{max}$  directly. Construct three equations, then solve to find the coefficient of friction.

#### 15

A test statistic has distribution B(30, p). Given that:

 $H_0: p = 0.4, H_1: p < 0.4$ 

(a) Find the critical region for the test statistic at the 5% significance level

A test statistic has distribution B(40, p). Given that:

 $H_0: p = 0.2, \quad H_1 : p \neq 0.2$ 

(b) Find the critical region for the test statistics at the 5% significance level

16

Complete this old spec paper

https://www.madasmaths.com/archive/iygb\_practice\_papers/c2\_practice\_paper s/c2\_r.pdf

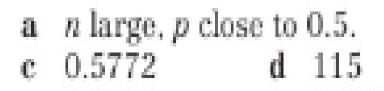
### 1 - Answers

Reject  $H_0$ . The greater the altitude the lower the temperature.

### 2 - Answers

- (a)  $\left(\frac{\pi}{6}, -\frac{1}{4}\right), \left(\frac{5\pi}{6}, -\frac{1}{4}\right)$ (b) (1, -1)
  - (c) (0, 0)





**b** 0.1593

### 4 - Answers

(a) 6000N

(b) Proof

### 5 - Answers

(a) 2.56 seconds

(b) 9.0ms<sup>-1</sup>

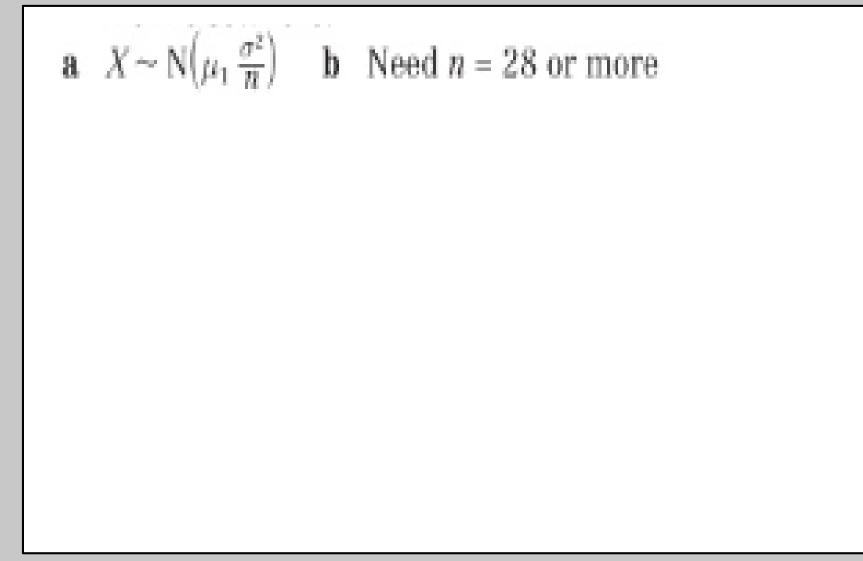
### 6 - Answers

(a) 243N

(b) 3.08ms<sup>-2</sup>

(c) 36.7m

7 - Answers



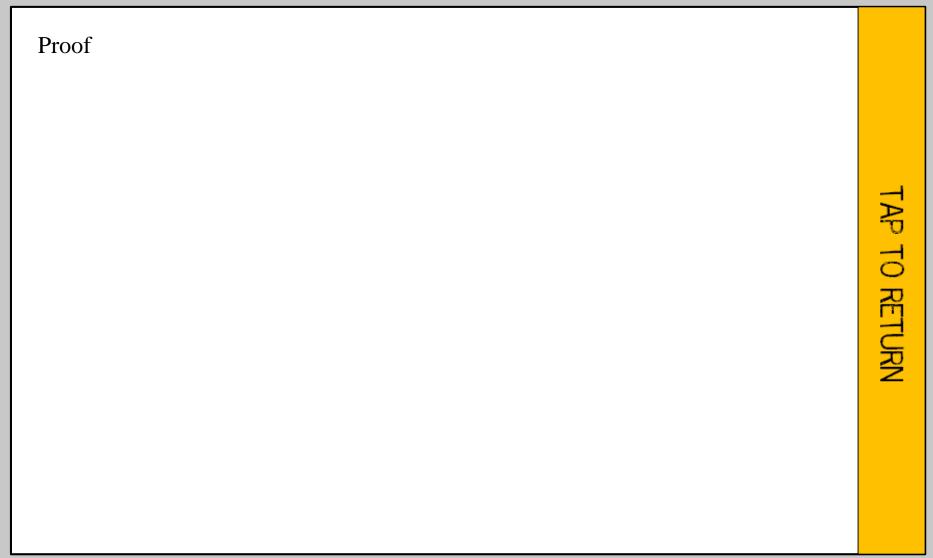
### 8 - Answers

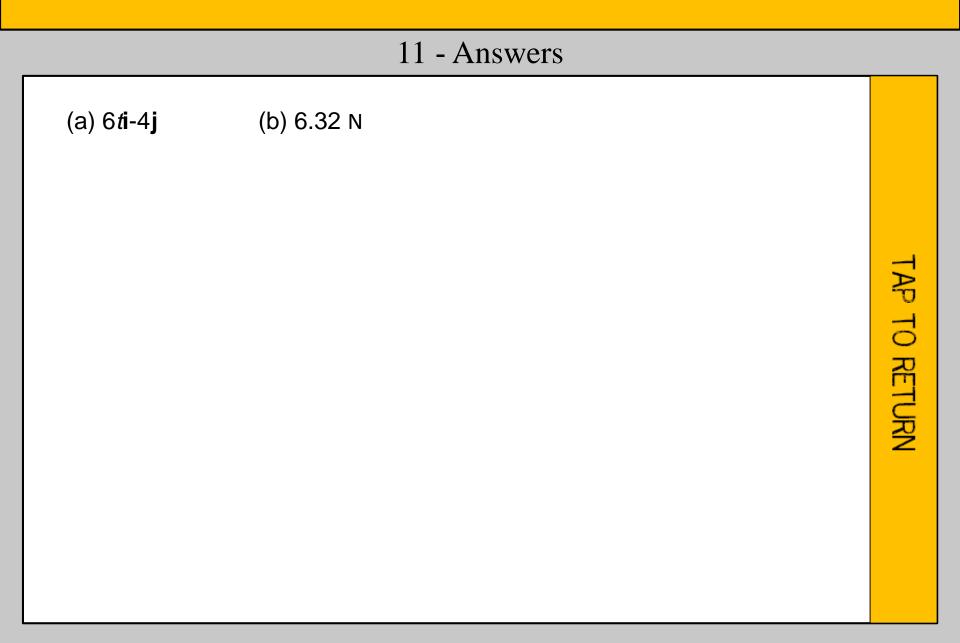
- (a) 60**i** 120**j**
- (b)  $\mathbf{p} = 20\mathbf{i} + 35\mathbf{j} + (60\mathbf{i} 120\mathbf{j})\mathbf{t}$
- (c) q = 96ti 72tj
- (d) 80 km

#### 9 – Answers

(a)  $\frac{1}{3}\cos(3x + 1) + c$ (b)  $8\sin(\frac{x}{2}) + c$ (c)  $-\ln(\cos x) + c$ (d)  $\frac{1}{4}\ln(\sin 4x) + c$ (e)  $\frac{1}{10}\sec^5 2x + c$ (f)  $-\frac{1}{5}\ln(\cos 5x) + c$ (g)  $\frac{1}{2}x - \frac{1}{24}\sin 12x + c$ (h)  $\frac{3}{2}x + \frac{3}{8}\sin 4x + c$ (i)  $\frac{3}{4}\tan 4x - 3x + c$ 

### 10 - Answers





### 12 - Answers

- (a)  $(x+3)^2 + (y-5)^2 = 16$
- (b) check desmos
- (c)  $(0, 5 + \sqrt{7}), (0, 5 \sqrt{7})$

13 - Answers

(a) 
$$\cos^4 x = (\cos^2 x)^2 = \left(\frac{1+\cos 2x}{2}\right)^2 = \frac{1}{4} + \frac{1}{2}\cos 2x + \frac{1}{4}\cos^2 2x = \frac{1}{4} + \frac{1}{2}\cos 2x + \frac{1}{4}\left(\frac{1+\cos 4x}{2}\right) = \frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$$

(b) 
$$\frac{1}{32}\sin 4x + \frac{1}{4}\sin 2x + \frac{3}{8}x + c$$

### 14 - Answers

μ = 0.22

### 15 - Answers

(a)  $x \leq 7$ 

(b)  $x \le 2$  and  $x \ge 14$ 

### 16 - Answers

https://www.madasmaths.com/archive/iygb\_practice\_papers/c2\_practice\_ papers/c2\_r\_solutions.pdf