Decision Mathematics D1

Advanced/Advanced Subsidiary

Tuesday 9 June 2015 – Afternoon

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Decision Mathematics D1), the paper reference (6689), your surname, other name and signature.
Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

Full marks may be obtained for answers to ALL questions.
This paper has 7 questions.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
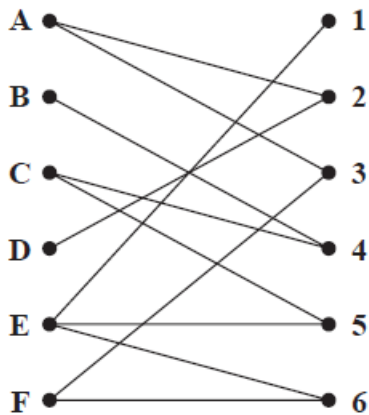Answers without working may not gain full credit.
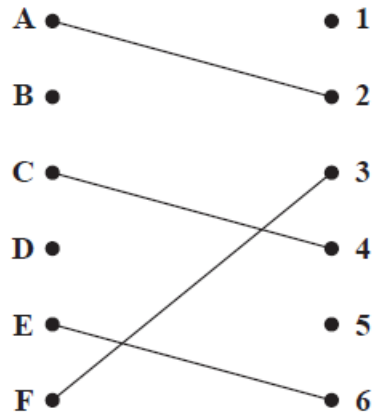
1.

Figure 1   **Figure 2**

A delivery firm has six vans, **A**, **B**, **C**, **D**, **E** and **F**, available for six deliveries, **1**, **2**, **3**, **4**, **5** and **6**. Each van must be assigned to just one delivery.

The bipartite graph shown in Figure 1 shows the possible matchings and Figure 2 shows an initial matching.

A complete matching is required, starting from the given initial matching.

(*a*) Explain why it is necessary to use the maximum matching algorithm twice.

(1)

There are three possible alternating paths that start at either **D** or **B**. One of these is

$$D - 2 = A - 3 = F - 6 = E - 5$$

(*b*) Find the other two alternating paths.

(2)

(*c*) List the improved matching generated by using the alternating path

$$D - 2 = A - 3 = F - 6 = E - 5$$

(1)

(*d*) Starting from the improved matching found in (*c*), use the maximum matching algorithm to find a complete matching. You should list the alternating path that you use and your complete matching.

(2)

2.   18   29   48   9   42   31   37   24   27   41

The numbers above are Alan's batting scores for the first 10 cricket matches of the season.

(*a*) Use a quick sort to sort this list of numbers into ascending order. You must make your pivots clear.

(4)

Alan's batting scores for the final 10 cricket matches of the same season were

72   53   89   91   68   67   90   77   83   75

(*b*) Carry out a bubble sort on this second list of numbers to produce a list of these scores in ascending order. You need only give the state of the list after each pass.

(4)

Alan's combined batting scores for the entire season were

9 18 24 27 29 31 37 41 42 48 53 67 68 72 75 77 83 89 90 91

(*c*) Use the binary search algorithm to locate 68 in the combined list of 20 scores. You must make your method clear.
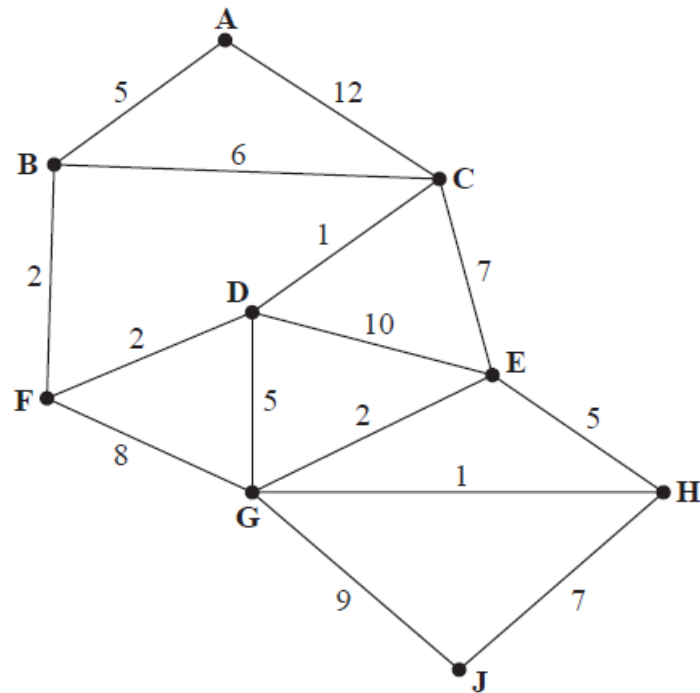
(3)

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3.

Figure 3

Figure 3 represents a network of roads. The number on each arc is the length, in km, of the corresponding road.

(*a*) Use Dijkstra's algorithm to find the shortest route from **A** to **J**. State the shortest route and its length.

(6)

(*b*) Explain how you determined the shortest route from your labelled diagram.

(2)

(*c*) Find the shortest route from **A** to **J** via **E** and state its length.
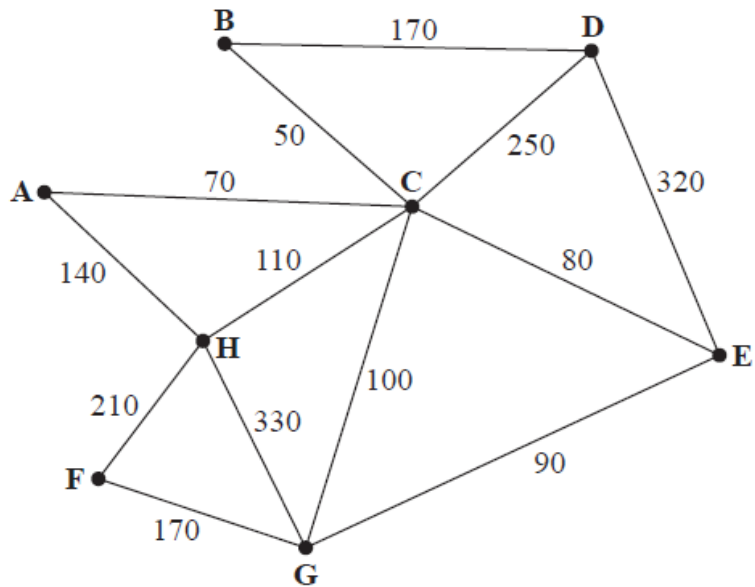
(2)

4.

Figure 4

[*The total weight of the network is 2090*]

(*a*) Explain why a network cannot have an odd number of vertices of odd valency.

(**2**)

Figure 4 represents a network of 13 roads in a village. The number on each arc is the length, in metres, of the corresponding road. A route of minimum length that traverses each road at least once needs to be found. The route may start at any vertex and finish at any vertex.

(*b*) Write down the vertices at which the route will start and finish.

(**1**)

A new road, **AB**, of length 130m is built. A route of minimum length that traverses each road, including **AB**, needs to be found. The route must start and finish at **A**.

(*c*) Use the route inspection algorithm to find the roads that will need to be traversed twice. You must make your method and working clear.

(**4**)

(*d*) Calculate the length of a possible shortest inspection route.

(**2**)

It is now decided to start and finish the inspection route at two distinct vertices. A route of minimum length that traverses each road, including **AB**, needs to be found. The route must start at **A**.

(*e*) State the finishing point so that the length of the route is minimised. Calculate how much shorter the length of this route is compared to the length of the route in (*d*). You must make your method and calculations clear.
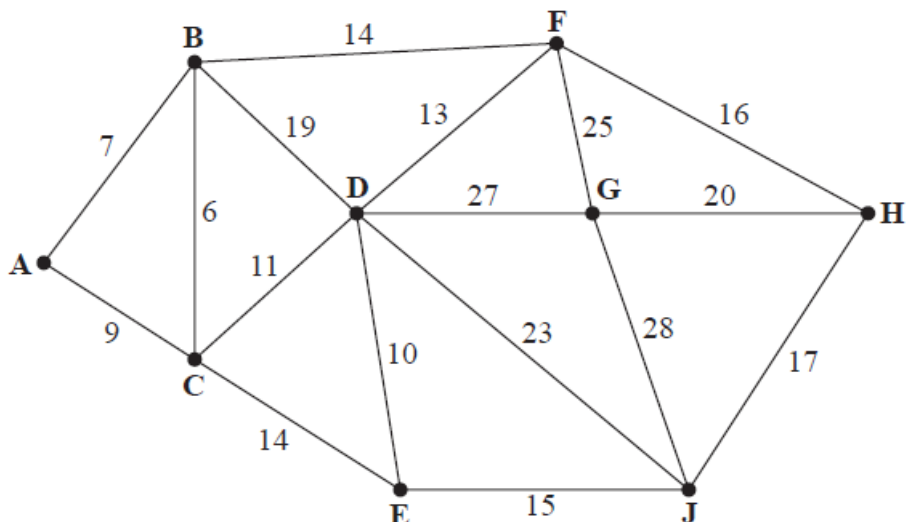
(**3**)

5.

Figure 5

The numbers on the 17 arcs in the network shown in Figure 5 represent the distances, in km, between nine nodes, **A**, **B**, **C**, **D**, **E**, **F**, **G**, **H** and **J**.

(*a*) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

(*b*) Starting at **G**, use Prim's algorithm to find a minimum spanning tree. You must clearly state the order in which you select the arcs of your tree.

(3)

(*c*) Find the weight of the minimum spanning tree.

(1)

A connected graph V has *n* nodes. The sum of the degrees of all the nodes in V is *m*. The graph T is a minimum spanning tree of V.

(*d*) (i) Write down, in terms of *m*, the number of arcs in V.

(ii) Write down, in terms of *n*, the number of arcs in T.

(iii) Hence, write down an inequality, in terms of *m* and *n*, comparing the number of arcs in T and V.

(3)

6. A linear programming problem in x and y is described as follows.

Minimise $C = 2x + 3y$

subject to

$$x + y \geq 8$$
$$x < 8$$
$$4y \geq x$$
$$3y \leq 9 + 2x$$

(a) Add lines and shading to Diagram 1 in the answer book to represent these constraints.

(4)

(b) Hence determine the feasible region and label it R.

(1)

(c) Use the objective line (ruler) method to find the exact coordinates of the optimal vertex, V, of the feasible region. You must draw and label your objective line clearly.

(3)

(d) Calculate the corresponding value of C at V.

(1)

The objective is now to maximise $2x + 3y$, where x and y are integers.

(e) Write down the optimal values of x and y and the corresponding maximum value of $2x + 3y$.

(2)

A further constraint, $y \leq kx$, where k is a positive constant, is added to the linear programming problem.

(f) Determine the least value of k for which this additional constraint does not affect the feasible region.

(2)

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7.

Activity	Time taken (days)	Immediately preceding activities
A	5	-
B	7	-
C	8	-
D	5	**A**
E	7	**A**
F	10	**B, C**
G	4	**B, C**
H	9	**C**
I	8	**G, H**
J	12	**G, H**
K	7	**D**
L	10	**E, F, I, J**

The table shows the activities required for the completion of a building project. For each activity the table shows the time taken, in days, and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.
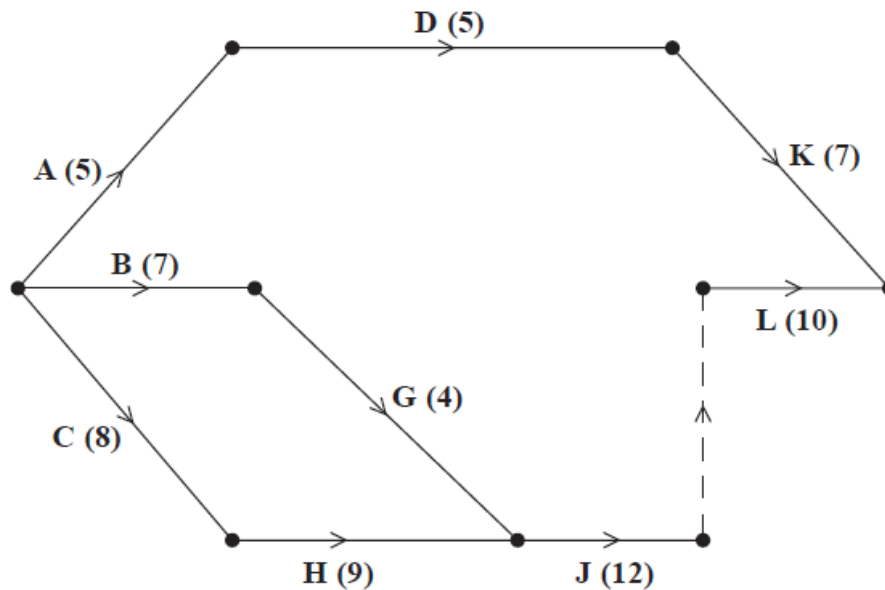

Figure 6

Figure 6 shows a partially completed activity network used to model the project. The activities are represented by the arcs and the numbers in brackets on the arcs are the times taken, in days, to complete each activity.

(*a*) Add activities, **E**, **F** and **I**, and exactly one dummy to Diagram 1 in the answer book.

(3)

(*b*) Complete Diagram 1 in the answer book to show the early event times and late event times.

(4)

(*c*) Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time. You must show your working.

(2)

(*d*) Schedule the activities, using the minimum number of workers, so that the project is completed in the minimum time.

(4)

TOTAL FOR PAPER: 75 MARKS

END