IYGB GCE

Core Mathematics C4

Advanced

Practice Paper Z

Difficulty Rating: 4.1067/2.1167

Time: 2 hours

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

$$f(x) \equiv \frac{1}{(2-3x)^3}, \quad |x| < \frac{2}{3}.$$

a) Find the series expansion of $f(x)$, up and including the term in x^2. **(4)**

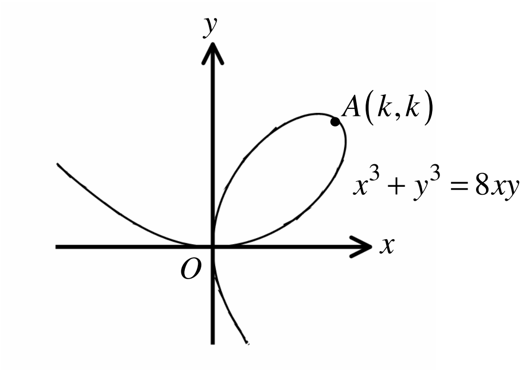
It is given that

$$\frac{2+px}{(2-3x)^3} \equiv \frac{1}{4} + \frac{1}{8}x + qx^2 + \dots$$

where p and q are non zero constants.

b) Determine the value of p and the value of q. **(5)**

Question 2



The figure above shows a curve known as "the folium of Descartes", with equation

$$x^3 + y^3 = 8xy.$$

The point $A(k,k)$, where k is a non zero constant, lies on the curve.

a) Find the value of k. **(2)**

b) Show that the gradient at A is -1. **(4)**

Question 3

A bubble is formed and its volume is increasing at the constant rate of $300 \text{ cm}^3\text{s}^{-1}$.

The shape of the bubble remains spherical at all times.

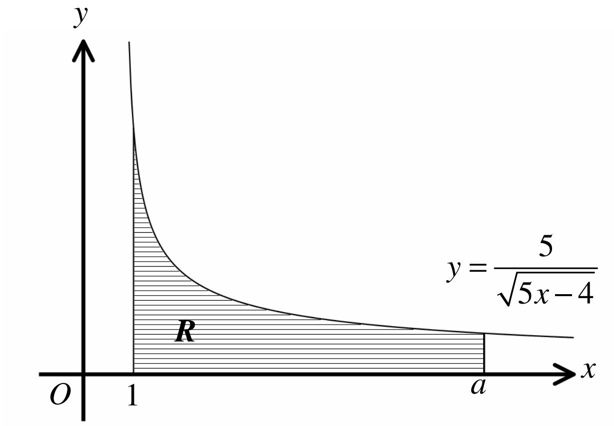
Find the rate at which the radius of the bubble is increasing …

a) … when the radius of the bubble reaches 15 cm . **(4)**

b) … ten seconds after the bubble was first formed. **(4)**

$\left[\text{volume of a sphere of radius } r \text{ is given by } \frac{4}{3}\pi r^3\right]$

Question 4



The figure above shows part of the graph of the curve C with equation

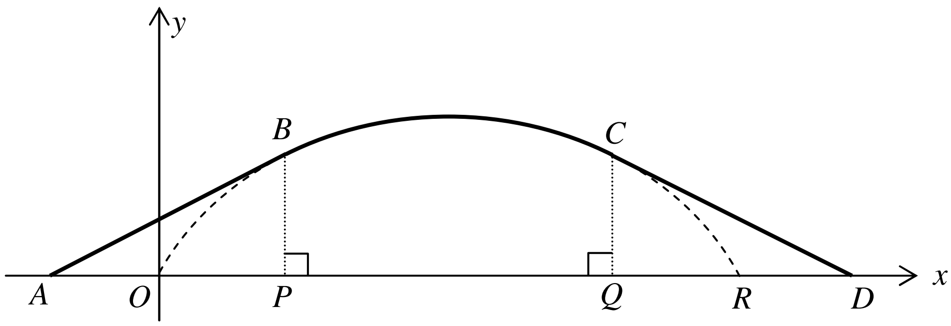
$$y = \frac{5}{\sqrt{5x-4}}, \quad x > \frac{4}{5}.$$

The shaded region R is bounded by the curve, the vertical straight lines $x = 1$ and $x = a$, and the x axis.

The region R is rotated by 2π radians about the x axis forming a solid of revolution.

Given that the area of R is 10 square units, show that the volume of the solid formed is $10\pi \ln 6$ cubic units. **(8)**

Question 5



The figure above shows a **symmetrical** design for a suspension bridge arch $ABCD$.

The curve $OBCR$ is a cycloid with parametric equations

$$x = 6(2\theta - \sin 2\theta), \quad y = 6(1 - \cos 2\theta), \quad 0 \le \theta \le \pi .$$

a) Show clearly that

$$\frac{dy}{dx} = \cot\theta \ . \qquad (4)$$

[continues overleaf]

[continued from overleaf]

The arch design consists of the curved part BC and the straight lines AB and CD.

The straight line AB is a tangent to the cycloid at the point B where $\theta = \frac{\pi}{3}$, and similarly the straight line CD is a tangent to the cycloid at the point C where $\theta = \frac{2\pi}{3}$.

b) Show further that …

i. … the tangent to the cycloid at B meets the x axis at $x = 4\pi - 12\sqrt{3}$. **(5)**

ii. … the length of AP is $9\sqrt{3}$. **(1)**

iii. ... the area between the x axis and the part of the cycloid between B and C is given by

$$36\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} 3 - 4\cos 2\theta + \cos 4\theta \; d\theta .$$ **(4)**

c) Hence find an exact value for the area enclosed by $ABCD$ and the x axis. **(5)**

Question 6

The position vectors and coordinates in this question are relative to a fixed origin O.

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 5\mathbf{i} + 3\mathbf{j} + 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

$$\mathbf{r}_2 = -\mathbf{i} + 5\mathbf{j} + a\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}),$$

where λ and μ are scalar parameters, and a is a scalar constant.

The point A lies on both l_1 and l_2.

a) Find the value of a and the coordinates of A. **(4)**

The point $P(11, p, 12)$, where p is a scalar constant, lies on l_1.

The point $Q(q, -9, -8)$, where q is a scalar constant, lies on l_2.

b) Find the value of p and the value of q. **(2)**

c) Determine the coordinates of the midpoint of PQ. **(2)**

d) Show that $|AP| = |AQ|$ **(2)**

e) Hence, or otherwise, find a vector equation of the angle bisector of $\measuredangle PAQ$. **(3)**

Question 7

The initial population of a city is 1 million.

Let P be the number of inhabitants in millions, t be the time in years, and treat P as a continuous variable.

The rate at which the population of this city is growing per year, is proportional to the product of its population and the difference of its population from 3 million.

a) By forming and solving a differential equation, show that

$$\frac{2P}{3-P} = e^{at},$$

where a is a positive constant. **(8)**

The city doubles its population to 2 million, after ten years.

b) Find the value of a in terms of $\ln 2$. **(2)**

c) Rearrange the answer in part **(a)** to show that

$$P = \frac{3}{1+2^{1-0.2t}}.$$ **(2)**
