1

Sketch the following functions: show clearly any asymptotes, vertical and horizontal, and any crossings with the coordinate axes.

- (a) $y = 1 2e^x$
- (b) $y = 2 + \ln(x + 1)$
- (c) $y = 10e^{2x}$

2

Corinne and her brother Dermot are lifted by their parents onto the two ends of a rope which is slung over a large horizontal branch. When their parents let go of them Dermot, whose mass is 54kg, begins to descend with an acceleration of 1ms⁻². By modelling the children as a pair of particles connected by a light inextensible string, and the branch as a smooth pulley,

- (a) Show that Corinne's mass is 44kg
- (b) Calculate the tension in the rope
- (c) Find the force on the branch

(d) In a more sophisticated model, the branch is assumed to be rough. Explain what effect this would have on the initial acceleration of the children.

3

A particle is projected from a point on level ground with speed u m s⁻¹ and angle of elevation α . The maximum height reached by the particle is 42 m above the ground and the particle hits the ground 196 m from its point of projection.

Find the value of α and the value of u.

4

In this question \mathbf{i} and \mathbf{j} are the unit vectors due east and due north respectively, and \mathbf{k} is the unit vector acting vertically upwards.

A BASE jumper descending with a parachute is modelled as a particle of mass 50 kg subject to forces describing the wind, **W**, and air resistance, **F**, where:

W = (20i + 16j) NF = (-4i - 3j + 45k) N

(a) With reference to the model, suggest a reason why the k component of F is greater than the other components.

(b) Taking g = 9.8 m s⁻², find the resultant force acting on the BASE jumper.

(c) Given that the BASE jumper starts from rest and travels a distance of 180 m before landing, find the total time of the descent.

5

- a) A uniform plank, of length 3 m and mass 5 kg, rests in a horizontal equilibrium on two supports, one a the end of the plane and the other 1 m from the other end. Find the reaction force supplied by each support.
- b) A uniform beam of length 2m and mass 20 kg is suspended horizontally by wires at either end. A painted of weight 80 kg is standing 0.5 m from one end of the beam. Find the tension in each of the ires

6

- a) A non-uniform plank *AB*, of length 5 m and mass 4 kg, rests in equilibrium on two supports, one at each end. The centre of mass of the plank is located 1.2 m from end *A*. Find the reaction force in each support.
- b) A non-uniform plank of length 3 m rests in equilibrium on two supports, located 60 cm and 80 cm from each end. The reaction forces in the two supports are equal. Find the position of the centre of mass of the plank

7

A uniform ladder of length 4 m and mass 30 kg rests against a smooth vertical wall. The ladder makes a 70° angle with the horizontal ground and is in limiting equilibrium. Find the coefficient of friction between the ladder and the ground.

8

a) The random variable X follows the normal distribution N(14,49). Find x if:
i) P(X < x) = 0.8
ii) P(X < x) = 0.46
b) The random variable X follows the normal distribution N(36.5, 10). Find x if:
i) P(X > x) = 0.9
ii) P(X > x) = 0.4
c) The random variable X follows the normal distribution N(0,12). Find x if:
i) P(|X| < x) = 0.5
ii) P(|X| < x) = 0.8

9

If $X \sim N(\mu, \sigma^2)$, find μ and σ when: *a)* P(X > 7) = 0.8 and P(X < 6) = 0.1 *b)* P(X > 150) = 0.3 and P(X < 120) = 0.4 *c)* P(X > 0.1) = 0.4 and $P(X \ge 0.6) = 0.25$ *d)* P(X > 700) = 0.8 and $P(X \ge 400) = 0.99$

TAP FOR ANSWERS

10

In each of the following situations it is believed that $X \sim N(\mu, 100)$. Find the acceptance region in each of the following cases.

<i>a</i>) $H_0: \mu = 60$	$H_1: \mu \neq 60$	5% significance	n=16
<i>b)</i> $H_0: \mu = 120$	$H_1: \mu \neq 120$	10% significance	n=30
<i>c</i>) $H_0: \mu = 80$	$H_1: \mu > 80$	1% significance	n=18
<i>d</i>) $H_0: \mu = 750$	$H_1: \mu > 750$	2% significance	n=45
<i>e)</i> $H_0: \mu = 80.$	4 $H_1: \mu < 80.4$	10% significance	n=120
<i>f</i>) $H_0: \mu = 93$	<i>H</i> ₁ : <i>µ</i> < 93	5% significance	n=400

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11

For a particle moving in two dimensions, the displacement vector from the starting point is given by

$$s = \begin{pmatrix} 3t^3 - 4t \\ t^4 - 2t^3 + t \end{pmatrix}$$

a) The components of the displacement vector give parametric equations of the trajectory of the particle, x = x(t), y = y(t). Use parametric differentiation to find the gradient of the tangent to this curve, $\frac{dy}{dx}$, when t = 3

b) Find the velocity vector when t = 3. What do you notice?

12

A particle moves with acceleration $a = ((12 \cos 2t)i - (12 \sin 2t)j) ms^{-2}$. Its initial velocity is $v(0) = (6j)ms^{-1}$

- a) Show that the speed of the particle is constant
- b) By considering the *x* and *y* components of the displacement vector, show that the particle moves in a circle.

13

Use proof by contradiction to prove the following

- a) Prove that if *ab* is even, with *a*, *b* integers, then at least one of them is even
- b) Prove that $\sqrt[3]{2}$ is irrational
- c) Prove that $\log_2 3$ is irrational

14

Find the first three terms of the binomial expansion of each of the following expressions, stating the range of convergence

a) $(1 + x)^{-2}$ PUT LATER – WE ONLY COVER AT THE END OF THIS WEEK!!! (I'd leave it in DJM)



15

Complete this old spec paper

https://www.madasmaths.com/archive/iygb_practice_papers/c1_practice_papers

<u>/c1 r.pdf</u>

1 - Answers

Check your graphs on google TAP TO RETURN

2 - Answers

(b) T=475N (3sf) (c) 950N (3sf) (d) Friction opposes motion, therefore accelerates less. TAP TO RETURN

3 - Answers

 $\alpha = 40.6^{\circ} (\text{nearest } 0.1^{\circ})$ u = 44 (2 s.f.)TAP TO RETURN

4 - Answers

(a) Air resistance acts in opposition to the motion of the BASE jumper. The motion downwards will be greater than the motion in the other directions.

(b) (16i + 13j - 40k) N

(c) 20 seconds

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- a) 37.5 n, 12.5N
- b) 300 N, 700N

- a) 9.41 N, 29.8N
- b) 80 cm from each support

7 answers

2m or 6m

8 answers				
a) i) b) i) c) i)	i)19.9 i)32.4 i) 2.34	ii) 13.3 ii) 37.3 ii)4.44		TAP FOR ANSWERS

9 answers

a) $\mu = 8.91, \sigma = 1.19$ b) $\mu = 130, \sigma = 38.6$ c) $\mu = -0.201, \sigma = 1.19$ d) $\mu = 870, \sigma = 202$

- a) $55.1 < \overline{X} < 64.0$ b) $117 < \overline{X} < 123$ c) $\overline{X} < 85.5$ d) $\overline{X} < 753$ e) $\overline{X} > 79.2$
- *f*) $\bar{X} > 92.2$







14 answers

a)
$$1 - 2x + 3x^2; |x| < 1$$

b) $1 - 3x + 6x^2; |x| < 1$
c) $1 + \frac{x}{3} - \frac{x^2}{9}; |x| < 1$
d) $1 + \frac{x}{4} - \frac{3x^2}{32}; |x| < 1$
e) $1 - x - \frac{x^2}{2}; |x| < \frac{1}{3}$
f) $1 - \frac{3x}{2} - \frac{9x^2}{8}; |x| < \frac{1}{3}$
g) $\frac{1}{4} + \frac{x}{16} + \frac{x^2}{64}; |x| < 4$
h) $\frac{1}{5} + \frac{x}{25} + \frac{x^2}{125}; |x| < 5$

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15 answers

a) <u>https://www.madasmaths.com/archive/iygb_practice_papers/c1_practice_p</u> <u>apers/c1_r_solutions.pdf</u>