BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 16A

1

Find:

(a) $\int 3sec4x\ tan4x\ dx$

(b) $\int -2\sec^4 2x\ tan2x\ dx$

(c) $\int \frac{2x-1}{x^2-x}dx$

(d) $\int \frac{2}{3-x}+\frac{6}{4x+1}dx$

TAP FOR ANSWERS

2

For each of the following functions, whose domain is the set of **positive** real numbers, sketch the
function and hence state the range.

For each function find its inverse

(a) $f(x) = \frac{1}{x+1}$

(b) $f(x) = (x+1)^2 - 1$

(c) $f(x) = x^2 + 4x + 5$

TAP FOR ANSWERS

3

(a)Sketch the two inequalities $y < (2 - x)(3 + x)$ and $y - x \geq 2$

(b) shade the region that satisfies both inequalities

TAP FOR ANSWERS

4

For each of the following functions, find the interval on which the function is:
(i) convex (ii) concave

(a) $f(x) = x^3 - 3x^2 + x - 2$

(b) $f(x) = x^4 - 3x^3 + 2x - 1$

(c) $f(x) = \sin x, 0 < x < 2\pi$

(d) $f(x) = x^2 + 3x - 7$

(e) $f(x) = e^x - x^2$

(f) $f(x) = \ln x, x > 0$

TAP FOR ANSWERS

5

Differentiate tan x from first principles

TAP FOR ANSWERS

6

Given that $e^{2x} + e^{2y} = xy$, find $\frac{dy}{dx}$ in terms of x and y.

TAP FOR ANSWERS

7

Given that $\int_0^{\theta} 4 \sin 2x \cos^4 2x \, dx = \frac{4}{5}$ where $0 < \theta < \pi$, find the exact value of θ.

TAP FOR ANSWERS

8

Evaluate the following

$$\lim_{\delta x \to 0} \sum_{x=1}^{2} \left(x^{\frac{3}{2}} - 8x^{-\frac{3}{2}}\right)^2 dx$$

TAP FOR ANSWERS

9

Find:

(a) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 4sin^2\left(\frac{x}{2}\right) dx$

(b) $\int \frac{2}{\cos^2\left(\frac{x}{2}\right)} dx$

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(a) Sketch the graph of $y = |2x + a|, a > 0$, showing the coordinates of the points where the graph meets the coordinate axes.

(b) On the same axes, sketch the graph of $y = \frac{1}{x}$

(c) Explain how your graphs show that there is only one solution of the equation $x|2x + a| - 1 = 0$

(d) Find, using algebra, the value of x for which $x|2x + a| - 1 = 0$.

TAP FOR ANSWERS

Solve the following equations in the range $0 \leq x \leq 360°$

(a) $\cos 2x = 3 \sin x + 2$

(b) $\sec^2 2x = 2tan2x$

(c) $cosec^2\left(\frac{x}{2}\right) = \sqrt{3}\cot\left(\frac{x}{2}\right) + 1$

TAP FOR ANSWERS

We can then write

$$4sin\theta + 3cos\theta = 5\sin(\theta + 36.87°)$$

Questions:

(a) Write $\sqrt{3}sin\theta + 3cos\theta$ in the form $R\sin(\theta + \alpha)$

(b) Write $12sin\theta + 5cos\theta$ in the form $R\sin(\theta + \alpha)$

TAP FOR ANSWERS

13

(a) Express $200\sin\theta - 150\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the value of α to 4 decimal places.

The electrical field strength, E V/m, in a microwave of width 25 cm can be modelled using the equation

$$E = 1700 + 200\sin\left(\frac{4\pi x}{25}\right) - 150\cos\left(\frac{4\pi x}{25}\right)$$

where x is the distance in cm from the left hand edge of the microwave oven.

(b) (i) Calculate the maximum value of E predicted by this model.
(ii) Find the values of x, for $0 \leq x < 25$, where this maximum occurs.

(c) Food in the microwave will heat best when the electric field strength at the centre of the food is above 1800 V/m. Find the range of possible locations for the centre of the food.

TAP FOR ANSWERS

14

What function approximates

$$\frac{cosx+tanx-1}{sinx}$$

when x is small?

Use your function to find the value of $x = \frac{\pi}{20}$ leaving your answer in terms of π.

TAP FOR ANSWERS

15

A curve C is given parametrically by the equations

$$x = 4t^2 + t, \quad y = \frac{1}{2}t^2 + 2t^3, \quad t \in \mathbb{R}.$$

The point $A\left(\frac{1}{2}, -\frac{1}{8}\right)$ lies on C.

a) Show that the gradient at A is $-\frac{1}{3}$.

b) By considering $\frac{y}{x}$, or otherwise, show that a Cartesian equation of C is

$$x^3 = 16y^2 + 2xy.$$

TAP FOR ANSWERS

15

Complete this old spec paper

https://www.madasmaths.com/archive/iygb_practice_papers/c3_practice_papers/c3_m.pdf

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 16A

1 - Answers

(a) $\frac{3}{4}sec4x + c$

(b) $-\frac{1}{4}sec^4 2x + c$

(c) $\ln|x^2 - x| + c$

(d) $-2\ln|3 - x| + \frac{3}{2}\ln|4x + 1| + c$

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 16A

2 – Answers

(a) $0 < f(x) < 1;\ f^{-1}(x) = \frac{1-x}{x}$

(b) $f(x) \geq 0;\ f^{-1}(x) = (x+1)^{\frac{1}{2}} - 1$

(c) $f(x) \geq 5;\ f^{-1}(x) = (x-1)^{\frac{1}{2}} - 2$

TAP TO RETURN

BHASVIC MαTHS
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3 - Answers

Proof

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 16A

4 - Answers

(a) (i) $(1, \infty)$ (ii) $(-\infty, 1)$

(b) (i) $(-\infty, 0) \cup \left(\frac{3}{2}, \infty\right)$ (ii) $\left(0, \frac{3}{2}\right)$

(c) (i) $(\pi, 2\pi)$ (ii) $(0, \pi)$

(d) (i) nowhere (ii) $(-\infty, \infty)$

(e) (i) $(\ln 2, \infty)$ (ii) $(-\infty, \ln 2)$

(f) (i) nowhere (ii) $(0, \infty)$

TAP TO RETURN

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5 - Answers

$\sec^2 x$

TAP TO RETURN

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A1 DOUBLES ASSIGNMENT 16A

6 - Answers

$\frac{dy}{dx} = \frac{y-2e^{2x}}{2e^{2y}-x}$

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7- Answers

$\theta = \frac{\pi}{2}$

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8 - Answers

$\frac{47}{4}$

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9 - Answers

(a) $\frac{\pi}{6} - \sqrt{3} + \sqrt{2}$

(b) $4\tan\left(\frac{x}{2}\right) + c$

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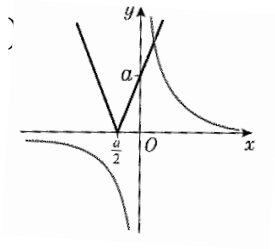
A1 DOUBLES ASSIGNMENT 16A

10 - Answers

(a& b)



(c) One intersection point

(d) $x = \frac{-a+\sqrt{a^2+8}}{4}$

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11 - Answers

(a) 210°, 330 °, 270 °

(b) 292.5, 202.5, 112.5, 22.5 degrees

(c) 60, 180 degrees

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12 - Answers

Check solutions by choosing a random value for θ (e.g. $\theta = 30°$) and substituting this back into the two different forms, in the same way you did at the beginning of this question.

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13 - Answers

(a) $R = 250, \alpha = 0.6435$

(b) (i) 1950 V/m (ii) x = 4.41 cm, x = 16.91 cm

(c) $2.10 \leq x \leq 6.71, 14.60 \leq x \leq 19.21$

TAP TO RETURN

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14 - Answers

$1 - \frac{x}{2}; \frac{1}{20}(1 - \frac{\pi}{2})$

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A1 DOUBLES ASSIGNMENT 16A

15 - Answers

Proof

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16 - Answers

https://www.madasmaths.com/archive/iygb_practice_papers/c3_practice_papers/c3_m_solutions.pdf

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