1 a

Sketch and state the ranges of the following functions (defined on $\mathbb{R}$): show asymptotes clearly

(a) $f(x) = \frac{1}{x+2} + 1$

(b) $g(x) = 1 - e^{2x}$

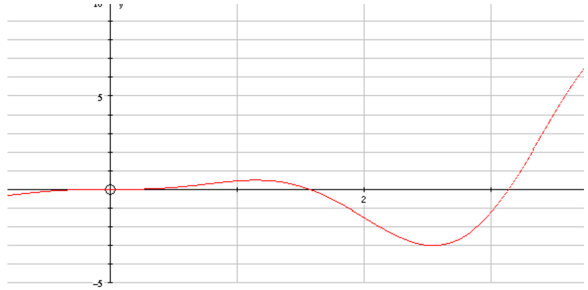
(c) $h(x) = \ln(1 + x)$

TAP FOR ANSWERS

1 b

The graph shown is the curve of $y = f(x)$.

The curve crosses the $x$ axis at $A\left(\frac{8}{5}, 0\right)$, $B\left(\frac{16}{5}, 0\right)$ and has a turning point at $C\left(\frac{7}{2}, -3\right)$



Sketch, showing the new coordinates of $A$, $B$ and $C$:

(a) $f(2x)$

(b) $3f(x)$

(c) $f(x) + 3$

TAP FOR ANSWERS

2

Given

$$f(x) = 2x^3 - 9x^2 - 11x + 30$$

(a) Show that $x = 5$ is a solution of the equation $f(x) = 0$

(b) Factorise $f(x)$ as a product of three linear factors

(c) Sketch the graph of $f(x)$

(d) Find the $x$ coordinate of the points where the line with equation $y = 7x + 30$ meets the graph of $f(x)$

TAP FOR ANSWERS

3

Differentiate each of the following functions

(a) $\ln 8x$

(b) $2e^x - 2\ln(x^2)$

(c) $\frac{3x}{1-\sin x}$

(d) $\frac{e^x}{\ln x}$

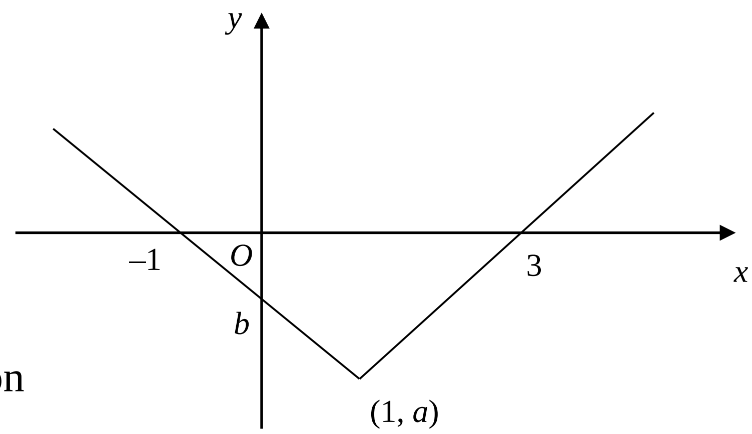
(e) $3\ln x - \ln 3x$

(f) $\ln\sqrt{x} - 2\ln({}^1/{}_x)$

TAP FOR ANSWERS

4

Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the $x$-axis at $(3, 0)$. The other line meets the $x$-axis at $(–1, 0)$ and the $y$-axis at $(0, b)$, $b < 0$.




In separate diagrams, sketch the graph with equation

(a) $y = f(x + 1)$,

(b) $y = f(|x|)$.

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $f(x) = |x – 1| – 2$, find

(c) the value of $a$ and the value of $b$,

(d) the value of $x$ for which $f(x) = 5x$.

TAP FOR ANSWERS

5

The points $A$ and $B$ have coordinates (4, 6) and (12, 2) respectively. The straight line $l_1$ passes through $A$ and $B$.

(a) Find an equation for $l_1$ in the form $ax + by = c$, where $a$, $b$ and $c$ are integers.

The straight line $l_2$ passes through the origin and has gradient –4.

(b) Write down an equation for $l_2$.

(c) The lines $l_1$ and $l_2$ intersect at the point $C$. Find the exact coordinates of the mid-point of $AC$.

TAP FOR ANSWERS

# BHASVIC MαTHS

## A1 DOUBLES ASSIGNMENT 14A

### 6a

Integrate the following

$$\int 5(x+3)^4\,dx$$

$$\int (x-2)^5\,dx$$

$$\int (4x-5)^7\,dx$$

$$\int \left(\frac{1}{8}x+1\right)^3\,dx$$

$$\int 4\left(3-\frac{1}{2}x\right)^6\,dx$$

$$\int (4-x)^8\,dx$$

TAP FOR ANSWERS

6b

Integrate the following

$$\int \frac{\tan 3x}{\cos 3x}\,dx$$

$$\int \frac{1}{\sin^2 x}\,dx$$

$$\int \sin 5x \cos x - \cos 5x \sin x \, dx$$

$$\int \frac{3 - \cos 2x}{\sin^2 2x}\,dx$$

$$\int \frac{\cos 2x}{\cos x + \sin x}\,dx$$

TAP FOR ANSWERS

7

(a) Prove that the derivative of sin (3x) is 3 cos(3x) from first principles

(b) Prove that the derivative of cos (3x) is -3 sin(3x) from first principles

TAP FOR ANSWERS

8

Solve the following equation on the interval $0 \leq \theta \leq 2\pi$. Give exact answers.

$$\sec^2 x + \tan x - 1 = 0$$
$$\sin 2x = 3 \sin x$$
$$\cos 2x - \sin^2 x = -2$$
$$5 \sin 2x = 3 \cos x$$
$$\tan 2x - \tan = 0$$

TAP FOR ANSWERS

9

A curve $C$ has equation $y = \frac{e^{2x}}{(x-2)^2}, x \neq 2$.

(a) Show that $\frac{dy}{dx} = \frac{Ae^{2x}(Bx-C)}{(x-2)^3}$ where $A$, $B$ and $C$ are integers to be found.

(b) Find the equation of the tangent of $C$ at the point $x = 1$.

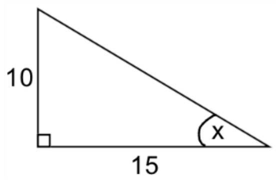
TAP FOR ANSWERS

# 10

NEW TECHNIQUES!

You will be familiar with using the inverse trig functions to calculate angles:
Eg. Calculate the angle $x$ in the triangle below:

We would write, $\tan x = \frac{opposite}{adjacent} = \frac{10}{15}$




$$x = tan^{-1}\left(\frac{10}{15}\right) = 34°$$

However, from now on we will use the notation $arctan$ instead of $tan^{-1}$ (although this is what most calculators use), to distinguish it from $\frac{1}{\tan x}$.

Therefore, the above would be written: $x = arctan\left(\frac{10}{15}\right) = 34°$

We know what the graphs of $sinx, cosx$ and $tanx$ look like, but what do the graphs of $arcsinx,\ arccosx$ and $\arctan x$ look like?

TAP FOR ANSWERS

10

NEW TECHNIQUES!

These can all be drawn by reflecting sections of the graphs sin(x), cos(x) and tan(x) respectively in the line y = x. This can be done following the steps below:

<u>Steps</u>

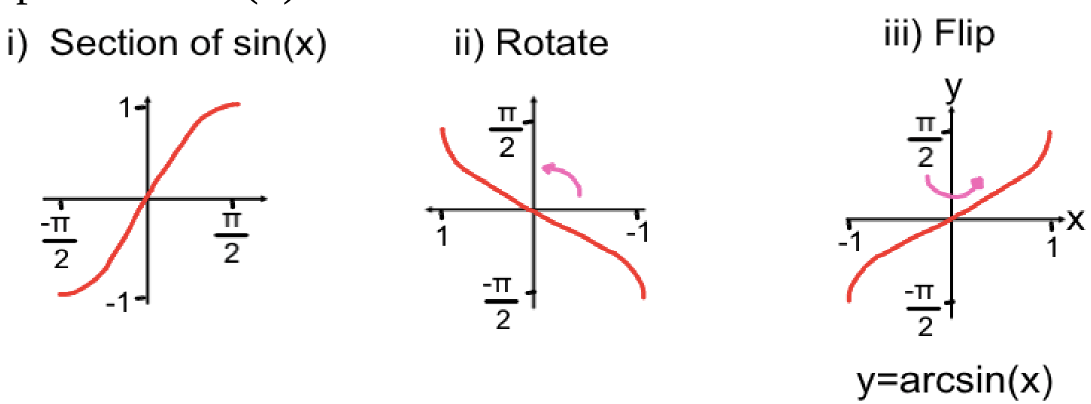
Draw section of original graph

Rotate 90° anticlockwise

Flip about y axis

Example,

Draw the graph of arcsin(x)

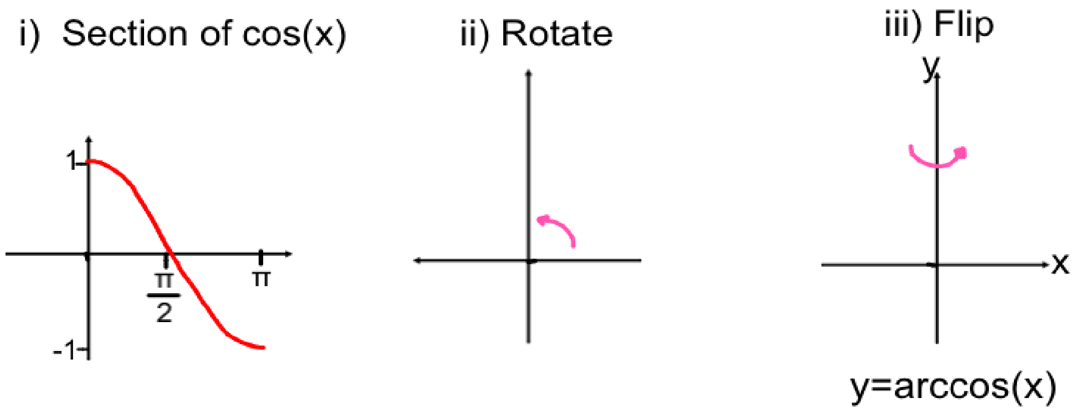



TAP FOR ANSWERS

NEW TECHNIQUES!

In the same way, complete the graphs for arccos(x) and arctan(x) below (don't forget to also rotate and flip your axes and any asymptotes!).

Draw the graph of arccos(x)

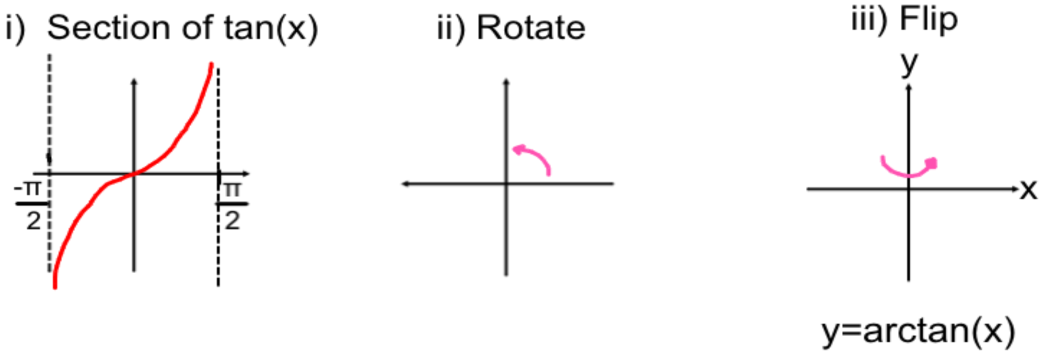



TAP FOR ANSWERS

10

NEW TECHNIQUES!

Draw the graph of arctan(x)




You need to be able to draw these confidently, so practise plotting each of these a couple of times.

(a) Which inverse function will never return a negative value? Explain your answer.

TAP FOR ANSWERS

11

Split the following into partial fractions:

$$\frac{x^2 + 5x + 7}{(x + 2)^3}$$

$$\frac{5x + 1}{x^3 + x^2}$$

$$\frac{1}{(x^2 - 4)(x - 2)}$$

TAP FOR ANSWERS

12

Curve $C$ has equation $x = (\arccos y)^2$. Show that

$$\frac{dy}{dx} = -\frac{\sqrt{1-\cos^2\sqrt{x}}}{2\sqrt{x}}$$

TAP FOR ANSWERS

13

Double angle and compound angle trig proofs

TAP FOR ANSWERS

14

$f(x) = 3x + 2$ and $g(x) = \frac{1}{x}$ with $x \neq 0$

(a) Find $f^{-1}(x)$, $g^{-1}(x)$ and $gf(x)$

(b) Show that $(gf)^{-1}(x) = f^{-1}g^{-1}(x) = \frac{1}{3}\left(\frac{1}{x} - 2\right)$

Note: you will need to show *both* that $f^{-1}g^{-1}(x) = \frac{1}{3}\left(\frac{1}{x} - 2\right)$ and that $(gf)^{-1}(x) = \frac{1}{3}\left(\frac{1}{x} - 2\right)$.

TAP FOR ANSWERS

15

Find the inverses of the following functions where each function is defined on its given domain, $x \in \mathbb{R}$

(a) $f(x) = (x-1)^2 + 4, x \geq 1$

(b) * $f(x) = x^2 + 4x - 1, x \geq -2$

(c) * $f(x) = x^2 + 4, x \geq -2$

* *complete the square first*

TAP FOR ANSWERS

15

Complete this old spec paper
https://www.madasmaths.com/archive/iygb_practice_papers/c1_practice_papers/c1_p.pdf

TAP FOR ANSWERS

# 1a - Answers

*Check sketches on Desmos*

(a) $f(x) \in \mathbb{R},\ f(x) \neq -2$

(b) $g(x) \in \mathbb{R},\ g(x) < 1$

(c) $h(x) \in \mathbb{R},$

TAP TO RETURN

# 1 b - Answers

(a) $A' = \left(\frac{4}{5}, 0\right), B' = \left(\frac{8}{5}, 0\right), C' = \left(\frac{7}{4}, -3\right)$

(b) $A' = \left(\frac{8}{5}, 0\right), B' = \left(\frac{16}{5}, 0\right), C' = \left(\frac{7}{2}, -9\right)$

(c) $A' = \left(\frac{8}{5}, 3\right), B' = \left(\frac{16}{5}, 3\right), C' = \left(\frac{7}{2}, 0\right)$

TAP TO RETURN

## 2 - Answers

(a) Proof

(b) $(x-5)(2x-3)(x+2)$

(c) Graph

(d) $x=-\frac{3}{2}, 0, 6$

TAP TO RETURN

# 3 - Answers

(a) $\frac{1}{x}$

(b) $2e^x - \frac{4}{x}$

(c) $\frac{3-3\sin x+3x\cos x}{(1-\sin x)^2}$

(d) $\frac{e^x(x\ln x-1)}{x(\ln x)^2}$

(e) $\frac{2}{x}$

(f) $\frac{5}{2x}$

TAP TO RETURN

# 4 - Answers

(a) and (b) use graph sketching app

(c) $a = -2, \quad b = -1$

(d) $x = -\frac{1}{6}$

TAP TO RETURN

# 5 - Answers

(a) $x + 2y - 16 = 0$

(b) $y = -4x$

(c) $\left(\frac{6}{7}, \frac{53}{7}\right)$

TAP TO RETURN

# 6a - Answers

$$(x+3)^5 + C$$

$$\frac{(x-2)^6}{6} + C$$

$$\frac{1}{32}(4x-5)^8 + C$$

$$2\left(\frac{1}{8}x+1\right)^4 + C$$

$$-\frac{8}{7}\left(3-\frac{1}{2}x\right)^7 + C$$

$$-\frac{1}{9}(4-x)^9 + C$$

TAP TO RETURN

# 6b - Answers

$$\frac{1}{3}\sec 3x + C$$

$$-\cot x + C$$

$$-\frac{1}{4}\cos 4x + C$$

$$\frac{1}{2}(-3\cot 2x + \operatorname{cosec} 2x) + C$$

$$\sin x + \cos x + C$$

TAP TO RETURN

7 - Answers

Proof

TAP TO RETURN

# 8 - Answers

$0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi$

$0, \pi, 2\pi$

$\frac{\pi}{2}$

$\frac{\pi}{2}, 0.305, 2.84$

$0, \pi, 2\pi$

TAP TO RETURN

# BHASVIC MαTHS
# A1 DOUBLES ASSIGNMENT 14A

## 9 - Answers

(a) $\frac{(x-2)^2(2e^{2x})-e^{2x}[2(x-2)]}{(x-2)^4}=\frac{2(x-2)^2e^{2x}-2e^{2x}(x-2)}{(x-2)^4}$

$=\frac{2(x-2)e^{2x}-2e^{2x}}{(x-2)^3}=\frac{2e^{2x}(x-2-1)}{(x-2)^3}=\frac{2e^{2x}(x-3)}{(x-2)^3}$

$A=2, B=1, C=3$

(b) $y=4e^2x-3e^2$

# 10 - Answers

Check your graphs are correct on desmos.com

(a) arccos(x), because its graph is never negative

TAP TO RETURN

# 11 - Answers

Check your answer by subbing in $x = 0.01$ into both sides. They should equal the same number correc to 6 dp.

TAP TO RETURN

# BHASVIC MαTHS
# A1 DOUBLES ASSIGNMENT 14A

## 12 - Answers

$$\frac{dy}{dx} = 2\arccos y \times -\frac{1}{\sqrt{1-y^2}} = -\frac{2\arccos y}{\sqrt{1-y^2}}$$

$$\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{2\arccos y} = -\frac{\sqrt{1-\cos^2\sqrt{x}}}{2\sqrt{x}}$$

TAP TO RETURN

# 13 - Answers

Proof

TAP TO RETURN

# 14 - Answers

$f^{-1}(x) = {}^{1}/_{3}(x - 2)$, $g^{-1}(x) = {}^{1}/_{x}$, $x \neq 0$, $gf(x) = {}^{1}/_{(3x + 2)}$, $x \neq -{}^{2}/_{3}$

TAP TO RETURN

# 15 - Answers

(a) $f^{-1}(x) = 1 + \sqrt{x - 4}$

(b) $f^{-1}(x) = -2 + \sqrt{x + 5}$

(c) $f^{-1}(x) = \sqrt{x + 4} - 2$

TAP TO RETURN

# BHASVIC MαTHS

# A1 DOUBLES ASSIGNMENT 14A

## 15 - answers

https://www.madasmaths.com/archive/iygb_practice_papers/c1_practice_papers/c1_p_solutions.pdf

TAP FOR ANSWERS