Question Number	Scheme	Marks
1.	$\int_1^3 x^2 \ \mathrm{d}x = \left[\frac{x^3}{3}\ln x\right] - \int \frac{x^3}{3}\frac{1}{x}\ \mathrm{d}x$	M1 A1
	$= \left[\frac{x^3}{3}\ln x\right] - \int \frac{x^2}{3}\ \mathrm{d}x$	M1
	$= \left[\frac{x^3}{3}\ln x\right] - \frac{1}{9}x^3$	A1
	$= 9\ln 3 - 3 + \frac{1}{9} = 9\ln 3 - 2\frac{8}{9}$	M1 A1 (6)
		(6 marks)
2. (*a*)	$\frac{\mathrm{d}V}{\mathrm{d}t} = \pm c\sqrt{V}$ *or* $\frac{\mathrm{d}V}{\mathrm{d}t} \propto \sqrt{V}$	M1
	As $V = Ah$, $\frac{dV}{dh} = A$ *or* $V \propto h$	M1
	$\therefore$ *use chain rule to obtain* $\frac{dh}{dt} = -\frac{c}{A}\sqrt{V} = \frac{-c}{\sqrt{A}}\sqrt{h} = -k\sqrt{h}$	A1 (3)
(*b*)	$\int \frac{dh}{h} = -\int kdt$	M1
	$2h^{\frac{1}{2}} = A - kt$	M1 A1
	$h^{\frac{1}{2}} = \frac{A}{2} - \frac{kt}{2}$	
	$h = (A - Bt)^2$ *	A1 (4)
(*c*)	$t = 0,\ h = 1:\quad A = 1$	
	$t = 5,\ h = 0.5:\quad 0.5 = (1 - 5B)^2$	B1
	$B = \frac{(1 - \sqrt{0.5})}{5}\ (B = 0.0586)$	B1
	$h = 0, t = \frac{A}{B} = \frac{5}{1 - \sqrt{0.5}} = 17.1\text{min}$	B1 (3)
(*d*)	$h = \frac{A^2}{4} = 0.25\text{m}$	M1 A1 (2)
		(12 marks)

Question Number	Scheme	Marks
3. (*a*)	$\cos(A+A)=\cos^2 A-\sin^2 A$	M1
	$=\cos^2 A-(1-\cos^2 A)=2\cos^2 A-1$	A1 (2)
(*b*)	$[x=2,\theta=\frac{\pi}{4};x=\sqrt{6},\theta=\frac{\pi}{3}]$	B1
	$x=2\sqrt{2}\sin\theta,\frac{\mathrm{d}x}{\mathrm{d}\theta}=2\sqrt{2}\cos\theta$	B1
	$\int\sqrt{8-x^2}\mathrm{d}x=\int 2\sqrt{2}\cos\theta 2\sqrt{2}\cos\theta\mathrm{d}\theta=\int 8\cos^2\theta\mathrm{d}\theta$	M1 A1
	Using $\cos 2\theta=2\cos^2\theta-1$ to give $\int 4(1+\cos 2\theta)\mathrm{d}\theta \;=4\theta+2\sin 2\theta$	M1
	Substituting limits to give $\frac{1}{3}\pi+\sqrt{3}-2$ or given result	A1 ft (7)
(*c*)	$\frac{dy}{d\theta}=\frac{-2\sin 2\theta}{1+\cos 2\theta}$	B1
	Using the chain rule, with $\frac{dx}{d\theta}=\sec\theta\tan\theta$ to give $\frac{\mathrm{d}y}{\mathrm{d}x}(=-2\cos\theta)$	M1
	Gradient at the point where $\theta=\frac{\pi}{3}$ is –1	A1 ft
	Equation of tangent is $y+\ln 2=-(x-2)$ (or equivalent answer)	M1 A1 (5)
		(14 marks)
4. (*a*)	*A*: y = 16, *B*: y = 2	B1 (1)
(*b*)	$y(x-3)=4,\quad yx-3y=4$	
	$x=\frac{3y+4}{y}$ (*)	M1 (1)
(*c*)	$x^2=\left(3+\frac{4}{y}\right)^2 = 9+\frac{24}{y}+\frac{16}{y^2}$	M1 A1
	$\int x^2\ \mathrm{d}y = \int(9+24y^{-1}+16y^{-2})\ \mathrm{d}y$	
	$= 9y-\frac{16}{y},\ +24\ln y$	M1 A1ft, A1ft
	$\left[9y\ +24\ln y\ -\ 16y^{-1}\right]_2^{16}$ = (144 + 24 ln 16 – 1) – (18 + 24 ln 2 – 8)	M1
	$V=\pi(133+24\ln 8)$	A1 (7)
(*d*)	$V\times 27\approx 15\,500$ (*)	M1 A1 (2)
		(11 marks)

Question Number	Scheme	Marks
5. (*a*)	$\overrightarrow{AB} = (2\mathbf{i} - \mathbf{j} + 2\mathbf{k}),\ \overrightarrow{CB} = (-2\mathbf{i} + 2\mathbf{j} + \mathbf{k}),$	M1 A1
	(or $\overrightarrow{BA},\overrightarrow{BC},$ *or* $\overrightarrow{AB},\overrightarrow{BC},$ stated in above form or column vector form)	
	$\cos ABC = \dfrac{CB \bullet AB}{\lvert CB\rvert\lvert AB\rvert} = -\dfrac{4}{9}$	M1 A1 (4)
(*b*)	Area of $\Delta ABC = \frac{1}{2}\times 3\times 3\times \sin B$	M1
	$\sin B = \sqrt{(1-\frac{16}{81})} = \dfrac{\sqrt{65}}{9}$	M1
	$\therefore \text{Area} = \frac{1}{2}\sqrt{65}$	A1 (3)
(*c*)	$\overrightarrow{AC} = \begin{pmatrix}4\\-3\\1\end{pmatrix}\ \overrightarrow{DC} = \begin{pmatrix}0\\-1\\-3\end{pmatrix}$	M1
	or given in alternative form with attempt at scalar product	
	$\overrightarrow{AC} \bullet \overrightarrow{DC} = 0$, therefore the lines are perpendicular.	A1 (2)
(*d*)	$\overrightarrow{AD} = \begin{pmatrix}4\\-2\\4\end{pmatrix}\ \overrightarrow{DB} = \begin{pmatrix}-2\\1\\-2\end{pmatrix}$ and $AD{:}DB$ = 2:-1 (allow 2:1)	M1 A1 (2)
		(11 marks)

Question Number	Scheme	Marks
6. (*a*)	Distance from one side (m): 0, 2, 4, 6, 8, 10; Height (m): 0, 6.13, **7.80**, **7.80**, **6.13**, 0	
	"y" = 7.80 when "x" = 4 or 6	B1
	Symmetry	B1 ft (2)
(*b*)	Estimate area = $\frac{2}{2}$[0 + 2(6.13 + 7.80 + 7.80 + 6.13)]	B1 M1 A1ft
	= 55.7 m^2	A1 (4)
(*c*)	140 – (*b*) = 84.3 m^2	A1 ft (1)
(*d*)	Over-estimate;	B1
	reason, e.g. area under curve is under-estimate (due to curvature)	B1 (2)
		(9 marks)
7. (*a*)	Method using either	
	$\frac{A}{(1-x)}+\frac{B}{(2x+3)}+\frac{C}{(2x+3)^2}$ **or** $\frac{A}{1-x}+\frac{Dx+E}{(2x+3)^2}$	M1
	$A = 1$, $C = 10$, $B = 2$ **or** $D = 4$ and $E = 16$	B1 A1 A1 (4)
(*b*)	$\int [\frac{1}{1-x}+\frac{2}{2x+3}+10(2x+3)^{-2}]dx$ **or** $\int \frac{A}{1-x}+\frac{Dx+E}{(2x+3)^2}dx$	M1
	$-\ln\lvert 1-x\rvert+\ln\lvert 2x+3\rvert-5(2x+3)^{-1}(+c)$ **or**	M1 A1ft A1ft A1ft (5)
	$-\ln\lvert 1-x\rvert+\ln\lvert 2x+3\rvert-(2x+8)(2x+3)^{-1}(+c)$	
(*c*)	$(1-x)^{-1}+2(3+2x)^{-1}+10(3+2x)^{-2}=$	M1
	$1+x+x^2+...$	A1
	$+\frac{2}{3}(1-\frac{2x}{3}+\frac{4x^2}{9}...)$	M1 A1
	$+\frac{10}{9}(1+(-2)(\frac{2x}{3})+\frac{(-2)(-3)}{2}(\frac{2x}{3})^2+...)$	A1
	$=\frac{25}{9}-\frac{25}{27}x+\frac{25}{9}x^2...$	M1 A1 (7)
		(16 marks)