BHASVIC MαTHS

C3 C4 S2/M2/D1

A2 MATHS

CONTINUING WITH CONFIDENCE

You must aim to complete this entire booklet by September 14th and bring it with you to your first lesson.

It is your responsibility to make sure that the completed booklet is available for your teacher to look at in every lesson if they request to see it.

Name:……………………………………………………

- **You need to complete this entire booklet before September 14th. Your teacher will expect this booklet to be 100% complete and correct.**

- **Make sure you fill in the checklist. If you do find that you need help re-watch your teacher's videos, post on the BHASVIC maths Facebook page, contact your friends and meet up to chat about it. You could also come to the support sessions during enrolment on 31st August, 1st September and 4th September.**

- **You will also need your notes for C1, C2 and the last 5 weeks of the Summer Term.**

- **There will be a test on these topics during the week beginning**
  Monday 25th September.

- **Make sure you keep all your solutions to exercises together so your teacher can check your progress.**

- **Don't leave everything to the last minute. If you spread your work out over the Summer, you will find it easy to complete the booklet. Here is a suggested schedule. Allow 90 minutes every week.**

Week	17/7	24/7	31/7	7/8	14/8	21/8	28/8	4/9	11/9
Exercise	1-4	5-6	7-9	10-11	12-13	14-15	16-17	18	Test Yourself

CHECK LIST

TOPIC	I AM FINE ON THIS TOPIC	I NEED TO DO SOME MORE PRACTICE	I MUST GET HELP AT THE BEGINNING OF TERM
A: INDICES			
B: LOGARITHMS			
C: CURVE SKETCHING			
D: TRIGONOMETRY			
E: DIFFERENTIATION			
F: ALGEBRAIC FRACTIONS			
G: e and ln			
H: Reverse Chain Rule			

Section A: Indices

Recap the basics

The following skills are very important C1 and C2 foundational skills. Focus on building up your speed and accuracy on these topics.

Exercise 1: Indices

1. $\frac{2}{3}ap^3 \times \frac{3}{4}pa^2 \times \frac{1}{2}aq^2 \times \frac{2}{5}qa^2$
2. $(a^2b^3)^4$
3. $x^0 \div x^{-4}$
4. $a^7 \div a^{-1}$
5. $15(a^2b)^3 \div (3ab)^2$
6. $18a^2b^2c^2 \times 2(ab)^{-1}$
7. $3a^2bc^3 \times 4a^2bc \div 2a^2b^4c^2$
8. $9x^6y^4 \times 2x^5yz^3 \div 6(x^2yz)^2$

Exercise 2: Use of indices in differentiation

Find $\mathrm{f}'(x)$

1. $\mathrm{f}(x) = \frac{x^3+2}{x}$
2. $\mathrm{f}(x) = x^{-2}(1+x)$
3. $\mathrm{f}(x) = \frac{x^2-7x+4}{x^3}$
4. $\mathrm{f}(x) = \sqrt{x}(x-1)$
5. $\mathrm{f}(x) = \frac{x-1}{\sqrt{x}}$
6. $\mathrm{f}(x) = 1+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}$

Find the exact gradient of the tangent at the given point.

7. $y = \frac{3x-1}{x}$ where $x = \frac{1}{2}$
8. $y = 2\sqrt{x}\left(1-\sqrt{x}\right)$ where $x = 4$
9. $y = \frac{\sqrt{x}-1}{\sqrt{x}}$ where $x = 9$

Write in this box two ways you would check your differentiation…

Exercise 3: Use of indices in integration

Integrate the following functions with respect to *x*

1 $\sqrt{x}+\frac{1}{\sqrt[3]{x}}$	2. $\frac{4}{x^3}-\frac{1}{x^2}-x^2$	3. $2x^{\frac{5}{2}}-x^{-\frac{2}{5}}$
4. $\frac{1}{2}x-\frac{2}{\sqrt{x}}-1$	5. $\frac{1}{x^4}+\frac{1}{\sqrt[4]{x}}-4$	6. $6\sqrt{x}-3x^3+x^{-2}+2$

Evaluate the following definite integrals giving exact answers

7. $\int_1^4 \sqrt{x}\ \mathrm{d}x$	8. $\int_2^5 \frac{3}{x^2}\ \mathrm{d}x$	9. $\int_4^9 \left(2x^{\frac{1}{2}}+3x^{-\frac{1}{2}}\right)\ \mathrm{d}x$
10. $\int_0^2 \left(x^{\frac{1}{2}}-2\right)^2\ \mathrm{d}x$	11. $\int_4^9 x^{\frac{1}{2}}\left(2x-3\right)\ \mathrm{d}x$	12. $\int_{-3}^{-1} \frac{x-1}{x^4}\ \mathrm{d}x$

Write in this box how you would check your integration…

HINT: It is always a good idea to neaten up your answers to integrals. It makes it much easier when you need to substitute limits.

Section B: Logarithms

Exercise 4: Logarithms – write the following as a single logarithm

1. $2\log_a 3 - 3\log_a 2 + 4\log_a 1$	2. $3\log_a 4 - \log_a 2 - 3\log_a 6$
3. $2\log_a 7 - 2\log_a a + 2\log_a 3$	4. $\log_a 5 + \frac{1}{2}\log_a 16 - \log_a 2$
5. $5\log_a a + \frac{1}{3}\log_a 27 + \log_a 2$	6. $\frac{1}{4}\log_a 81 + 3\log_a\left(\frac{1}{4}\right) - 2\log_a\left(\frac{3}{4}\right)$
7. $2\log_{10} x + \log_{10} y - 3\log_{10} z$	8. $\log_2(x-3) - 2\log_2(2x+1) + \frac{1}{2}\log_2 x$

Exercise 5: Solving log equations

Find an expression or value for *x*.

1. $2\log_{10} x + \log_{10}\frac{1}{3} - \log_{10}\frac{3}{4} = 2$

2. $\log_x 3 + \log_x 27 = 2$

3. $\log_5 x = 16\log_x 5$

4. $\log_4(x+3) - \log_4 x = 3$

5. $\log_5(3x+95) = 2 + \log_5(x+3)$

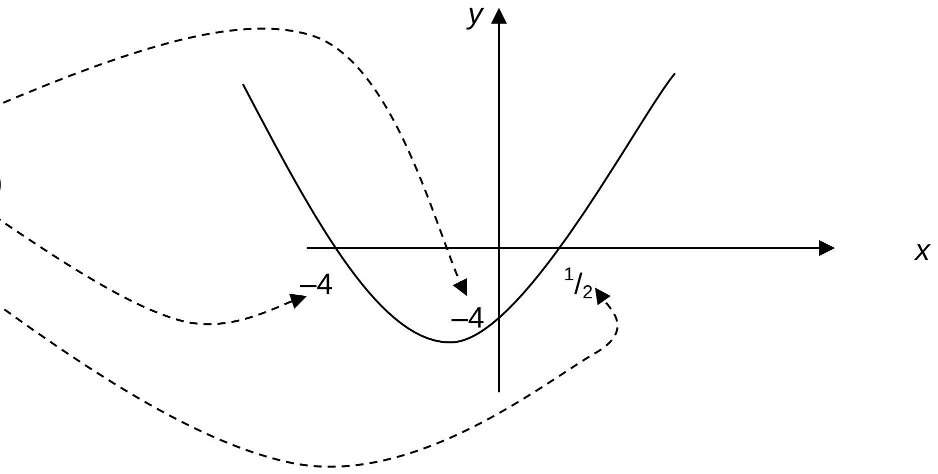
Write in this box how you would check your solutions…

Section C: Curve Sketching

You should be able to sketch graphs of the following functions WITHOUT a calculator.

1. QUADRATICS

$y = 2x^2 + 7x - 4$
$y = (2x - 1)(x + 4)$



For the minimum point ○ use calculus
○ complete the square
○ use symmetry

Make sure you can use all 3 methods to show that the minimum point is at $\left(-\frac{7}{4}, -\frac{81}{8}\right)$

2. CUBICS

$$y = x^3 - x^2 - 10x - 8$$

To factorise use the factor theorem $\mathrm{f}(x) = x^3 - x^2 - 10x - 8$

$$\mathrm{f}(x) = x^3 - x^2 - 10x - 8$$

$$\mathrm{f}(1) = 1 - 1 - 10 - 8 \neq 0$$

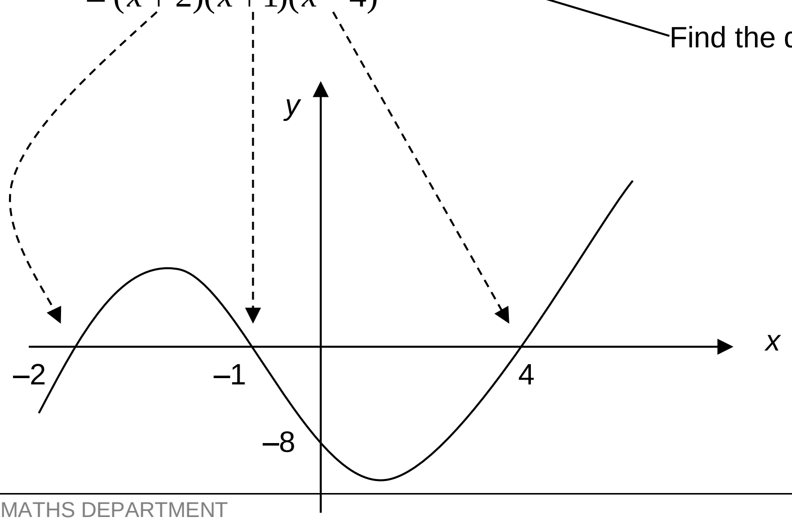
$$\mathrm{f}(2) = 8 - 4 - 20 - 8 \neq 0$$

$$\mathrm{f}(-2) = -8 - 4 + 20 - 8 = 0 \qquad \therefore (x+2) \text{ is a factor}$$

$$\text{so } y = (x+2)(x^2 - 3x - 4)$$

$$= (x+2)(x+1)(x-4)$$

Find the quadratic factor by inspection or long division



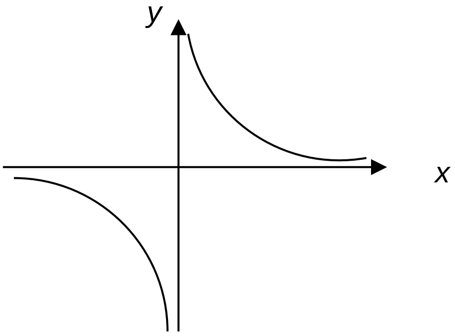
$y = +x^3$..........

So as $x \to \infty$

$y \to \infty$

3. RECIPROCAL GRAPHS

$y = \dfrac{3}{x}$



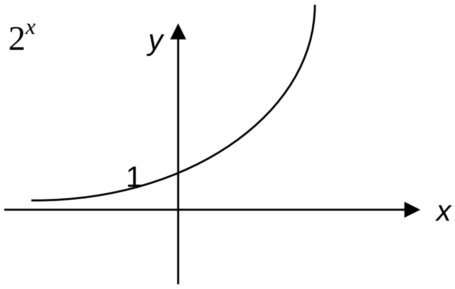
The ASYMPTOTES are

$x = 0$ (the y-axis)

and $y = 0$ (the x-axis)

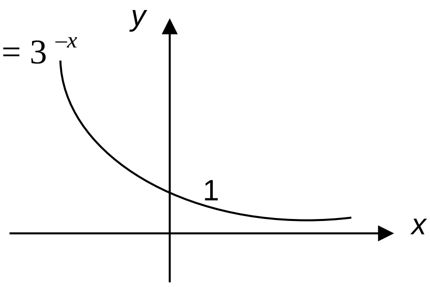
4. EXPONENTIAL GRAPHS ($y = a^x$ for $a > 1$)

$y = 2^x$



exponential growth

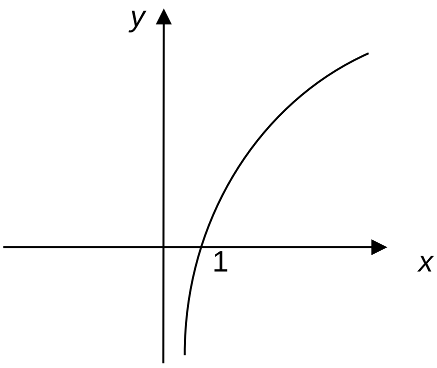
$y = 3^{-x}$



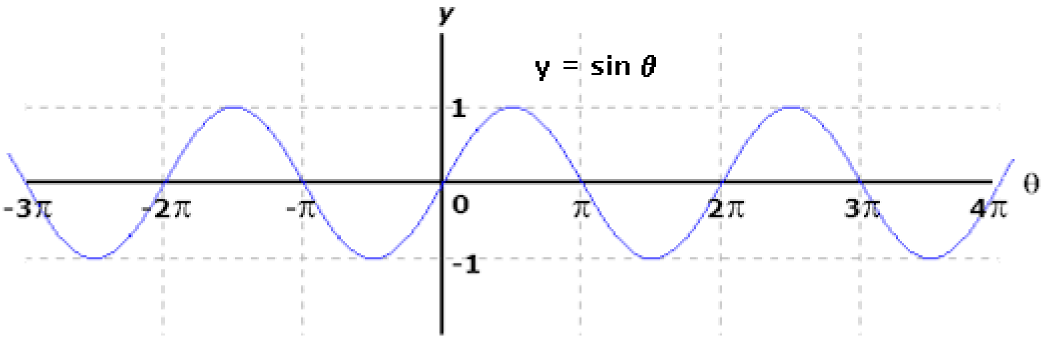
exponential decay

5. LOG GRAPHS (any base b > 1)

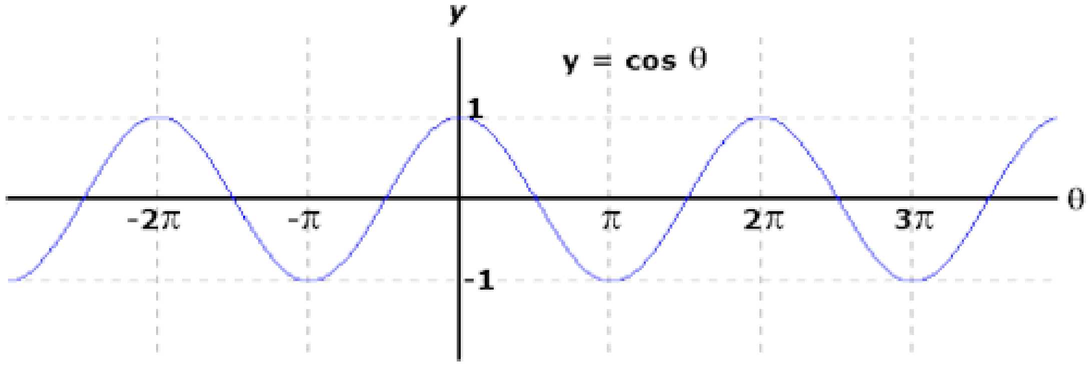
$y = \log_b x$



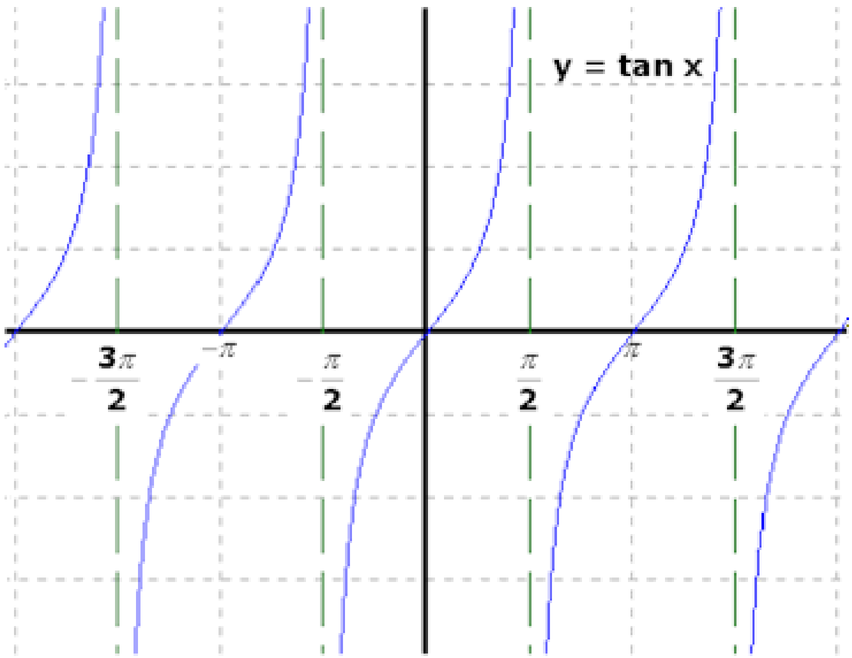
6. SINE GRAPH



7. COSINE GRAPH



8. TAN GRAPH



Transformations

You will need to be able to perform transformations on all these curves.

The following 3 transformations are all in the direction of the *y*-axis

- $y = -\mathrm{f}(x)$ — reflection in the *x*-axis
- $y = \mathrm{f}(x) + a \quad (a > 0)$ — moves up by *a*
- $y = a\mathrm{f}(x) \quad (a > 0)$ — sketch of scale factor *a* in *y*-direction

The following 3 transformations are all in the direction of the *x*-axis

- $y = \mathrm{f}(-x)$ — reflection in *y*-axis
- $y = \mathrm{f}(x + a) \quad (a > 0)$ — *a* in direction of –*x* (i.e., ←)
- $y = \mathrm{f}(ax) \quad (a > 0)$ — sketch of scale factor $\frac{1}{a}$ in *x*-direction (i.e., a squash)

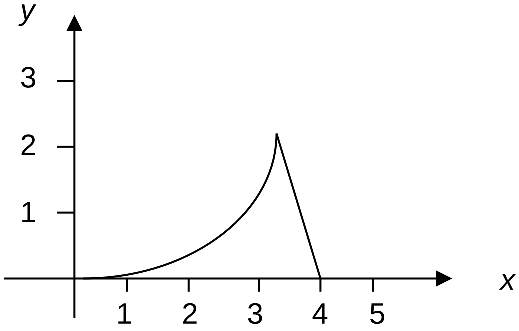
Exercise 6 Graph Sketching

Sketch the following graphs

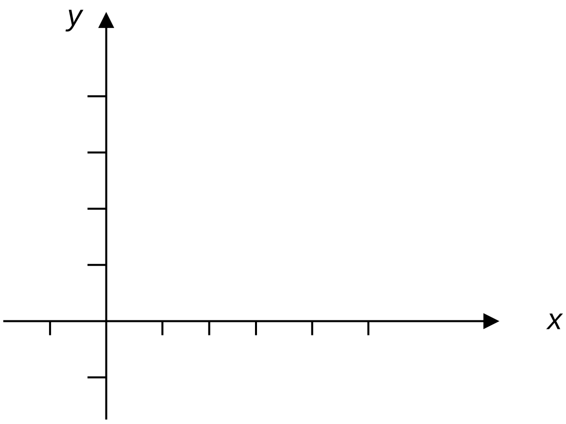
1. $y = \dfrac{1}{x-3}$
2. $y = 4^x - 2$
3. $y = -\log_3 x$
4. $y = \sin 3x \quad 0 \le x \le 2\pi$
5. $y = \dfrac{1}{2}\cos x \quad -\pi \le x \le \pi$
6. $y = \tan(-x) \quad -\pi \le x \le \pi$
7. $y = \sec x + 1 \quad 0 \le x \le 2\pi$
8. $y = \mathrm{cosec}\left(x - \frac{\pi}{2}\right) \quad -\pi < x < \pi$

Write in this box how you could check your graph transformations

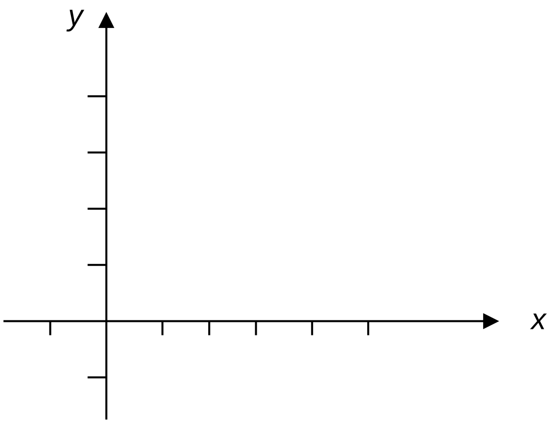
9. The diagram shows the curve with the equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = 0$ for $x < 0$ or $x > 4$



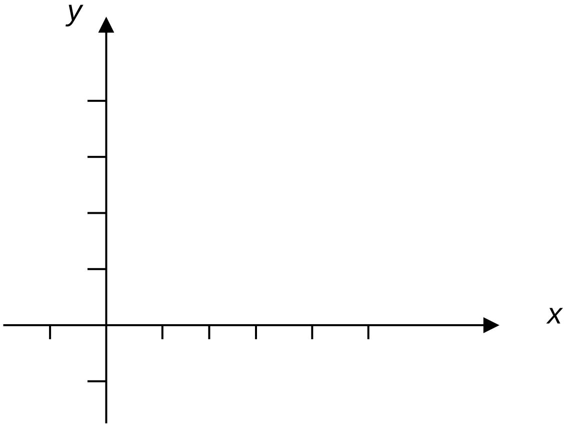
a) $y = \mathrm{f}(x) - 1$



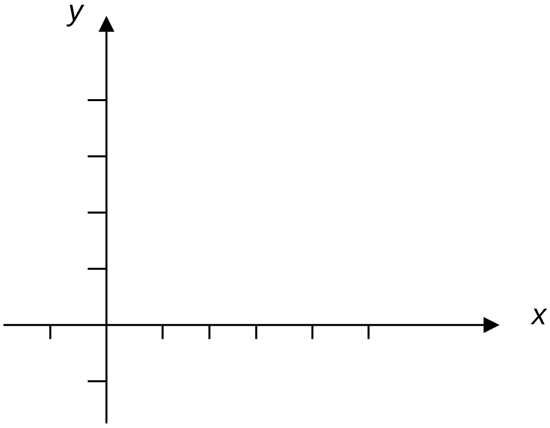
b) $y = \frac{1}{2}\mathrm{f}(x)$



c) $y = \mathrm{f}(2x)$



d) $y = \mathrm{f}(x+1)$



Section D: Trigonometry

In C3 and C4 we still use degrees to solve some equations, but radians will be used much more, so in this booklet angles will always be measured in radians. It is VITAL that you understand these questions.

Exercise 7: Trig Equations

These are revision of C2. Be sure to find all the solutions!

1. Solve the equations for $0 \le \theta \le 2\pi$

   a) $\sin 2\theta = 1$ b) $\cos 2\theta = 0$

   c) $\tan 2\theta = 1.5$ d) $\cos\frac{\theta}{2} = 0.4$

2. Solve the equations for $0 \le \theta \le 2\pi$

   a) $\cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{2}$ b) $\sin\left(\theta - \frac{\pi}{6}\right) = \frac{1}{\sqrt{2}}$

   c) $\tan\left(\theta - \frac{\pi}{6}\right) = \sqrt{3}$ d) $\sin\left(\theta - 0.5^c\right) = 0.9$

NB It is NOT helpful to expand using the compound angle formulae

Solve the following equations for $0 \le x \le \pi$. Give your solutions in terms of π or as decimals correct to 3 decimal places.

3. $\cos^2 x = \frac{1}{4}$

4. $2\sin x = 3\cos x$

5. $\tan^2 x = 1$

6. $\sin^2 x = 2\sin x\cos x$

7. $3\tan x = \cos x$

8. $\tan x = 2\sin x$

9. $\sin^2 x + \sin x = 2$

10. $\tan^2 x = \tan x + 2$

11. $\sin 2x = \cos 2x$

12. $1 + \sin x = 2\cos^2 x$

13. $\tan\left(x - \frac{\pi}{4}\right) = 1$

14. $\cos\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$

Write down how you would check your solutions to these problems

You must learn:

- $\tan x \equiv \dfrac{\sin x}{\cos x}$
- $\sec x \equiv \dfrac{1}{\cos x}$
- $\operatorname{cosec} x \equiv \dfrac{1}{\sin x}$
- $\cot x \equiv \dfrac{1}{\tan x} \equiv \dfrac{\cos x}{\sin x}$
- $\sin^2 x + \cos^2 x \equiv 1$
- $\tan^2 x + 1 \equiv \sec^2 x$
- $1 + \cot^2 x \equiv \operatorname{cosec}^2 x$

Using trig identities in proofs

Don’t forget to set this out properly and to show every stage of your working.

Example

Prove $\cot\theta(\operatorname{cosec}\theta - \cot\theta) \equiv \dfrac{\cos\theta}{1+\cos\theta}$

NB The right hand side has only cosθ so change everything into cosθ or sin θ

$$\text{LHS} \equiv \cot\theta(\operatorname{cosec}\theta - \cot\theta)$$

$$\equiv \frac{\cos\theta}{\sin\theta}\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)$$

$$\equiv \frac{\cos\theta}{\sin\theta}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$\equiv \frac{\cos\theta(1-\cos\theta)}{\sin^2\theta}$$

$$\equiv \frac{\cos\theta(1-\cos\theta)}{1-\cos^2\theta}$$

$$\equiv \frac{\cos\theta(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$\equiv \frac{\cos\theta}{1+\cos\theta}$$

$$\equiv \text{RHS}$$

$\therefore$ Proof complete

Exercise 8 Trig proofs

Prove the identities

1. $(\sin\theta+\cos\theta)^2-2\sin\theta\cos\theta\equiv 1$

2. $\text{cosec}\,\theta(1-\cos\theta)(1+\cos\theta)\equiv\sin\theta$

3. $\frac{1}{\sec^2\theta}+\frac{1}{\text{cosec}^2\theta}\equiv 1$

4. $\sin^2\theta+2\cos^2\theta\equiv 2-\sin^2\theta$

5. $\text{cosec}\,\theta-\sin\theta\equiv\cot\theta\cos\theta$

6. $\text{cosec}^2\theta+\sec^2\theta\equiv\text{cosec}^2\theta\sec^2\theta$

7. $\frac{\sin\theta}{1+\sin\theta}\equiv\tan\theta(\sec\theta-\tan\theta)$

8. $\frac{\sec\theta}{\tan\theta+\cot\theta}\equiv\sin\theta$

9. $\sin^4 x-\cos^4 x\equiv\sin^2 x-\cos^2 x$

10. $\frac{\sin\theta}{1-\cos\theta}+\frac{\sin\theta}{1+\cos\theta}\equiv 2\,\text{cosec}\,\theta$

Exercise 8a More Trig proofs

We strongly recommend that you do this exercise because trig proofs can be tough, especially if you do not know your formulae. The more of these questions you complete, the easier you are going to find the C3 work.
However, if you are confident with this work it will be OK if you **don't** do Exercise 8a.

Prove the identities

1. $tan\theta \operatorname{cosec}\theta \equiv \sec\theta$
2. $\cot\theta \sec\theta \tan\theta\sqrt{(1-\sin^2\theta)} \equiv 1$
3. $\sec\theta - \cos\theta \equiv \sin\theta\tan\theta$
4. $\tan\theta + \cot\theta \equiv \frac{1}{\sin\theta\cos\theta}$
5. $\cos^2\theta - \sin^2\theta \equiv 2\cos^2\theta - 1$
6. $\frac{\sin^2\theta}{1-\cos\theta} \equiv 1 + \cos\theta$
7. $\operatorname{cosec}^2\theta(\tan^2\theta - \sin^2\theta) \equiv \tan^2\theta$
8. $\frac{\cos\theta}{\sin\theta+1} + \frac{\sin\theta+1}{\cos\theta} \equiv 2\sec\theta$
9. $\sec^4\theta - \tan^4\theta \equiv 1 + 2\tan^2\theta$
10. $\frac{\cot^2\theta}{1+\cot^2\theta} \equiv \cos^2\theta$
11. $\frac{2\tan\theta}{1+\tan^2\theta} \equiv 2\sin\theta\cos\theta$
12. $\frac{1-\tan^2\theta}{1+\tan^2\theta} \equiv 1 - 2\sin^2\theta$
13. $\frac{\cot\theta+\tan\theta}{\operatorname{cosec}\theta+\sec\theta} \equiv \frac{1}{\cos\theta+\sin\theta}$
14. $\sin^3 x + \cos^3 x \equiv (\sin x + \cos x)(1 - \sin x \cos x)$

Using Trig Identities to Solve Equations

Example Solve $\operatorname{cosec}^2\theta = 1 + 2\sec^2\theta$ for $-\pi \le \theta \le \pi$

$$\frac{1}{\sin^2\theta} = 1 + \frac{2}{\cos^2\theta}$$

$$\cos^2\theta = \sin^2\theta\cos^2\theta + 2\sin^2\theta$$

$$1 - \sin^2\theta = \sin^2\theta(1 - \sin^2\theta) + 2\sin^2\theta$$

$$1 - \sin^2\theta = \sin^2\theta - \sin^4\theta + 2\sin^2\theta$$

$$\sin^4\theta - 4\sin^2\theta + 1 = 0$$

$$\sin^2\theta = \frac{4 \pm \sqrt{16-4}}{2}$$

$$= \frac{4 \pm \sqrt{12}}{2}$$

$$\sin^2\theta = 3.732 \text{ or } 0.2679$$

$$\sin\theta = \pm 1.93 \qquad \pm 0.5176$$

no solutions

$\sin\theta = 0.5176$ $\qquad$ $\sin\theta = -0.5176$

$\theta = 0.544 \text{ or } 2.60$ $\qquad$ $\theta = -0.544 \text{ or } -2.6$

Exercise 9 Trig Equations

Give answers in terms of π where appropriate

1. Solve the equations for $0 \le x \le 2\pi$

a) $\sec x = 2$ b) $\cot x = \sqrt{3}$ c) $\operatorname{cosec} x = \sqrt{2}$

d) $\sec x = 1.2$ e) $\cot x = 3$ f) $\operatorname{cosec} x = 1$

2. Solve the equations

a) $2\cot^2\theta - 3\cot\theta + 1 = 0$ for $0 \le \theta \le 2\pi$

b) $\sec^2\theta - \tan\theta = 1$ for $-\pi \le \theta \le \pi$

c) $\cot^2\theta - 3\operatorname{cosec}\theta + 3 = 0$ for $0 \le \theta \le \pi$

3. Solve the equations

a) $\sec 2x = 3$ for $0 \le x \le 2\pi$

b) $3\operatorname{cosec}^2 2x = 4$ for $0 \le x \le \pi$

Write down how you would check your solutions to these problems

Compound and Double Angles

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

THE FOLLOWING FORMULAE MUST BE LEARNT AND YOU MUST ALSO BE ABLE TO PROVE THEM USING THE FORMULAE ABOVE

$$\sin 2A \equiv 2\sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$$

Example

Prove $\dfrac{\sin 2\theta + \sin\theta}{\cos 2\theta + \cos\theta + 1} \equiv \tan\theta$

$$\text{LHS} = \frac{2\sin\theta\cos\theta + \sin\theta}{(2\cos^2\theta - 1) + \cos\theta + 1}$$

$\cos 2\theta \equiv 2\cos^2\theta - 1$ chosen from the 3 possibilities so that we get $-1 + 1 = 0$

$$= \frac{2\sin\theta\cos\theta + \sin\theta}{2\cos^2\theta + \cos\theta}$$

$$= \frac{\sin\theta(2\cos\theta + 1)}{\cos\theta(2\cos\theta + 1)}$$

Remember:
To cancel fractions you must factorise first

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta \qquad = \text{RHS}$$

Exercise 10 More Trig proofs

Prove the following identities:

1. $\sin(\theta + 90^\circ) \equiv \cos\theta$

2. $\cos x \cos 2x + \sin x \sin 2x \equiv \cos x$

3. $\dfrac{\sin(A+B)}{\sin(A-B)} \equiv \dfrac{\tan A + \tan B}{\tan A - \tan B}$

4. $\sin(A+B)\sin(A-B) \equiv \sin^2 A - \sin^2 B$

5. $\dfrac{\sin A}{\sin B} + \dfrac{\cos A}{\cos B} \equiv \dfrac{2\sin(A+B)}{\sin 2B}$

6. $\tan A + \cot A \equiv 2\operatorname{cosec} 2A$

7. $\dfrac{\sin 2A}{1+\cos 2A} \equiv \tan A$

8. $\cos 2A \equiv \dfrac{1-\tan^2 A}{1+\tan^2 A}$

9. $\cos 3A \equiv 4\cos^3 A - 3\cos A$

10. $\cos^4\theta - \sin^4\theta \equiv \cos 2\theta$

Exercise 10a More Trig proofs

We strongly recommend that you do this exercise because trig proofs can be tough, especially if you do not know your formulae. The more of these questions you complete, the easier you are going to find the C3 work.
However, if you are confident with this work it will be OK if you **don't** do Exercise 10a.

1. $\sin(A+B) + \sin(A-B) \equiv 2\sin A \cos B$

2. $\sin 2x \cos x - \cos 2x \sin x \equiv \sin x$

3. $\tan A + \tan B \equiv \dfrac{\sin(A+B)}{\cos A \cos B}$

4. $\dfrac{\cos 2A}{\cos A - \sin A} \equiv \cos A + \sin A$

5. $\dfrac{\sin A}{\sin B} - \dfrac{\cos A}{\cos B} \equiv \dfrac{2\sin(A-B)}{\sin 2B}$

6. $\cot A - \tan A \equiv 2\cot 2A$

7. $\sin 2A \equiv \dfrac{2\tan A}{1+\tan^2 A}$

8. $\sin 3A \equiv 3\sin A - 4\sin^3 A$

9. $\cot\theta \equiv \dfrac{\sin 2\theta}{1-\cos 2\theta}$

10. $\cot 2A + \operatorname{cosec} 2A \equiv \cot A$

11. $\tan 3\theta \equiv \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

12. $\sin^4\theta + \cos^4\theta \equiv \frac{1}{4}(\cos 4\theta + 3)$

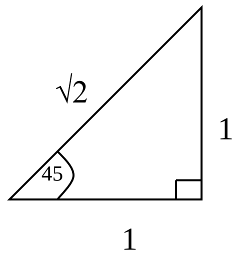
Example

Solve: $\sin\left(\theta + \frac{\pi}{4}\right) = \sqrt{2}\cos\theta$ for $0 \le \theta \le 2\pi$

$$\sin\theta\cos\tfrac{\pi}{4} + \cos\theta\sin\tfrac{\pi}{4} = \sqrt{2}\cos\theta$$

$$\sin\theta \times \tfrac{1}{\sqrt{2}} + \cos\theta \times \tfrac{1}{\sqrt{2}} = \sqrt{2}\cos\theta$$

→ using



$$\sin\theta + \cos\theta = 2\cos\theta$$

$$\sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = 1$$

$$\tan\theta = 1$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

Exercise 11 Trig Equations

Solve these equations for θ (or x) between 0 and 2π inclusive.

1. $\sin(x + 60^\circ) + \cos(x + 30^\circ) = \frac{1}{2}$
2. $\sin 2\theta - \sin\theta = 0$
3. $2\sin 2\theta + \cos\theta = 0$
4. $4\cos 2\theta - 6\cos\theta + 5 = 0$
5. $\sin 2\theta = 2\cos 2\theta$
6. $\cos 2\theta + \sin\theta - 1 = 0$
7. $3\cos 2\theta - \cos\theta + 2 = 0$
8. $\tan 2\theta + \tan\theta = 0$
9. $\sin\theta + \sin\dfrac{\theta}{2} = 0$
10. $\sin\dfrac{\theta}{2} = \sin\theta$
11. $2\cos 2x = 5 - 13\sin x$

NB: In questions 10 and 11 you need to work with $\dfrac{\theta}{2}$

so can use $\sin 2A \equiv 2\sin A\cos A$

to write $\sin\theta = 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$

Write down how you would check your solutions to these problems

Section E: Differentiation

You must know

$$y = x^n \qquad \frac{dy}{dx} = nx^{n-1}$$

$$y = \sin x \qquad \frac{dy}{dx} = \cos x$$

$$y = \cos x \qquad \frac{dy}{dx} = -\sin x$$

$$y = tanx \qquad \frac{dy}{dx} = \sec^2 x$$

$$y = secx \qquad \frac{dy}{dx} = secxtanx$$

$$y = cosecx \qquad \frac{dy}{dx} = -cosecxcotx$$

$$y = cotx \qquad \frac{dy}{dx} = -cosec^2 x$$

You must look in your notes for examples. Here are three exercises on the three techniques you must know.

Function of a Function using the chain rule

i.e., $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

for example: $y = \sin\left(3x^2 - 2\right)$

$y = \sin t \qquad$ where $\quad t = 3x^2 - 2$

$\frac{dy}{dt} = \cos t \qquad \frac{dt}{dx} = 6x$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \cos t \times 6x$$

$$= 6x\cos\left(3x^2 - 2\right)$$

BUT you should be aiming to write the answers down straight away.

Exercise 12 Chain Rule

1. Find $\frac{dy}{dx}$

a) $y=(3x-4)^4$ b) $y=(8x+11)^5$ c) $y=(x^2-3)^2$

d) $y=(3x^3+1)^3$ e) $y=(1-3x)^4$ f) $y=(3-x)^{10}$

2. Differentiate the following:

a) $y=(1+3x)^{-1}$ b) $y=\frac{1}{(2x+1)^2}$ c) $y=\frac{1}{(5x+2)^3}$

d) $y=\frac{3}{(4x-1)^2}$ e) $y=\frac{5}{(x^2+1)}$ f) $y=\frac{10}{(1-x)^4}$

3. a) If $f(x)=\sqrt{2x^2-1}$, show that $f'(x)=\frac{2x}{\sqrt{2x^2-1}}$

b) Find $f'(2)$

4. Find $f'(x)$

a) $f(x)=\sqrt{5x^2+3}$ b) $f(x)=(3x+1)^{\frac{1}{3}}$

c) $f(x)=\frac{1}{\sqrt{x^2-3}}$ d) $f(x)=\frac{1}{x^3+1}$

e) $f(x)=\frac{1}{\sqrt{8x+7}}$ f) $f(x)=\frac{1}{\sqrt{x}+2}$

5. Differentiate the following with respect to *x*. Use the correct notation.

a) $\sin x$ b) $\cos 3x$ c) $4\tan x$

d) $\sin 6x$ e) $\cos\frac{3}{2}x$ f) $5\sin 2x$

g) $10\cos x+\sin x$ h) $\sin(x+1)$ i) $-\cos\frac{1}{2}x$

j) $\tan(3x+1)$ k) $-\sec(4x^2+x)$ l) $-4\cot\frac{2x}{3}$

6. Find $\frac{dy}{dx}$ in the following:

a) $y=(\sin x)^2$ b) $3(\cos x)^2$ c) $y=\sin^3 x$

d) $y=2\cos^3 x$ e) $y=\sqrt{\cos x}$ f) $y=\cos^2 4x$

g) $y=x+\sqrt{\sin x}$ h) $y=(\sin 2x)^{\frac{3}{2}}$

i) $y=cosec^3(4x)$ j) $-4\tan^{\frac{1}{2}}(5x+1)$

Product Rule

If $y = uv$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Example $y = x^3 \sin x$

$$\frac{dy}{dx} = x^3 \cos x + 3x^2 \sin x$$

Exercise 13 Product Rule

Differentiate the following with respect to *x* and simplify your answers.

1. $y = x(2x+1)^2$
2. $y = x^2(3x-1)^2$
3. $y = x^3(x-1)^2$
4. $y = (x+2)^2(x+3)^2$
5. $y = 2x(1-x)^3$
6. $y = (4x^3+1)(x-1)^2$
7. $y = 5x(1+2x)^4$
8. $y = (3x+1)^4(x-3)$
9. $y = \left(x+2\right)x^{\frac{3}{2}}$
10. $y = (4x+1)^2\sqrt{x}$
11. $y = x^2\sqrt{2x+1}$
12. $y = x^3\sqrt{4x-1}$

Differentiate the following with respect to *x*:

13. $x \sin x$
14. $x \cos 2x$
15. $x^2 \sin x$
16. $\sin x \cos x$
17. $sinx\ tan2x$
18. $secxcosec3x$

Exercise 14 Integration

Do these integrals, check by differentiation.

1. $\int 5(x+7)^4\, dx$
2. $\int 9(3x-1)^2\, dx$
3. $\int 8(2x-5)^3\, dx$
4. $\int 6x(x^2+1)^2\, dx$
5. $\int 2cos2x\ \ dx$
6. $\int sin3x\, dx$

Write in this box two ways you would check your differentiation…

Quotient Rule

If $y = \frac{u}{v}$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$$

Example

$$y = \frac{4x^2}{\sin x}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sin x \times 8x - 4x^2\cos x}{\sin^2 x}$$

$$= \frac{8x\sin x - 4x^2\cos x}{\sin^2 x}$$

Exercise 15 Quotient Rule

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$

1. $y = \frac{2x}{x+1}$
2. $y = \frac{x+7}{2x-1}$
3. $y = \frac{x^2}{3x+1}$
4. $y = \frac{4x+5}{x^2+1}$
5. $y = \frac{(x+1)^2}{x^2}$
6. $y = \frac{2x+1}{(x+1)^3}$
7. $y = \frac{(3x+2)^2}{(4x+1)^3}$
8. $y = \frac{x^3}{(x-3)^4}$
9. $y = \frac{(3x-4)^3}{(2x+1)^2}$
10. $y = \frac{\sqrt{x+1}}{x}$
11. $y = \frac{x}{\sqrt{(4x+3)}}$
12. $y = \frac{3x+1}{\sqrt{2x-1}}$
13. $y = \frac{\cos x}{x}$
14. $y = \frac{\sin 2x}{x^2}$
15. $y = \frac{x}{\sin x}$
16. $y = \frac{x^3}{\cos x}$
17. $y = \frac{\cos x}{\sin x}$
18. $y = \frac{cosecx}{x^2+1}$

*

Section F: Algebraic Fractions

Exercise 16 Algebraic Fractions

Simplify:

1. $\frac{1}{4a} + \frac{1}{7a}$

2. $\frac{2}{y+1} - \frac{1}{y^2+y}$

3. $\frac{8x^2y}{5} \div 2xy^3$

4. $\frac{3p}{4q^2} \times \frac{2p}{q^3}$

5. $\frac{x+2}{2x+1} \times \frac{6x^2-7x-5}{x^2-4}$

6. $\frac{1}{2x-3} - \frac{x-1}{2x^2-15x+18}$

7. $\frac{x^2+5x+6}{x^2-4}$

8. $\frac{2x^3-2x}{x^2-2x+1}$

9. $\frac{x-3}{x+2} - \frac{x-2}{x-3}$

10. $x - \frac{3}{x+2}$

11. $\frac{6}{x^2+6x+5} + \frac{5}{x^2+7x+6}$

12. $\frac{x^2-2x-15}{x^2-4} \div \frac{x^2-3x-18}{x^2+4x+4}$

13. $\frac{x^2+4x}{x^2-9} \times \frac{x-3}{x^2+5x+4}$

14. $1 + \frac{1}{x+2}$

15. Show that $\frac{x^4+x+1}{x^2+1}$ can be put in the form $Ax^2 + B + \frac{Cx+D}{x^2+1}$.
Find the values of *A*, *B*, *C* and *D*.

16. Find the value of the constants of *A*, *B*, *C* and *D* in the identity

$$x^3 - 2x^2 - 5 \equiv (Ax^2 + Bx + C)(x-3) + D$$

17. $f(x) = \dfrac{x^3 - 3x^2 + 5x - 2}{(x+1)^2}$

Show that f(x) can be written as

$f(x) = x + A + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$

where *A*, *B* and *C* are constants to be found.

18. $f(x) = \dfrac{2x}{x+1} - \dfrac{x+7}{x^2 - x - 2}$

Show that $f(x) = 2 - \dfrac{3}{x-2}$

19. Given that $\dfrac{4x^4+4x^3-23x^2-4}{x^2+x-6} \equiv Ax^2 + Bx + C - \dfrac{D}{x+E}$,

find the value of each of the constants A, B, C, D and E.

20. Find the solution of the equation

$$\frac{2x^2+x-1}{x^2-x} + \frac{2}{x} = \frac{3x-1}{x-1}$$

Write in this the method you would use to check your answers…

Section G: e and ln

Exercise 17 e and ln

1. Differentiate the following expressions

a) e^x b) e^{2x} c) $e^{5x^3-2x^2+x-2}$ d) e^{sinx}

e) ln x f) ln 2x g) ln ax h) $\ln(5x^3+2x^2+x-2)$

i) ln (sin x) j) ln (sec x) k) –ln (cos x) l) ln (sec x + tan x)

m) 3^x n) a^x o) a^{sinx} p) $a^{\sin^2(3x^2-2)}$

2. Sketch the following curves

a) $y = e^x$ b) $y = -e^x$ c) $y = e^{-x}$ d) $y = e^{x-2}$

e) y = ln x f) y = ln(-x) g) y = -ln x h) y = ln (x-2)

3. A heated cube is dropped into a liquid. As the ball cools, its temperature, T °C, t minutes after it enters the liquid, is given by $T = 200\, e^{-0.1t} + 30 \quad t \geq 0$.

(*a*) Find the temperature of the ball as it enters the liquid.
(*b*) Find the value of t for which T = 150, giving your answer to 3 significant figures.
(*c*) Find the rate at which the temperature of the ball is decreasing at the instant when t = 50. Give your answer in °C per minute to 3 significant figures.
(*d*) From the equation for temperature T in terms of t, given above, explain why the temperature of the ball can never fall to 20 °C.

Section H: Reverse Chain Rule

Example

$\int x(3x^2-5)^3 dx$

Use the Guess Differentiate Adjust method

G: $(3x^2-5)^4$ (why was $(3x^2-5)^4$ a sensible guess?

D: $\frac{d}{dx}(3x^2-5)^4 = 24x(3x^2-5)^3$

A: That answer is 24 times too big. So $\frac{d}{dx}(\frac{1}{24}(3x^2-5)^4) = x(3x^2-5)^3$

$\therefore \int x(3x^2-5)^3 dx = \frac{1}{24}(3x^2-5)^4 + c$

Exercise 18 Reverse Chain Rule

1) $\int x(3x^2-5)^6 dx$
2) $\int 3x(3x^2-5)^6 dx$
3) $\int 4x(3x^2-5)^{10} dx$
4) $\int 5x^2(3x^3-5)^4\, dx$
5) $\int \cos x \sin^4 x\, dx$
6) $\int \sin x \cos^9 x\, dx$
7) $\int \sec^2 x\ \tan^4 x\, dx$
8) $\int \operatorname{cosec}^6 x \cot x\, dx$

Test Yourself

- You must attempt all these questions.
- Mark your attempts using the mark scheme provided at the back of this pack
- Remember, this test and the rest of the booklet all needs to be handed in (marked !) when you arrive for your **first lesson back** in September. This will be on (or after) September 14th

1. Simplify $\dfrac{x^3 - 3x^2}{x^3 - 9x}$ (4)

2. Express $\dfrac{3}{x^2 + 2x} + \dfrac{x-4}{x^2 - 4}$ as a single fraction in its simplest form. (5)

3. a) Differentiate $\sqrt{1 + x^2}$ (4) b) Differentiate $\dfrac{1+x}{1-x}$ (3)

4. Fig 1 shows the graph of $y = x\sqrt{1+x}$. The point *P* on the curve is on the *x*-axis.

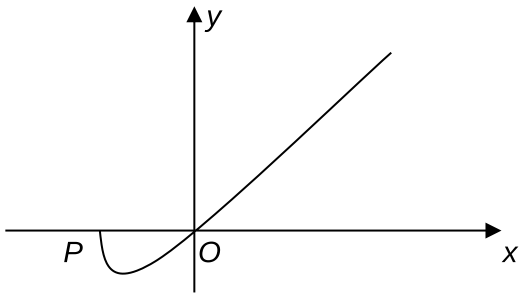


Fig 1

i) Write down the coordinates of *P* (1)

ii) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3x+2}{2\sqrt{1+x}}$. (4)

iii) Hence find the coordinates of the turning point on the curve.
What can you say about the gradient of the curve at *P*. (4)

5. Find the equation of the tangent to the curve $y = \dfrac{2+x}{\cos x}$ at the point on the curve where $x = 0$. (6)

6. A curve has equation $y = \frac{2x}{\sin x},\ 0 < x < \pi$

a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. (2)

b) The point *P* on the curve has coordinates $\left(\frac{\pi}{2}, \pi\right)$

i) Show that the equation of the tangent to the curve at *P* is $y = 2x$ (3)

ii) Find the equation of the normal to the curve at *P*, giving your answer in the form $y = mx + c$ (3)

7. Find the solutions of

$$6\tan\theta - \sec^2\theta = 7$$

in the interval $0 \le \theta < 2\pi$, giving each answer in radians to one decimal place. (5)

8. i) Express $\sec^2\theta + \mathrm{cosec}^2\theta$ in terms of $\sin\theta$ and $\cos\theta$, giving your answer as a single fraction as simply as possible. (3)

ii) Hence prove that $\sec^2\theta + \mathrm{cosec}^2\theta \equiv 4\mathrm{cosec}^2 2\theta$ (3)

iii) Find the values of θ, for $0 < \theta < \frac{1}{2}\pi$, such that $\sec^2\theta + \mathrm{cosec}^2\theta = 10$ (5)

9. a) Prove that $\frac{2\tan x}{1+\tan^2 x} \equiv \sin 2x$ (4)

b) Hence or otherwise find the exact value of tan 15° in the form $a + b\sqrt{3}$, where *a* and *b* are integers. (4)

10. a) Show that $\frac{\cot^2\theta}{1+\cot^2\theta} \equiv \cos^2\theta$ (3)

b) Hence solve $\frac{\cot^2\theta}{1+\cot^2\theta} = 2\sin 2\theta$ for $0^\circ \le \theta \le 360^\circ$ (6)

11. i) Given that $\sin\left(\theta + 45^\circ\right) = 2\sin\theta$, show that

$$\tan\theta = \frac{1}{2\sqrt{2}-1}$$ (4)

ii) Hence solve the equation

$$\sin\left(\theta + 45^\circ\right) = 2\sin\theta$$

for value of θ between 0° and 360°, giving your answers correct to the nearest degree. (2)

12. i) Write down the formula for $\tan 2x$ in terms of $\tan x$. (1)

ii) By letting $\tan x = t$, show that the equation

$$4\tan 2x + 3\cot x \sec^2 x = 0$$

becomes $3t^4 - 8t^2 - 3 = 0$ (4)

iii) Find all the solutions of the equation

$$4\tan 2x + 3\cot x \sec^2 x = 0$$

which lie in the range $0 \le x \le 2\pi$ (4)

13. a) For each of the following expressions, find dy/dx

i) $y = 3x^4 sec4x$ (2)

ii) $y = 5\tan^7(4x^2)$ (2)

iii) $y = \frac{4x^2}{cotx}$ (3)

iv) $y = e^{sinx - 3cos2x}$ (2)

b) Integrate each of the following expressions

i) $\int \sec 2x \tan 2x \, dx$ (2)

ii) $\int \cos 2x \, (\sin^2 2x) \, dx$ (2)

Total score is **100**. If you scored over **90** you are **READY FOR A2 MATHS!**

If you did not score more than 90, you are **NOT** ready for A2 Maths. You need to go back and do some more practice questions. You should then take the “Are you ready for A2 Maths” Test again and keep taking it until you get more than 90.

ANSWERS TO EXERCISES

Exercise 1

1) $\frac{1}{10}a^6p^4q^3$ 2) a^8b^{12} 3) x^4 4) a^8

5) $\frac{5}{3}a^4b$ 6) $36abc^2$ 7) $6a^2b^{-2}c^2$ 8) $3x^7y^3z$

Exercise 2

(Q1-6: equivalent forms are acceptable)

1) $2x - \frac{2}{x^2}$ 2) $-2x^{-3} - x^{-2}$ 3) $-\frac{1}{x^2} + \frac{14}{x^3} - \frac{12}{x^4}$ 4) $\frac{3}{2}\sqrt{x} - \frac{1}{2\sqrt{x}}$

5) $\frac{1}{2x^{\frac{1}{2}}} + \frac{1}{2x^{\frac{3}{2}}}$ 6) $-\frac{1}{x^2} - \frac{2}{x^3} - \frac{3}{x^4}$ 7) 4 8) $-\frac{3}{2}$

9) $\frac{1}{54}$

Exercise 3

1) $\frac{2}{3}x^{\frac{3}{2}} + \frac{3}{2}x^{\frac{2}{3}} + c$ 2) $-\frac{2}{x^2} + \frac{1}{x} - \frac{x^3}{3} + c$ 3) $\frac{4}{7}x^{\frac{7}{2}} - \frac{5}{3}x^{\frac{3}{5}} + c$

4) $\frac{1}{4}x^2 - 4\sqrt{x} - x + c$ 5) $-\frac{1}{3}x^{-3} + \frac{4}{5}x^{\frac{3}{4}} - 4x + c$ 6) $4x^{\frac{3}{2}} - \frac{3}{4}x^4 - x^{-1} + 2x + c$

7) $\frac{14}{3}$ 8) $\frac{9}{10}$ 9) $31\frac{1}{3}$

10) $10 - \frac{16\sqrt{2}}{3}$ 11) $130\frac{4}{5}$ 12) $-\frac{62}{81}$

Exercise 4

1) $\log_a \frac{9}{8}$ 2) $\log_a \frac{4}{27}$ 3) $\log_a \frac{441}{a^2}$ 4) $\log_a 10$

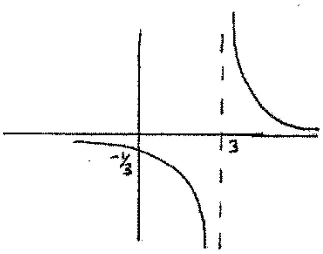
5) $\log_a 6a^5$ 6) $\log_a \frac{1}{12}$ 7) $\log_{10} \frac{x^2y}{z^3}$ 8) $\log_2 \frac{\sqrt{x}(x-3)}{(2x+1)^2}$

Exercise 5

1) $x = 15$ 2) $x = 9$ 3) $x = 625$ 4) $x = \frac{1}{21}$ 5) $x = \frac{10}{11}$

Exercise 6

1.



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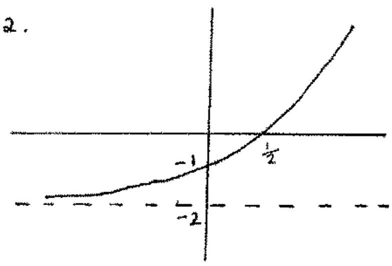

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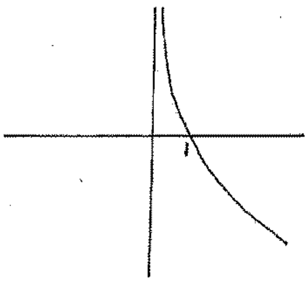

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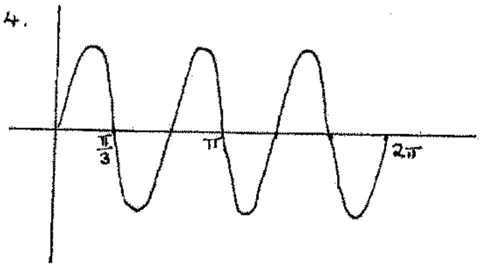

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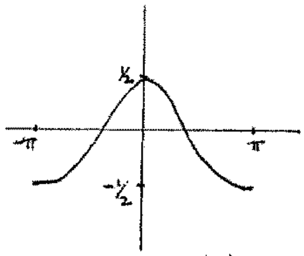

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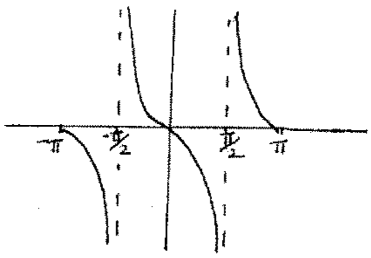

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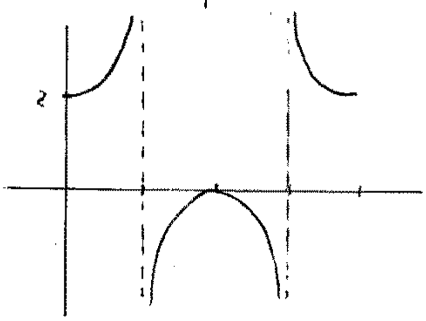

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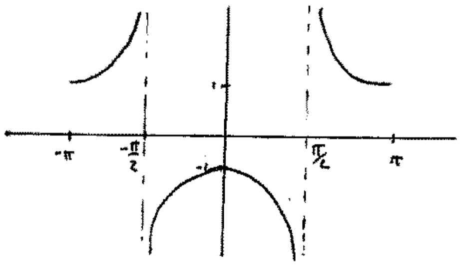

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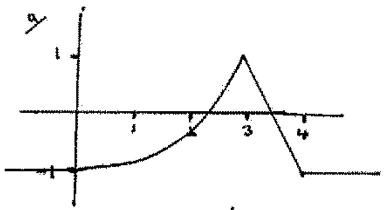

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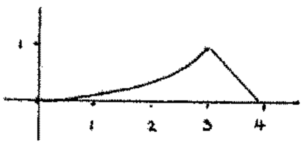

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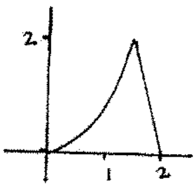

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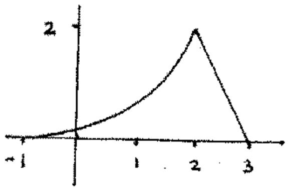


Exercise 7

1a) $\frac{\pi}{4}, \frac{5\pi}{4}$ 1b) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ 1c) $0.491, 2.062, 3.633, 5.204$ 1d) 2.319

2a) $\frac{7\pi}{12}, \frac{23\pi}{12}$ 2b) $\frac{5\pi}{12}, \frac{11\pi}{12}$ 2c) $\frac{\pi}{2}, \frac{3\pi}{2}$ 2d) $1.620, 2.522$

3a) $\frac{\pi}{3}, \frac{2\pi}{3}$ 4) 0.983 5) $\frac{\pi}{4}, \frac{3\pi}{4}$ 6) $0, \pi, 1.107$ 7) $0.308, 2.834$

8) $0, \pi, \frac{\pi}{3}$ 9) $\frac{\pi}{2}$ 10) $1.107, \frac{3\pi}{4}$ 11) $\frac{\pi}{8}, \frac{5\pi}{8}$ 12) $\frac{\pi}{6}, \frac{5\pi}{6}$ 13) $\frac{\pi}{2}$ 14) $\frac{\pi}{2}$

Exercise 8

Proofs

Exercise 9

1a) $\frac{\pi}{3}, \frac{5\pi}{3}$ 1b) $\frac{\pi}{6}, \frac{7\pi}{6}$ 1c) $\frac{\pi}{4}, \frac{3\pi}{4}$ 1d) $0.586, 5.70$ 1e) $0.321, 3.46$ 1f) $\frac{\pi}{2}$

2a) $\frac{\pi}{4}, \frac{5\pi}{4}, 1.11, 4.25$ 2b) $-\pi, 0, \pi, -\frac{3\pi}{4}, \frac{\pi}{4}$ 2c) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

3a) $0.616, 2.53, 3.76, 5.67$ 3b) $\frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$

Exercise 10

Proofs

Exercise 11

1) $1.28, 5.01$ 2) $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$ 3) $\frac{\pi}{2}, \frac{3\pi}{2}, 3.39, 6.03$

4) $\frac{\pi}{3}, \frac{5\pi}{3}, 1.32, 4.97$ 5) $0.553, 2.12, 3.69, 5.27$ 6) $0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, 2\pi$

7) $\frac{\pi}{3}, \frac{5\pi}{3}, 1.91, 4.37$ 8) $0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$ 9) $0, \frac{4\pi}{3}, 2\pi$

10) $0, \frac{2\pi}{3}, 2\pi$ 11) $0.253, 2.89$

Exercise 12

1. $a)\ 12(3x-4)^3$ $b)\ 40(8x+11)^4$ $c)\ 4x(x^2-3)$
   $d)\ 27x^2(3x^3+1)^2$ $e)\ 12(1-3x)^3$ $f)\ -10(3-x)^9$
2. a) $-3(1+3x)^{-2}$ $b)\ -4(2x+1)^{-3}$ $c)\ -15(5x+2)^{-4}$
   $d)\ -24(4x-1)^{-3}$ $e)\ -10x(x^2+1)^{-2}$ $f)\ 40(1-x)^{-5}$
3. b) $\frac{4}{7}\sqrt{7}$
4. $a)\ \frac{5x}{\sqrt{(5x^2+3)}}$ $b)\ (3x+1)^{-\frac{2}{3}}$
   $c)\ -x(x^2-3)^{-\frac{3}{2}}$ $d)\ \frac{3x^2}{(x^3+1)^2}$
   $e)\ -4(8x+7)^{-\frac{3}{2}}$ $f)\ -\frac{1}{\sqrt{2}(\sqrt{x}+2)^2}$
5. a) cos x b) -3sin 3x c) 4sec^{2}x
   d) 6cos 6x e) $-\frac{3}{2}\sin\frac{3}{2}x$ f) 10 cos 2x
   g) -10sinx + cos x h) cos(x + 1) i) $\frac{1}{2}\sin\frac{1}{2}x$
   j) $3\sec^2(3x+1)$ k) $-(8x+1)\sec(4x^2+x)\tan(4x^2+x)$
   l) $\frac{8}{3}cosec^2\left(\frac{2x}{3}\right)$
6. a) 2 sin x cos x b) -6 sin x cos x c) 3 sin^{2}x cosx
   d) -6 sin^2 x cos x e) $-½\ (cos\ x)^{-\frac{1}{2}} cos\ x$ f) -8 cos 4x sin 4x
   g) $1+\frac{cosx}{2\sqrt{sinx}}$ h) $3(sin\ 2x)^{\frac{1}{2}}\cos 2x$
   $i)\ -12cosec^3 4xcot4x$ $j)\ -10\sec^2(5x+1)\tan^{-\frac{1}{2}}(5x+1)$

Exercise 13

1. $(2x + 1)(6x + 1)$
2. $2x(3x – 1)(6x – 1)$
3. $x^2(x – 1)(5x – 3)$
4. $2(x + 2)(x + 3)(2x + 5)$
5. $2(1 – x)^2(1 – 4x)$
6. $2(x – 1)(10x^3 – 6x^2 + 1)$
7. $5(1 + 2x)^3(10x + 1)$
8. $(3x + 1)^3(15x – 35)$
9. $\frac{\sqrt{x}}{2}(5x+6)$
10. $\frac{(4x+1)}{2\sqrt{x}}(20x+1)$
11. $\frac{x}{\sqrt{2x+1}}(5x+12)$
12. $\frac{x^2}{\sqrt{4x-1}}(14x-3)$
13. $\sin x + x \cos x$
14. $\cos 2x – 2x \sin 2x$
15. $x^2 \cos x + 2x \sin x$
16. $\cos^2 x – \sin^2 x$
17. $2\sin(x)\sec^2 2x + \cos(x)\tan(2x)$
18. $\sec(x)\tan(x)\,cosec(3x) - 3\sec(x)\,cosec(3x)\cot(3x)$

Exercise 14

1. $(x + 7)^5 + c$
2. $(3x - 1)^3 + c$
3. $(2x - 5)^4 + c$
4. $(x^2 + 1)^3 + c$
5. $\sin 2x + c$
6. $-\frac{1}{3}\cos 3x + c$

Exercise 15

1. $\frac{2}{(x+1)^2}$
2. $-\frac{15}{(2x-1)^2}$
3. $\frac{3x^2+2x}{(3x+1)^2}$
4. $\frac{4-10x-4x^2}{(x^2+1)^2}$
5.. $\frac{-2(x+1)}{x^3}$
6. $\frac{-4x-1}{(x+1)^4}$
7. $\frac{-6(3x+2)(2x+3)}{(4x+1)^4}$
8. $\frac{-x^3-9x^2}{(x-3)^5}$
9. $\frac{(3x-4)^2(6x+25)}{(2x+1)^3}$
10. $\frac{-x-2}{2x^2\sqrt{x+1}}$
11. $\frac{2x+3}{(4x+3)^{\frac{3}{2}}}$
12. $\frac{3x-4}{(2x-1)^{\frac{3}{2}}}$
13. $-\left(\frac{x\sin x+\cos x}{x^2}\right)$
14. $\frac{2x\cos 2x-2\sin 2x}{x^3}$
15. $\frac{\sin x-x\cos x}{\sin^2 x}$
16. $\frac{3x^2\cos x+x^3\sin x}{\cos^2 x}$
17. $-\frac{1}{\sin^2 x} = -cosec^2 x$
18. $\frac{dy}{dx} = \frac{\left((x^2+1)cosec(x)\cot(x)-2x\,cosec(x)\right)}{(x^2+1)^2}$

Exercise 16

1. $\frac{11}{28a}$
2. $\frac{2y-1}{y(y+1)}$
3. $\frac{4x}{5y^2}$
4. $\frac{3p^2}{5q^5}$
5. $\frac{3x-5}{x-2}$
6. $\frac{-5}{(2x-3)(x-6)}$
7. $\frac{x+3}{x-2}$
8. $\frac{2x(x+1)}{x-1}$.
9. $\frac{13-6x}{(x+2)(x-3)}$
10. $\frac{(x-1)(x+3)}{x+2}$
11. $\frac{11x+61}{(x+5)(x+1)(x+6)}$
12. $\frac{(x-5)(x+2)}{(x-2)(x-6)}$
13. $\frac{x}{(x+3)(x+1)}$
14. $\frac{x+3}{x+2}$
15. $x^2 - 1 + \frac{x+2}{x^2+1}$
16. $x^2 + x + 3 + \frac{4}{x-3}$
17. $x - 5 + \frac{14}{x+1} - \frac{11}{(x+1)^2}$
18. Proof
19. $4x^2 + 1 - \frac{1}{x+3}$
20. $x = 3, (x \neq 1)$

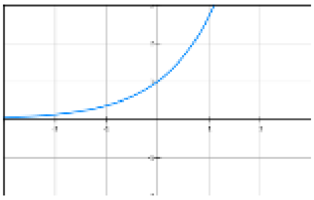
Exercise 17

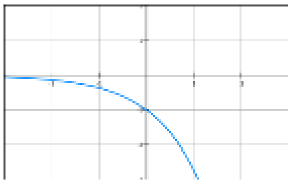
1 a) e^x b) $2e^{2x}$ c) $(15x^2-4x+1)e^{5x^3+2x^2+x-2}$ d) $\cos x\ e^{sinx}$

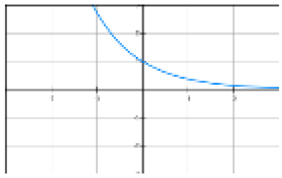
e) $\frac{1}{x}$ f) $\frac{1}{x}$ g) $\frac{1}{x}$ h) $\frac{(15x^2+4x+1)}{5x^3+2x^2+x-2}$

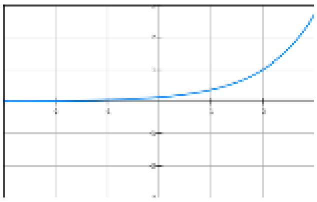
i) $\cot x$ j) $\tan x$ k) $\tan x$ l) $\sec x$

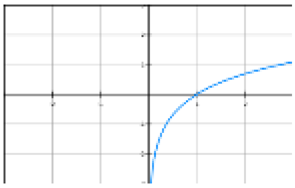
m) $3^x ln\,3$ n) $a^x \ln a$ o) $cosx\ a^{sinx} \ln a$ p) $6x\sin(6x^2-4)\ a^{\sin^2(3x^2-2)} \ln a$

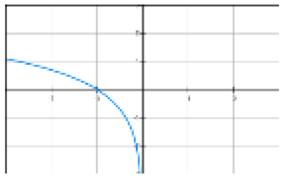
2a)


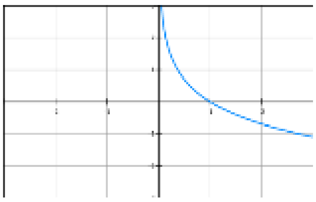
2b)


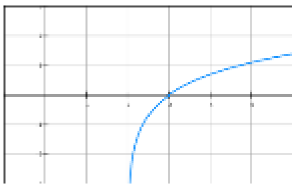
2c)


2d)


2e)


2f)


2g)


2h)


3a) 230°C b) 5.11 c) -0.135°C min^{-1} d) $20 = 200\ e^{-0.1t} + 30$ has no solutions

Exercise 18

1) $\frac{1}{42}(3x^2-5)^7+c$ 2) $\frac{1}{14}(3x^2-5)^7+c$ 3) $\frac{2}{33}(3x^2-5)^{11}+c$

4) $\frac{1}{9}(3x^3-5)^5+c$ 5) $\frac{1}{5}\sin^5 x+c$ 6) $-\frac{1}{10}\cos^{10} x+c$

7) $\frac{1}{5}\tan^5 x+c$ 8) $-\frac{1}{6}cosec^6 x+c$

CWC Test Yourself Mark scheme.

1. $\frac{x^2(x-3)}{x(x^2-9)}$ M1 fac top, M1 fac bottom

$= \frac{x(x-3)}{(x-3)(x+3)}$ M1 cancel 'x' & fac bottom

$= \frac{x}{x+3}$ A1 [4]

2. $\frac{3}{x(x+2)} + \frac{x-4}{(x+2)(x-2)}$ M1M1 fac denoms

$= \frac{3(x-2)+x(x-4)}{x(x+2)(x-2)}$ M1 common denominator

$= \frac{3x-6+x^2-4x}{x(x+2)(x-2)}$

$= \frac{x^2-x-6}{x(x+2)(x-2)}$ M1 correct quadratic on numerator

$= \frac{(x+2)(x-3)}{x(x+2)(x-2)}$

$= \frac{x-3}{x(x-2)}$ A1 [5]

3. a) $y=(1+x^2)^{\frac{1}{2}}$ M1 index $\frac{1}{2}$

$\frac{dy}{dx} = \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \times (2x)$ M1M1 CR

$= x(1+x^2)^{-\frac{1}{2}}$

$= \frac{x}{\sqrt{1+x^2}}$ A1

b) $y = \frac{1+x}{1-x}$

$\frac{dy}{dx} = \frac{1-x-(1-x)(-1)}{(1-x)^2}$ M1M1 QR

$= \frac{1-x+1+x}{(1-x)^2} = \frac{2}{(1-x)^2}$ A1 [7]

KEY Fac = factorise ID = identity
CR = Chain Rule
PR = Product Rule
QR = Quotient Rule

4. $y = x\sqrt{1+x}$

i) $y=0$ $x=-1$ $(-1,0)$ B1

ii) $y = x(1+x)^{\frac{1}{2}}$

$\frac{dy}{dx} = x\,\frac{1}{2}(1+x)^{-\frac{1}{2}} + (1+x)^{\frac{1}{2}}$ M1M1 PR

$= \frac{x}{2\sqrt{1+x}} + \frac{1+x}{\sqrt{1+x}}$ M1 denoms.

$= \frac{x+2+2x}{2\sqrt{1+x}} = \frac{3x+2}{2\sqrt{1+x}}$ A1

iii) At tp $\frac{dy}{dx}=0$

$\Rightarrow 3x+2=0$ M1

$x = -\frac{2}{3}$ A1

$y = -\frac{2}{3}\sqrt{1-\frac{2}{3}} = \frac{-2}{3\sqrt{3}}$ A1

Gradient is infinite at P B1 [9]

5. $y = \frac{2+x}{\cos x}$

$\frac{dy}{dx} = \frac{\cos x-(2+x)(-\sin x)}{\cos^2 x}$ M1M1 QR

$= \frac{\cos x + 2\sin x + x\sin x}{\cos^2 x}$ A1

At $x=0$ $y = \frac{2}{\cos 0} = 2$ M1

$\frac{dy}{dx} = \frac{1}{1} = 1$ M1

Eqn of tangent

$y-2 = x$ A1 [6]

6. $y = \frac{2x}{\sin x}$

a) $\frac{dy}{dx} = \frac{2\sin x - 2x\cos x}{\sin^2 x}$ M1 A1 QR

b) i) At P $\frac{dy}{dx} = \frac{2}{1} = 2$ B1

Eqn tangent $y - \pi = 2(x - \frac{\pi}{2})$ M1

$y - \pi = 2x - \pi$

$y = 2x$ A1

ii) Gradient of normal $-\frac{1}{2}$ B1

Eqn normal $y - \pi = -\frac{1}{2}(x - \frac{\pi}{2})$ M1

$2y - 2\pi = -x + \frac{\pi}{2}$

$2y = \frac{5}{2}\pi - x$

$y = \frac{5}{4}\pi - \frac{1}{2}x$ A1

[8]

7. $6\tan\theta - \sec^2\theta = 7$

$6\tan\theta - (1 + \tan^2\theta) = 7$ M1 ID

$\tan^2\theta - 6\tan\theta + 8 = 0$ M1 quadratic

$(\tan\theta - 4)(\tan\theta - 2) = 0$ M1 fac

$\tan\theta = 4 \quad \tan\theta = 2$

$\theta = 1.3, 4.5, 1.1, 4.2$ M1 Any 2 solutions, A1 all 4 solutions

[5]

8. i) $\sec^2\theta + \csc^2\theta$

$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$ M1 ID

$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin^2\theta}$ M1 common denom

$= \frac{1}{\cos^2\theta\sin^2\theta}$ A1 ID

ii) $\sec^2\theta + \csc^2\theta \equiv 4\csc^2 2\theta$

LHS $= \frac{1}{\cos^2\theta\sin^2\theta}$ M1 (i)

8ii) continued

$= \frac{4}{(2\cos\theta\sin\theta)^2}$ M1

$= \frac{4}{\sin^2 2\theta}$

$= 4\csc^2 2\theta =$ RHS A1

iii) $\sec^2\theta + \csc^2\theta = 10$

$4\csc^2 2\theta = 10$ M1 (ii)

$\csc 2\theta = \sqrt{\frac{5}{2}}$ M1

$\sin 2\theta = \sqrt{\frac{2}{5}}$

$2\theta = 0.684, 2.45$ M1

$\theta = 0.342, 1.23$ M1 one solution, A1 both solutions

[11]

9. a) $\frac{2\tan x}{1 + \tan^2 x} \equiv \sin 2x$

LHS $= \frac{2\tan x}{\sec^2 x}$ M1 ID

$= \frac{\frac{2\sin x}{\cos x}}{\frac{1}{\cos^2 x}}$ M1 ID

$= \frac{2\sin x}{\cos x} \times \frac{\cos^2 x}{1}$ M1

$= 2\sin x\cos x$

$= \sin 2x =$ RHS A1

For my proofs you may have a slightly different answer....

9b) $\tan 15°$

$\frac{2\tan 15}{1+\tan^2 15} = \sin 30$ M1 substitution

$\frac{2\tan 15}{1+\tan^2 15} = \frac{1}{2}$

$4\tan 15 = 1+\tan^2 15$ M1 quadratic

$\tan^2 15 - 4\tan 15 + 1 = 0$

$\tan 15 = \frac{4 \pm \sqrt{16-4}}{2}$ M1 quadratic formula

$= 2-\sqrt{3}$ A1 [8]

10. i) $\frac{\cot^2\theta}{1+\cot^2\theta} \equiv \cos^2\theta$

LHS $= \frac{\frac{\cos^2\theta}{\sin^2\theta}}{1+\frac{\cos^2\theta}{\sin^2\theta}}$ M1 ID

$= \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}}$ M1 denom.

$= \frac{\cos^2\theta}{\sin^2\theta} \times \frac{\sin^2\theta}{1}$

$= \cos^2\theta =$ RHS A1

ii) $\frac{\cot^2\theta}{1+\cot^2\theta} = 2\sin 2\theta$

$\cos^2\theta = 2\sin 2\theta$ M1 i)

$\cos^2\theta = 4\sin\theta\cos\theta$ M1 ID

$\cos\theta(\cos\theta - 4\sin\theta) = 0$ M1 fac

$\cos\theta = 0$ $\tan\theta = \frac{1}{4}$ M1

$\theta = 90, 270, 14, 194$ M1 any 2 solut A1 all 4 solution [9]

11. i) $\sin(\theta+45) = 2\sin\theta$

$\sin\theta\cos 45 + \cos\theta\sin 45 = 2\sin\theta$ M1 ID

$\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = 2\sin\theta$ M1

$\frac{1}{\sqrt{2}}\cos\theta = \sin\theta\left(2-\frac{1}{\sqrt{2}}\right)$ M1

$\tan\theta = \frac{\frac{1}{\sqrt{2}}}{\frac{2\sqrt{2}-1}{\sqrt{2}}} = \frac{1}{2\sqrt{2}-1}$ A1

ii) $\theta = \tan^{-1}\left(\frac{1}{2\sqrt{2}-1}\right)$

$= 29, 209$ M1, A1 [6]

12. i) $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$ B1

ii) $4\left(\frac{2t}{1-t^2}\right) + \frac{3}{t}(1+t^2) = 0$ M1 M1

$\frac{8t^2 + 3(1+t^2)(1-t^2)}{t(1-t^2)} = 0$

$8t^2 + 3(1-t^4) = 0$ M1

$8t + 3 - 3t^4 = 0$

$3t^4 - 8t - 3 = 0$ A1

iii) $(3t^2+1)(t^2-3) = 0$ M1 fac

$t^2 = -\frac{1}{3}$ $t^2 = 3$ M1

No soln $\tan\theta = \pm\sqrt{3}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ M1 2 soln, A1 4 soln. [9]

13. a) i) $y = 3x^4 \sec 4x$

$\frac{dy}{dx} = 3x^4(4\sec 4x \tan 4x) + 12x^3 \sec 4x$ M1

$= 12x^4 \sec 4x \tan 4x + 12x^3 \sec 4x$ A1

ii) $y = 5\tan^7(4x^2)$

$\frac{dy}{dx} = 35 \tan^6(4x^2) \times 8x$ M1

$= 280x \sec^2(4x^2)$ A1

iii) $y = \frac{4x^2}{\cot x}$

$\frac{dy}{dx} = \frac{\cot x(8x) - 4x^2(-\cot x \operatorname{cosec} x)}{\cot^2 x}$ M1 M1

$= \frac{8x + 4x^2 \operatorname{cosec} x}{\cot x}$ A1

iv) $(\cos x + 6\sin 2x)e^{\sin x - 3\cos 2x}$ M1 A1

b) i) $\int \sec 2x \tan 2x \, dx = \frac{1}{2}\sec 2x + c$ M1 A1

$\left(y = \sec 2x, \quad \frac{dy}{dx} = 2\sec 2x \tan 2x\right)$

ii) $\frac{1}{6}\sin^3 2x + c$ M1 A1

[13]

TOTAL SCORE IS (100) IF YOU SCORED OVER (90) YOU ARE

READY FOR A2 MATHS!

IF YOU DID NOT ACHIEVE 90 GO BACK AND DO SOME MORE PRACTICE OTHERWISE YOU WILL BE STRUGGLING THROUGHOUT THE COURSE.