BHASVIC MαTHS

A2 Doubles summer assignment *2*

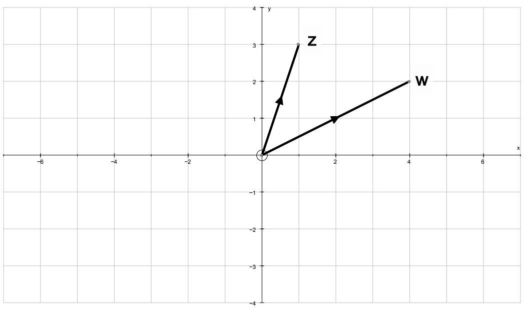
Section: *Core*

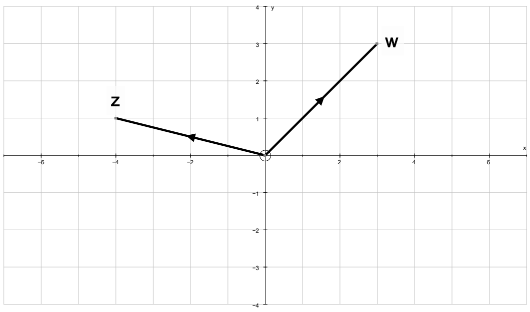
<u>Past</u>

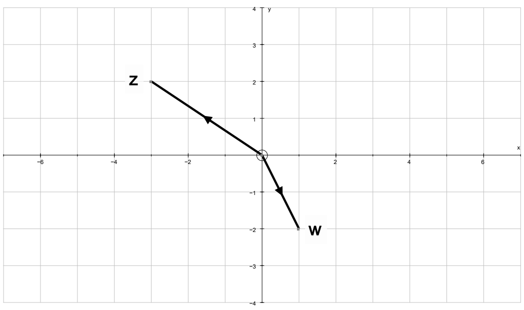
1. Solve these quadratic equations and represent the solutions on an Argand diagram:

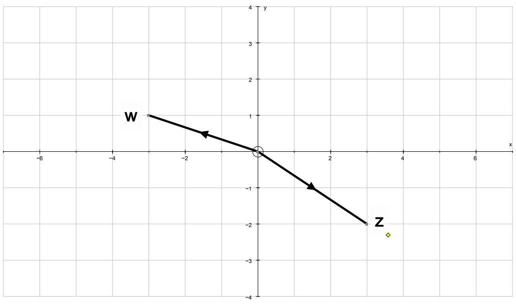
   (a) $z^2 - 4z + 13 = 0$ (b) $z^2 - 10z + 26 = 0$ (c) $z^2 + 2z + 17 = 0$ (d) $z^2 + 6z + 13 = 0$

2. Each diagram shows two complex numbers, z and w. Copy each diagram and add the complex numbers corresponding to z^*, -w, and z + w

(a) 

(b) 

(c) 

(d) 

3. Write each complex number in the form r(cosθ +isinθ) where $-\pi < \theta \ll \pi$

   (a) z = 4i (b) z = -5 (c) z = $2 - 2\sqrt{3}i$ (d) z =$\frac{\sqrt{3}+i}{3}$

4. Find the modulus and argument of the following:

   (a) z = $4(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right))$ (b) z = $3(\cos\left(\frac{\pi}{8}\right) - i\sin\left(\frac{\pi}{8}\right))$

   (c) z = $-10(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right))$ (d) z = $6(\cos\left(-\frac{\pi}{10}\right) + i\sin\left(\frac{\pi}{10}\right))$

5. (a) If z = $3(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right))$ and w = $5(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right))$ write zw in the form r(cosθ +isinθ) where $-\pi < \theta \ll \pi$

(b) If z = $6(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right))$ and w = $3(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right))$ write $\frac{z}{w}$ in the form x + iy

6. If z = 1 + i, write down the complex conjugate z^*, and express $\frac{z}{z^*}$ in the form a + bi.

7. If $z = \frac{1}{2-3i}$, express z in the form a + bi, write down z^* and hence evaluate zz^*.

8. Find, in the form *a* + *b*i with *b* > 0, the complex number *z* satisfying simultaneously the equations $zz^* = 25$, $z + z^* = 6$

Present

1. Illustrate each locus on an Argand diagram (shade the areas the inequalities cover).
   a) i) $|z - 2i| = 5$ ii) $|z - 3| = 5$
   b) i) $|z + 4| = 1$ ii) $|z + 3i| = 2$
   c) i) $|z - i| \leq 2$ ii) $|z + i| > 3$
   d) i) $|z - 3 + i| > 2$ ii) $|z + 1 - 2i| \leq 1$

2. The set of points from question **1 a i** can be described as $\{x + iy: x^2 + (y-2)^2 = 25\}$. Write the sets of points for the rest of question 1 using similar notation.
   a) ii) $|z - 3| = 5$
   b) i) $|z + 4| = 1$ ii) $|z + 3i| = 2$
   c) i) $|z - i| \leq 2$ ii) $|z + i| > 3$
   d) i) $|z - 3 + i| > 2$ ii) $|z + 1 - 2i| \leq 1$

3. Sketch each locus on an Argand diagram.
   a) i) $|z - 2| = |z + 2i|$ ii) $|z + 1| = |z - i|$
   b) i) $|z - 3i| < |z + i|$ ii) $|z + 2| > |z - 3|$
   c) i) $\text{Re}(z) = 5$ ii) $\text{Im}(z) > -2$

4. Sketch each locus on an Argand diagram.
   a) i) $\arg(z) = \frac{\pi}{3}$ ii) $\arg(z) = \frac{\pi}{4}$
   b) i) $\arg(z) = -\frac{\pi}{6}$ ii) $\arg(z) = -\frac{3\pi}{4}$
   c) i) $\arg(z) = \frac{\pi}{2}$ ii) $\arg(z) = \pi$

5. Shade the region on an Argand diagram where $1 < |z - 3i| \leq 3$.

Future

1. The roots of the equations $z^2 + 4z + 29 = 0$ are z_1 and z_2. Show that $|z_1| = |z_2|$ and calculate in degrees the argument of z_1 and the argument of z_2.
In an Argand diagram, O is the origin and z_1 and z_2 are represented by the points P and Q. Calculate the radius of the circle passing through the points O, P and Q.