BHASVIC MαTHS

A1 DOUBLES ASSIGNMENT 15A

1

Solve the following equations on the interval $0 \leq x \leq 2\pi$

(a) $\sin 3x = -1$

(b) $\cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{2}}$

(c) $\tan\left(x + \frac{3\pi}{2}\right) = -1.4$

TAP FOR ANSWERS

2

Solve the following quadratic inequalities:

(a) $x^2 - 3x - 10 \leq 0$

(b) $x^2 + 7x + 12 \geq 0$

(c) $2x - 4x^2 + 6 < 0$

TAP FOR ANSWERS

3

Prove the identities

(a) $\frac{\sin\theta}{1+\cos\theta}+\frac{1-\cos\theta}{\sin\theta}\equiv\frac{2\sin\theta}{1+\cos\theta}$

(b) $\tan x+\cot x\equiv\sec x\operatorname{cosec} x$

(c) $\frac{\sin\theta}{1-\cos\theta}-\frac{\sin\theta}{1+\cos\theta}\equiv 2\cot\theta$

(d) $\cot 2x=\frac{\cot^2 x-1}{2\cot x}$

TAP FOR ANSWERS

Simplify the following expressions

$$\frac{x^2 - 8x + 15}{x^2 - 9} \times \frac{2x^2 + 6x}{(x-5)^2}$$

$$\frac{x^2 + 3x}{3x + 2} \div (x + 3)$$

$$\left(\frac{1}{x-3} - \frac{1}{x}\right) \div \frac{6x}{x^2 - 9}$$

TAP FOR ANSWERS

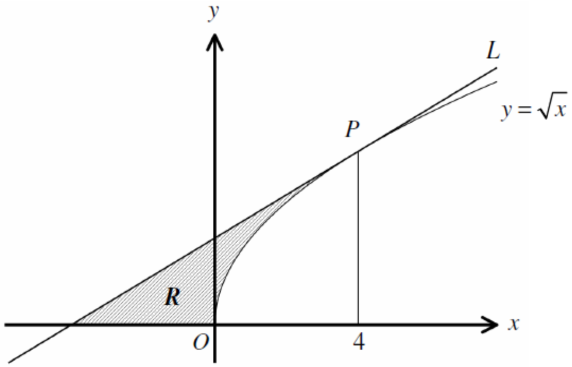
5

Assuming standard results for sin x and cos x, prove that:

(a) The derivative of arccos x is $-\frac{1}{\sqrt{1-x^2}}$

(b) The derivative of arctan x is $\frac{1}{1+x^2}$

TAP FOR ANSWERS



The figure above shows the graph of the curve C with equation

$$y = \sqrt{x}, \qquad x \geq 0$$

The point P lies on C where $x = 4$.
The straight line L is the tangent to C at P.

(a) Find an equation of L

The finite region R, shown shaded in the figure above, is bounded by the curve C, the tangent L and the x axis.

(b) Find the exact area of R

TAP FOR ANSWERS

7

The curve C has equation $y = \frac{1}{\cos x \sin x}, 0 < x \leq \pi$

(a) Find $\frac{dy}{dx}$

(b) Determine the number of stationary points of the curve C.

(c) Find the equation of the tangent at the point where $x = \frac{\pi}{3}$, giving your answer in the form $ax + by + c = 0$, where a, b and c are exact constants to be determined.

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 15A

8

(a) $\int 8cosec^2 x\, dx$

(b) $\int 4\cos\left(\frac{1}{2}x\right) dx$

(c) $\int -2sec4xtan4x\, dx$

(d) $\int cosx \sin^3 x\, dx$

(e) $\int \sin^2 x\, dx$

(f) $\int \tan^2 x\, dx$

TAP FOR ANSWERS

9

Given that $f(x) = \frac{2}{x-1} - \frac{6}{(x-1)(2x+1)}$, $x > 1$,

(a) Prove that $f(x) = \frac{4}{2x+1}$

(b) Find the range of f

(c) Find $f^{-1}(x)$ and state its domain

(d) State the range of $f^{-1}(x)$

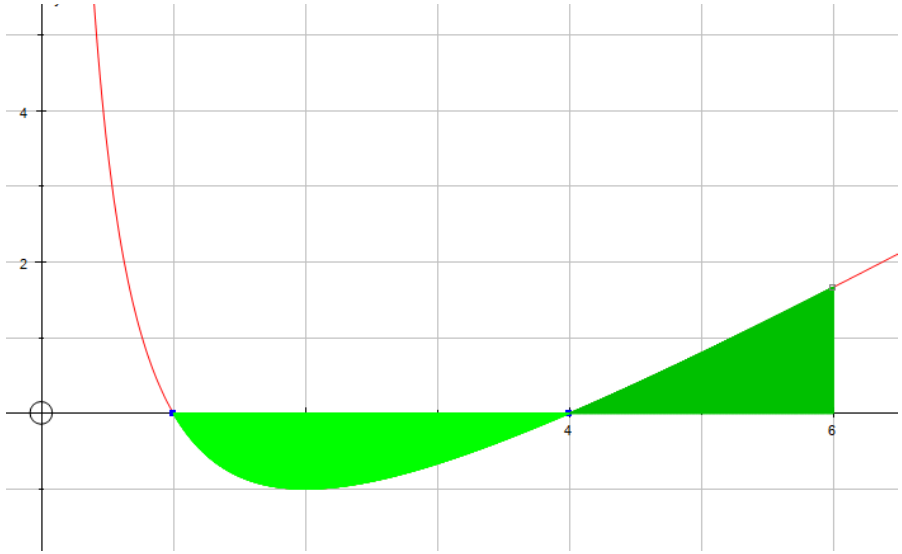
TAP FOR ANSWERS

Find the gradient of each curve at the given point

a) $x^2 + 3y^2 = 7$ at $(2, -1)$
b) $2x^3 - y^3 = -6$ at $(1,2)$
c) $\cos x + \sin y = 0$ at $(0, \pi)$
d) $\tan x + \tan y = 2$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$

TAP FOR ANSWERS

The diagram shows a part of the curve with equation $y = \frac{4}{x} + x - 5$



a) Show that the curve crosses the x-axis at $x = 1$ and $x = 4$

b) Find the exact value of the shaded area. Give your answers in the form $p - 4\ln q$, where p and q are rational numbers

TAP FOR ANSWERS

(a) Show that $\sin(A+B) + \sin(\mathrm{A}-\mathrm{B}) = 2\sin \mathrm{A}\cos \mathrm{B}$

(b) Hence, solve the equation $\sin\left(x+\frac{\pi}{6}\right) + \sin\left(x-\frac{\pi}{6}\right) = 3\cos x\,,\ 0 \le \theta \le \pi$

TAP FOR ANSWERS

13

The graph of

$$y = \frac{x - a}{x + 2}$$

Has a gradient 1 at point $(a, 0)$ and $a \neq -2$. Find the value of a

TAP FOR ANSWERS

14

Let $f(x) = \ln(x-1) + \ln 3$, for $x > 1$

a) Find $f^{-1}(x)$

Let $g(x) = e^x$ for $x \in \mathbb{R}$

b) Find $gf(x)$ giving your answer in the form $ax + b$, where $a, b, \in \mathbb{Z}$. Find the domain and range of $gf(x)$

TAP FOR ANSWERS

15

Complete this old spec paper

https://www.madasmaths.com/archive/iygb_practice_papers/c1_practice_papers/c1_q.pdf

TAP FOR ANSWERS

Answers -1

(a) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(b) $\frac{\pi}{2}$

(c) 0.62, 3.76

TAP TO RETURN

Answers - 2

(a) $-2 \leq x \leq 5$

(b) $x \leq 4$ or $x \geq -3$

(c) $x < -1$ or $x > 3/2$

TAP TO RETURN

Answers - 3

Proof

TAP TO RETURN

Answers - 4

$$\frac{2x}{x-5}$$

$$\frac{x}{3x+2}$$

$$\frac{x+3}{2x^2}$$

TAP TO RETURN

Answers - 5

(a) (a) Let $y = \arccos x \Rightarrow \cos y = x \Rightarrow \frac{dy}{dx} = -\sin y$

$\frac{dy}{dx} = -\frac{1}{\sin y} = -\frac{1}{\sqrt{1-\cos^2 y}} = -\frac{1}{\sqrt{1-x^2}}$

(b) Let $y = \arctan x$

Then, $\tan y = x$

$\frac{dy}{dx} = \sec^2 y$

$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$

TAP TO RETURN

Answers - 6

(a) $\frac{1}{4}x + 1$

(b) $\frac{8}{3}$

TAP TO RETURN

Answers - 7

(a) $\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x}$

(b) 2

(c) $24x - 9y + 12\sqrt{3} - 8\pi = 0$

TAP TO RETURN

Answers - 8

(a) $-8cotx + c$

(b) $8\sin\left(\frac{1}{2}x\right) + c$

(c) $-\frac{1}{2}sec4x + c$

(d) $\frac{1}{4}\sin^4 x + c$

e) $\frac{1}{2}x - \frac{1}{4}sin2x + c$

f) $tanx - x + c$

TAP TO RETURN

Answers - 9

(a) Proof

(b) $0 < f(x) < \frac{4}{3}$

(c) $f^{-1}(x) = \frac{4-x}{2x}, 0 < x < \frac{4}{3}$ (= the range of f)

(d) $f^{-1}(x) > 1$ (= the domain of f)

TAP TO RETURN

Answers - 10

(a) $\frac{2}{3}$
(b) $\frac{1}{2}$
(c) 0
(d) -1

TAP TO RETURN

BHASVIC MαTHS

A1 DOUBLES ASSIGNMENT 15A

Answers - 11

$$\frac{15}{2} - 4\ln\left(\frac{8}{3}\right)$$

TAP TO RETURN

Answers - 12

(a) Proof

(b) $\frac{\pi}{3}$

TAP TO RETURN

Answers - 13

(a) $a = -1$

TAP TO RETURN

Answers - 14

(a) $f^{-1}(x) = \frac{e^x}{3} + 1$

(b) $gf(x) = 3x - 3$, domain $x > 1$ range $gf(x) > 0$

TAP TO RETURN

Answers - 15

(a) https://www.madasmaths.com/archive/iygb_practice_papers/c1_practice_papers/c1_q_solutions.pdf

TAP TO RETURN