1

Find the gradient of the tangent to the following functions at $x = 1$:

(a) $y = \frac{4x+3}{2x^2}$

(b) $y = \frac{3-\sqrt{x}}{x^3}$

(c) $y = \left(\frac{2x-1}{\sqrt{x}}\right)^2$

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 13A

1. Differentiate these functions with respect to x:

(a) $y = 3\sec(6x^2 + 5)$ (b) $y = \tan\left(x^2 + 3\right)$ (c) $e^{-3x}cotx$

2. Find $\frac{dy}{dx}$, in terms of y, given that

(a) $x = \tan y$ (b) $x = y^3 \sin y$ (c) $x = 3y\sec y$

(Hint: Find $\frac{dx}{dy}$ first then use the special case of the chain rule where $\frac{dy}{dx} = \frac{1}{\frac{dy}{dx}}$)

3. By using the quotient rule, find the derivatives of the following:

(a) $\frac{x+3}{2x+1}$ (b) $\frac{3x^2}{(2x-1)^2}$ (c) $\frac{x^4}{cos3x}$

TAP FOR ANSWERS

3

Find the value of k for which $y = 3x + 1$ is a tangent to the curve $x^2 + y^2 = k$

TAP FOR ANSWERS

4

The function k is defined by $\mathrm{k}(x) = \frac{a}{x^2}, a > 0, x \in \mathbb{R}, x \neq 0$

(a) Sketch the graph of $y = \mathrm{k}(x)$.

(b) Explain why it is not necessary to sketch $y = |\mathrm{k}(x)|$ and $y = \mathrm{k}(|x|)$.

The function m is defined by $\mathrm{m}(x) = \frac{a}{x^2}, a < 0, x \in \mathbb{R}, x \neq 0$.

(c) Sketch the graph of $y = \mathrm{m}(x)$

(d) State with a reason whether the following statements are true or false.

(i) $|\mathrm{k}(x)| = |\mathrm{m}(x)|$ (ii) $\mathrm{k}(|x|) = \mathrm{m}(|x|)$ (iii) $\mathrm{m}(x) = \mathrm{m}(|x|)$

TAP FOR ANSWERS

5

Prove the following identities:

(a) $\dfrac{\operatorname{cosec} A}{\operatorname{cosec} A - \sin A} \equiv \sec^2 A$

(b) $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} \equiv \dfrac{2}{\sin\theta}$

TAP FOR ANSWERS

Use algebraic division to express these improper fractions in the form $ax^2 + bx + c + \frac{R}{\text{divisor}}$

(a) $\frac{x^3+2x^2+3x-4}{x-1}$

(b) $\frac{2x^3+3x^2-4x+5}{x+3}$

(c) $\frac{x^4+3x^2-4}{x^2+1}$

TAP FOR ANSWERS

7

(a) Prove, from first principles, that the derivative of $2x^3$ is $6x^2$

(b) Prove, from first principles, that the derivative of sin 2x is 2cos2x

TAP FOR ANSWERS

8

a) Use the identify $\sin^2 A + \cos^2 A \equiv 1$ to show that
$\sin^4 A + \cos^4 A \equiv \frac{1}{2}(2 - \sin^2 2A)$.

b) Deduce that $\sin^4 A + \cos^4 A \equiv \frac{1}{4}(3 + cos4A)$.

c) Hence solve $8\sin^4\theta + 8\cos^4\theta = 7$ for $0 < \theta < \pi$.

TAP FOR ANSWERS

Given that $\mathrm{f}(x) = \frac{2x}{x+5} + \frac{6x}{x^2+7x+10}, x > 10$

(a) Show that $f(x) = \frac{2x}{x+2}$

(b) Hence find $\mathrm{f}'(3)$

TAP FOR ANSWERS

i) Integrate the following functions by working out what has been differentiated:

(Remember to differentiate the function back to check)

(a) $\int \sin 4x \, dx$ (b) $\int (3x+2)^5 \, dx$ (c) $\int \cos(x+2) \, dx$

ii) Find the following integrals by considering what has been differentiated:

(Remember to differentiate the function back to check)

$(a) \int \sec 3x \tan 3x dx$ (b) $\int \operatorname{cosec} x \cot x dx$ (c) $\int \sec^2 2x dx$

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 13A

The normal to the curve $y = \sec^2 x$ at the point $P\left(\frac{\pi}{4}, 2\right)$ meets the line $y = x$ at the point Q. Find the exact coordinates of Q.

TAP FOR ANSWERS

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 13A

Integrate the following by using reverse chain rule:

a) $\int (e^{2x} - \frac{1}{2}\sin(2x-1))\,dx$

b) $\int \sin^5 3x \cos 3x\,dx$

c) $\dfrac{\cos 2x}{3 + \sin 2x}$

d) $\dfrac{\sin 2x}{(3 + \cos 2x)^3}$

e) xe^{x^2}

f) $\sec^2 x \tan^2 x$

g) $\sec^2 x(1 + \tan^2 x)$

TAP FOR ANSWERS

Sketch the graph of $y = |x - 2a|$ (where a is a positive constant) showing the points of the intersection with coordinate axes. Solve $|x - 2a| = \frac{1}{3}x$ for x in terms of a.

TAP FOR ANSWERS

Integrate the following with respect to *x*:

(a) $\displaystyle\int \frac{\sec^2 x}{(1+\tan x)^3}\,dx$

(b) $\displaystyle\int 2\sin x\cos^3 x\,dx$

(c) $\displaystyle\int \frac{x}{(1-x^2)^5}\,dx$

TAP FOR ANSWERS

Complete this old spec paper

https://www.madasmaths.com/archive/iygb_practice_papers/c1_practice_papers/c1_o.pdf

TAP FOR ANSWERS

1 - Answers

(a) -5

(b) $-\frac{13}{2}$

(c) 3

TAP TO RETURN

2 - Answers

1. (a) $36x\sec(6x^2+5)\tan(6x^2+5)$ (b) $2x\sec^2(x^2+3)$ (c) $-e^{-3x}\left(3\cot x+\operatorname{cosec}^2 x\right)$

2. (a) $\cos^2 y$ (b) $1/y^2(3\sin y+y\cos y)$ (c) $\dfrac{\cos y}{3\left(1+y\tan y\right)}$

3. (a) $\dfrac{-5}{(2x+1)^2}$ (b) $\dfrac{-6x}{(2x-1)^3}$ (c) $\dfrac{x^3(3x\sin 3x+4\cos 3x)}{\cos^2 x}$

TAP TO RETURN

3 - Answers

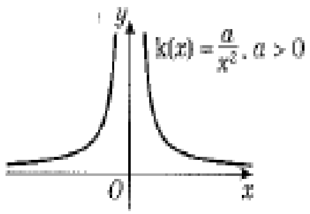
$$k = \frac{1}{10}$$

TAP TO RETURN

BHASVIC MαTHS
A1 DOUBLES ASSIGNMENT 13A

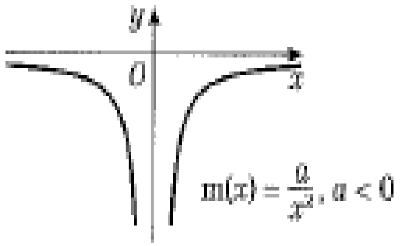
4 - Answers

(a)



(b) Both these graphs would match the original graph.

(c)



(d) (i) True, $|k(x)| = \left|\frac{a}{x^2}\right| = \left|\frac{-a}{x^2}\right| = |m(x)|$

(ii) False, $k(|x|) = \frac{a}{|x|^2} \neq \frac{-a}{|x|^2} = m(|x|)$

(iii) True, $m(|x|) = \frac{-a}{|x|^2} = \frac{-a}{x^2} = m(x)$

TAP TO RETURN

5 - Answers

Proof

TAP TO RETURN

6 - Answers

(a) $x^2 + 3x + 6 - \frac{2}{x-1}$

(b) $2x^2 - 3x + 5 - \frac{10}{x+3}$

(c) $x^2 + 2 - \frac{6}{x^2+1}$

TAP TO RETURN

7 - Answers

Proof

TAP TO RETURN

8 - Answers

c) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$

TAP TO RETURN

9 - Answers

(a) $\frac{2x}{x+5}+\frac{6x}{(x+5)(x+2)}=\frac{2x(x+2)}{(x+5)(x+2)}+\frac{6x}{(x+5)(x+2)}$
$=\frac{2x(x+2+3)}{(x+5)(x+2)}=\frac{2x(x+5)}{(x+5)(x+2)}=\frac{2x}{(x+2)}$

(b) $\frac{4}{25}$

TAP TO RETURN

10 - Answers

i) (a) $-\frac{1}{4}\cos 4x + c$ (b) $\frac{1}{18}(3x+2)^6 + c$ (c) $\sin(x+2)+c$

ii) (a) $\frac{1}{3}\sec 3x + c$ (b) -cosec x + c (c) $\frac{1}{2}\tan 2x + c$

TAP TO RETURN

11 - Answers

$(1.757, 1.757)$

TAP TO RETURN

12 - Answers

a) $\frac{1}{2}e^{2x} + \frac{1}{4}\cos(2x - 1) + c$

b) $\frac{1}{18}\sin^6 3x + c$

c) $\frac{1}{2}\ln|3 + sin2x| + c$

d) $\frac{1}{4}(3 + cos2x)^{-2} + c$

e) $\frac{1}{2}e^{x^2} + c$

f) $\frac{1}{3}\tan^3 x + c$

g) $tanx + \frac{1}{3}\tan^3 x + c$

TAP TO RETURN

13 - Answers

Use Desmos to check graph

$x = 3a$ or $\frac{3}{2}a$

TAP TO RETURN

14 - Answers

a) $-\dfrac{1}{2}(1+\tan x)^{-2}+c$

b) $-\dfrac{1}{2}\cos^4 x+c$

c) $\dfrac{1}{8}(1-x^2)^{-4}+c$

TAP TO RETURN

15 - Answers

https://www.madasmaths.com/archive/iygb_practice_papers/c1_practice_papers/c1_o_solutions.pdf

TAP TO RETURN