BHASVIC MαTHS

A2 Doubles summer CWC

Section: *Core*

1. $f(x) = 2x^3 - 8x^2 + 7x - 3$.

Given that $x = 3$ is a solution of the equation $f(x) = 0$, solve $f(x) = 0$ completely.

2. Given that $z_1 = 3 + 2i$ and $z_2 = \frac{12 - 5i}{z_1}$,

(*a*) find z_2 in the form $a + ib$, where a and b are real.

(*b*) Show, on an Argand diagram, the point P representing z_1 and the point Q representing z_2.

(*c*) Given that O is the origin, show that $\angle POQ = \frac{\pi}{2}$.

The circle passing through the points O, P and Q has centre C. Find

(*d*) the complex number represented by C,

(*e*) the exact value of the radius of the circle.

3. $f(x) = (x^2 + 4)(x^2 + 8x + 25)$

(*a*) Find the four roots of $f(x) = 0$.

(*b*) Find the sum of these four roots.

4. The complex numbers z_1 and z_2 are given by $z_1 = 2 - i$ and $z_2 = -8 + 9i$

(*a*) Show z_1 and z_2 on a single Argand diagram.

Find, showing your working,

(*b*) the value of $|z_1|$,

(*c*) the value of arg z_1, giving your answer in radians to 2 decimal places,

(*d*) $\frac{z_2}{z_1}$ in the form $a + bi$, where a and b are real.

5. $z = 5\sqrt{3} - 5\text{i}$

Find (*a*) $|z|$, (*b*) $\arg(z)$, in terms of π.

$$w = 2\left(\cos\frac{\pi}{4} + \text{i}\sin\frac{\pi}{4}\right)$$

Find (*c*) $\left|\frac{w}{z}\right|$, (*d*) $\arg\left|\frac{w}{z}\right|$, in terms of π.

6. The point *P* represents a complex number *z* on an Argand diagram, where $|z + 1| = |z - \text{i}|$

and the point *Q* represents a complex number *w* on the Argand diagram, where $|w| = |w - 1 + \text{i}|$

Find the exact coordinates of the points where the locus of *P* intersects the locus of *Q*.

7. Use standard series to evaluate

$$\sum_{r=1}^{25}(5r - 1)$$

8. (a) Use standard results to show that

$$\sum_{r=1}^{n}(4r + 1)(4r - 1) = \frac{1}{3}n(4n + 1)(4n + 5)$$

for all positive integers n.

(b) Hence evaluate the sum

$$\sum_{r=9}^{28}(4r + 1)(4r - 1)$$

9. (a) Use standard results to find an expression for

$$\sum_{r=1}^{n}r(3r - 1)$$

Factorise your answer as far as possible.

(b) Hence, show that for any integer $N > 1$,

$$\sum_{r=1}^{N^2-1}r(3r - 1)$$

is a perfect square.

10. Prove that

$$\sum_{r=1}^{n}6(r^2 - 1) \equiv (n - 1)n(2n + 5)$$

11. Prove that

$$\sum_{r=1}^{n}(r-1)(r+2) \equiv \frac{1}{3}(n-1)n(n+4)$$

12. (a) Simplify $(r+2)^2 - r^2$

(b) Hence, using the method of differences, show that

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

13. (a) Show that $\frac{1}{r^2} - \frac{1}{(r+1)^2} \equiv \frac{2r+1}{r^2(r+1)^2}$

(b) Hence find an expression for

$$\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2}$$

in the form $A + \frac{B}{(n+1)^2}$ for constants A and B to be stated.

(c) Evaluate the series

$$\sum_{r=4}^{19} \frac{2r+1}{r^2(r+1)^2}$$

(d) Show that

$$\sum_{r=n}^{2n-1} \frac{2r+1}{r^2(r+1)^2} = \frac{3}{4n^2}$$

for all positive integers n.

14. (a) Show that $\frac{1}{r} - \frac{1}{r+1} + 4 = \frac{(2r+1)^2}{r(r+1)}$ for all $r > 0$.

(b) Hence find, in terms of the positive integer n, and expression for

$$\sum_{r=1}^{n} \frac{(2r+1)^2}{r(r+1)}$$

factorising your answer as far as possible.

(c) Evaluate the series

$$\sum_{r=r}^{14} \frac{(2r+1)^2}{r(r+1)}$$

15. (a) Express $\frac{2}{r(r^2-1)}$ in the form $\frac{A}{(r+1)} + \frac{B}{r} + \frac{C}{(r-1)}$ for constants A, B and C.

(b) Hence show that

$$\sum_{r=2}^{n} \frac{4}{r(r^2-1)} = \frac{(n-1)(n+2)}{n(n+1)}$$

16. Find $\frac{dy}{dx}$:

(a) $y = x^7(2-5x^3)^4$ (b) $y = x^2\sqrt{3-4x}$ (c) $y = \dfrac{(5x+1)^3}{\sqrt{x}}$

(d) $y = (x^2-3x+1)^5(2x-3)^3$ (e) $y = \sqrt{\dfrac{x^2+1}{x^3-3}}$ (f) $y = \dfrac{x(x-1)^3}{x-3}$

(g) $y = \dfrac{x\sqrt{5-x^2}}{6-x}$

17. Find the function $f'(x)$ where $f(x)$ is:

a) $\frac{x^2}{\tan x}$ b) $\frac{1+\sin x}{\cos x}$ c) $e^{2x}\cos x$ d) $e^x \sec 3x$

e) $\frac{\sin 3x}{e^x}$ f) $e^x \sin^2 x$ g) $\frac{\ln x}{\tan x}$ h) $\frac{e^{\sin x}}{\cos x}$

18. (*a*) Differentiate with respect to *x*,

(i) $e^{3x}(\sin x + 2\cos x)$,

(ii) $x^3 \ln(5x+2)$.

Given that $y = \dfrac{3x^2+6x-7}{(x+1)^2}$, $x \neq 1$,

(*b*) Show that $\dfrac{dy}{dx} = \dfrac{20}{(x+1)^3}$.

(*c*) Hence find $\dfrac{d^2y}{dx^2}$ and the real values of *x* for which $\dfrac{d^2y}{dx^2} = -\dfrac{15}{4}$.

19. Find the equation of the normal to the curve $y = \dfrac{1}{1+2\sin x}$ at the point where $x = \dfrac{\pi}{6}$.

20. A curve has equation $f(x) = \dfrac{x+1}{\sqrt{x-1}}$. Find $f'(x)$ and hence find the coordinates of the turning point of the curve. Determine whether the turning point is a maximum or a minimum point.

21. (*a*) Express $\dfrac{2}{4-y^2}$ in partial fractions.

(*b*) Hence obtain the solution of $2\cot x \dfrac{dy}{dx} = (4-y^2)$

for which $y = 0$ at $x = \dfrac{\pi}{3}$, giving your answer in the form $\sec^2 x = g(y)$.

22. During a chemical reaction, a compound is being made from two other substances.

At time *t* hours after the start of the reaction, *x* g of the compound has been produced.

Assuming that $x = 0$ initially, and that $\frac{dx}{dt} = 2(x-6)(x-3)$

(a) Show that it takes approximately 7 minutes to produce 2 g of the compound.

(b) Explain why it is not possible to produce 3 g of the compound.

23. Liquid is pouring into a container at a constant rate of $20\text{cm}^3\text{s}^{-1}$ and is leaking out at a rate proportional to the volume of the liquid already in the container.

a) Write a differential equation to model this situation
b) The container is initially empty. Solve the differential equation, giving your answer in terms of k
c) Given that $\frac{dV}{dt} = 10 \ when \ t = 5$, find the volume in the container at 10 seconds after the start

24. In a certain pond, the rate of increase of the number of fish is proportional to the number of fish, n, present at time t.

a) Assuming that n can be regarded as a continuous variable, write down a differential equation relating n and t, and hence show that $n = Ae^{kt}$

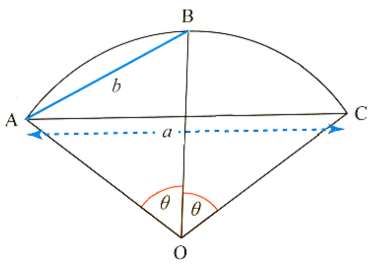
b) In a revised model, it is assumed also that fish are removed from the pond, by anglers and by natural wastage, at the constant rate of p per unit time, so that

$$\frac{dn}{dt} = kn - p$$

Given that k = 2, p = 100 and that initially there were 500 fish in the pond, solve this differential equation, expressing n in terms of t

c) Give a reason why this revised model is not satisfactory for large values of t

25. An approximate rule used by builders to find the length, c, of a circular arc ABC is $c = \frac{8b-a}{3}$, where a and b are as shown in the diagram.



(a) If O is the centre of the circle, show that $b = 2r\sin\left(\frac{\theta}{2}\right)$ and $a = 2r\sin\theta$.

(b) Using the cubic approximation to $\sin x$, show that $8b - a = 6r\theta$. Hence verify the rule.

(c) Find the percentage error caused by using this rule when $\theta = \frac{\pi}{3}$.

26. (a) Write down the Maclaurin expansions of

(i) $\cos\theta$ (ii) $\sin\theta$ (iii) $\cos\theta + i\sin\theta$

giving your answers in ascending powers of θ.

(b) Substitute $x = i\theta$ in the Maclaurin series for e^x and simplify the terms.

(c) Show that your answers to (a)(iii) and (b) are the same, and hence

$$e^{i\theta} = \cos\theta + i\sin\theta$$

27. (a) Write down the first four non-zero terms in Maclaurin series for $\cos u$.

(b) Substitute $u = 2x$ in this series to obtain the first four non-zero terms in the Maclaurin series for $\cos 2x$.

28. (a) Given that $f(x) = e^{2x}\sin 3x$, show that $f''(x) = 4f'(x) - 13f(x)$.

(b) Differentiate this result twice to find expressions for $f^{(3)}(x)$ and $f^{(4)}(x)$ in terms of lower derivatives.

(c) Hence find the Maclaurin expansion for $f(x)$ up to the term in x^4.