BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 17A

1

(a) Show that $\cos\theta - \sqrt{3}\sin\theta$ can be written in the form $R\cos(\theta + \alpha)$, with $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(b) Hence sketch the graph of $y = \cos\theta - \sqrt{3}\sin\theta$, $0 < \alpha < 2\pi$, giving the coordinates of points of intersection with the axes.

(c) Express $7\cos\theta - 24\sin\theta$ in the form $R\cos(\theta + \alpha)$, with $R > 0$ and $0 < \alpha < 90°$.

(d) The graph of $y = 7\cos\theta - 24\sin\theta$ meets the y-axis at P. State the coordinates of P.

(e) Write down the maximum and minimum values of $7\cos\theta - 24\sin\theta$.

(f) Deduce the number of solutions, in the interval $0 < \theta < 360°$, of the following equations:

(i) $7\cos\theta - 24\sin\theta = 15$
(ii) $7\cos\theta - 24\sin\theta = 26$

TAP FOR ANSWERS

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Solve the following equations, in the intervals given in brackets.

(a) $6\sin x + 8\cos x = 5\sqrt{3}$, $[0, 360°]$

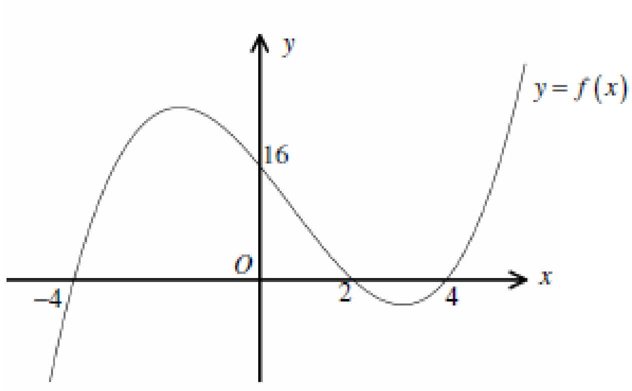
(b) $2\cos 3\theta - 3\sin 3\theta = -1$, $[0, 90°]$

(c) $8\cos\theta + 15\sin\theta = 10$, $[0, 360°]$

(d) $5\sin\frac{x}{2} - 12\cos\frac{x}{2} = 6.5$, $[-360°, 360°]$

TAP FOR ANSWERS

3



The figure above shows the graph of the curve with equation $y = f(x)$.
The curve crosses at the points $(-4,0)$, $(2,0)$ and $(4,0)$ and the y axis at the points $(0,16)$
Determine the equation of $f(x)$ in the form

$$f(x) = ax^3 + bx^2 + cx + d$$

Where a, b, c and d are constants.

TAP FOR ANSWERS

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4

A curve *C* has parametric equations

$$x = 6\cos 2t, \qquad y = 2\sin t, \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \lambda \operatorname{cosec} t$, giving the exact value of the constant λ.

(b) Find an equation of the normal to *C* at the point where $t = \frac{\pi}{3}$

Give your answer in the form $y = mx + c$, where m and c are simplified surds.

The cartesian equation for the curve *C* can be written in the form

$$x = \mathrm{f}(y), \quad -k < y < k$$

where f(y) is a polynomial in y and k is a constant.

(c) Find f(y)

(d) State the value of k.

TAP FOR ANSWERS

The curve C has equation

$$2\cos 3x \sin y = 1, \quad 0 \le x, y \le \pi.$$

a) Show that

$$\frac{dy}{dx} = 3\tan 3x \tan y.$$

The point $P\left(\frac{\pi}{12}, \frac{\pi}{4}\right)$ lies on C.

b) Show that an equation of the tangent to C at P is

$$y = 3x.$$

TAP FOR ANSWERS

Daily mean windspeed is modelled as being normally distributed with a standard deviation of 3.1 knots.

A random sample of 25 recorded daily mean windspeeds is taken at Heathrow in 2015.

Given that the mean of the sample is 12.2 knots, test at the 2.5% level of significance whether the mean of the daily mean windspeeds is greater than 9.5 knots.

State your hypotheses clearly.

TAP FOR ANSWERS

7

(a) Express $1.4\sin\theta - 5.6\cos\theta$ in the form $R\sin(\theta-\alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Round R and α to 3 decimal places.

(b) Hence find the maximum value of $1.4\sin\theta - 5.6\cos\theta$ and the smallest positive value of θ for which this maximum occurs.

The length of daylight $d(t)$ at a location in northern Scotland can be modelled using the equation $d(t) = 12 - 5.6\cos\left(\frac{360t}{365}\right) + 1.4\sin\left(\frac{360t}{365}\right)$

Where t is the numbers of days into the year:

(c) Calculate the minimum number of daylight hours in northern Scotland as given by this model.

(d) Find the value of t when this minimum number of daylight hours occurs.

TAP FOR ANSWERS

Express the following in the form given, where α is in radians and $0 < \alpha < \frac{\pi}{2}$.
Give your answer to 3 significant figures.

(a) $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$

(b) $2cosx - sinx$ in the form $R\cos(x + \alpha)$

TAP FOR ANSWERS

By splitting this integral into partial fractions first, and using your reverse chain rule methods, evaluate:

$\int_0^2 \frac{25x+1}{(2x-1)(x+1)^2}\,dx$ * hint – remember that $ln f(x)$ $differentiates\ to$ $\frac{f'(x)}{f(x)}$ *

TAP FOR ANSWERS

Solve the equations

(a) $\log_2 x + 4\log_x 2 = 5$

(b) $\log_3(2 - 3x) = \log_9(6x^2 - 19x + 2)$

TAP FOR ANSWERS

The term independent of x in the expansion of $\left(x^3 + \frac{a}{x^2}\right)^5$ is *-80*

Find the value of the constant a

TAP FOR ANSWERS

Sketch the following modulus graphs, writing down the co-ordinates of any points at which the graph meets the coordinate axes.

(a) $y = |2x + 3|$
(b) b) $y = |2x^2 + 5x - 12|$
(c) y= $|2^x - 2|$
(d) $y = |\sin 2x|$

TAP FOR ANSWERS

An oil spillage on the surface of the sea remains circular at all times.

The radius of the spillage , $r$km, is increasing at the constant rate of 0.5kmh^{-1}

(a) Find the rate at which the area of the spillage, A km^2, is increasing when the circle's radius has reached 10km

A different oil spillage on the surface of the sea also remains circular at all times.

The area of this spillage, Akm^2, is increasing at the rate 0.5km^{2}h^{-1}.

(b) Show that when the area of the spillage has reached 10km^2, the rate at which the radius, r, of the spillage is increasing is

$$\frac{1}{4\sqrt{10\pi}} kmh^{-1}$$

TAP FOR ANSWERS

The function t is defined by $\mathrm{t}: x \mapsto 5 - 2x$

Solve the equation $\mathrm{t}^2(x) - \left(\mathrm{t}(x)\right)^2 = 0$

TAP FOR ANSWERS

14

Complete this old spec paper

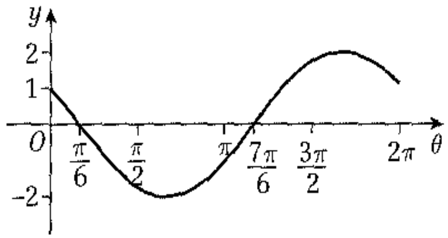
https://www.madasmaths.com/archive/iygb_practice_papers/c3_practice_papers/c3_n.pdf

TAP FOR ANSWERS

1 - Answers

(a) $\cos\theta - \sqrt{3}\sin\theta \equiv R\cos(\theta + \alpha)$ gives $R = 2, \alpha = \frac{\pi}{3}$

(b) $y = 2\cos\left(\theta + \frac{\pi}{3}\right)$



(c) $25\cos(\theta + 73.7°)$

(d) (0, 7)

(e) 25, -25

(f) (i) 2   (ii) 0

TAP TO RETURN

2 - Answers

(a) 6.9°, 66.9°

(b) 16.6°, 65.9°

(c) 8.0 °, 115.9°

(d) -165.2°, 74.8°

TAP TO RETURN

3 - Answers

$$y = -\frac{1}{2}x^3 - x^2 - 8x + 16$$

TAP TO RETURN

4 - Answers

ANS:

a) $\lambda = -\frac{1}{12}$

b) $y = 6\sqrt{3}x + 19\sqrt{3}$

c) $f(y) = 6 - 3y^2$

d) k = 2

TAP TO RETURN

5 - Answers

PROOF

TAP TO RETURN

6 - Answers

H_0: $\mu = 9.5, H_1$: $\mu > 9.5$. Critical region is $\bar{X} \geq 10.715$. $\bar{x} = 12.2 > 10.715$, so reject H_0 and conclude that the mean daily windspeed is greater than 9.5 knots.

TAP TO RETURN

7 - Answers

(a) $R = 5.772, \alpha = 75.964°$

(b) 5.772 when $\theta = 166.0°$

(c) 6.228 hours

(d) 350.8 days

TAP TO RETURN

8 - Answers

Check solutions by choosing a random value for θ (e.g. $\theta = 0.7rad$) and substituting this back into the two different forms

TAP TO RETURN

9 - Answers

$$\frac{16}{3}$$

TAP TO RETURN

10 - Answers

(a) 2, 16

(b) $-\frac{1}{3}, -2$

TAP TO RETURN

11 - Answers

$a = -2$

TAP TO RETURN

12 - Answers

Use desmos to check.

TAP TO RETURN

BHASVIC MαTHS

A1 DOUBLES ASSIGNMENT 17A

13 - Answers

(a) $10\pi\ km^2h^{-1}$ $(31.4km^2h^{-1})$

TAP TO RETURN

14 - Answers

$3 \pm \frac{\sqrt{6}}{2}$

TAP TO RETURN

15 - Answers

https://www.madasmaths.com/archive/iygb_practice_papers/c3_practice_papers/c3_n_solutions.pdf

TAP TO RETURN