Made into our C4 Practice paper 7

Paper Reference (complete below)									Centre No.					
	6	6	6	6	/	0	1		Candidate No.					

Surname	Initial(s)
Signature	

Paper Reference(s)

6666

Edexcel GCE

Core Mathematics C4

Advanced Subsidiary

Set A: Practice Paper 2

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
Total	

Materials required for examination
Mathematical Formulae

Items included with question papers
Nil

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. You must write your answer for each question in the space following the question. If you need more space to complete your answer to any question, use additional answer sheets.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner.
Answers without working may gain no credit.

Turn over

1. Use integration by parts to find the exact value of $\int_1^3 x^2 \ln x \, dx$.

(6)

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2. Fluid flows out of a cylindrical tank with constant cross section. At time t minutes, $t \geq 0$, the volume of fluid remaining in the tank is V m^3. The rate at which the fluid flows, in m^3 min^{-1}, is proportional to the square root of V.

(*a*) Show that the depth h metres of fluid in the tank satisfies the differential equation

$$\frac{dh}{dt} = -k\sqrt{h}, \qquad \text{where } k \text{ is a positive constant.}$$

(3)

(*b*) Show that the general solution of the differential equation may be written as

$$h = (A - Bt)^2, \qquad \text{where } A \text{ and } B \text{ are constants.}$$

(4)

Given that at time $t = 0$ the depth of fluid in the tank is 1 m, and that 5 minutes later the depth of fluid has reduced to 0.5 m,

(*c*) find the time, T minutes, which it takes for the tank to empty.

(3)

(*d*) Find the depth of water in the tank at time $0.5T$ minutes.

(2)

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3. (*a*) Use the identity for $\cos(A + B)$ to prove that $\cos 2A = 2\cos^2 A - 1$.

(2)

(*b*) Use the substitution $x = 2\sqrt{2} \sin \theta$ to prove that

$$\int_2^{\sqrt{6}} \sqrt{(8 - x^2)} \, dx = \tfrac{1}{3}(\pi + 3\sqrt{3} - 6).$$

(7)

A curve is given by the parametric equations

$$x = \sec \theta, \quad y = \ln(1 + \cos 2\theta), \quad 0 \leq \theta < \frac{\pi}{2}.$$

(*c*) Find an equation of the tangent to the curve at the point where $\theta = \frac{\pi}{3}$.

(5)

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4. **Figure 2**

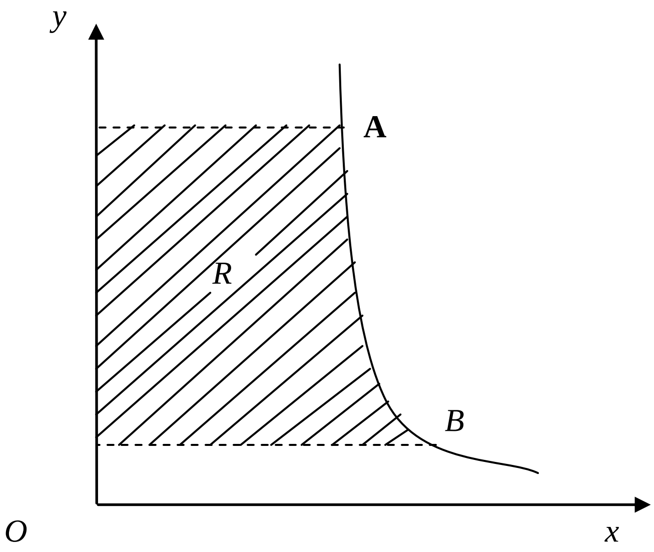

Figure 2 shows a sketch of the curve C with equation $y = \dfrac{4}{x-3}$, $x \neq 3$.

The points A and B on the curve have x-coordinates 3.25 and 5 respectively.

(a) Write down the y-coordinates of A and B.

(1)

(b) Show that an equation of C is $\dfrac{3y+4}{y}$, $y \neq 0$.

(1)

The shaded region R is bounded by C, the y-axis and the lines through A and B parallel to the x-axis. The region R is rotated through 360° about the y-axis to form a solid shape S.
Given that the volume is $\int \pi y^2 \, dx$

(c) Find the volume of S, giving your answer in the form $\pi(a + b \ln c)$, where a, b and c are integers.

(7)

The solid shape S is used to model a cooling tower. Given that 1 unit on each axis represents 3 metres,

(d) show that the volume of the tower is approximately 15 500 m^3.

(2)

5. Relative to a fixed origin O, the point A has position vector $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, the point B has position vector $5\mathbf{i} + \mathbf{j} + \mathbf{k}$, and the point C has position vector $7\mathbf{i} - \mathbf{j}$.

(*a*) Find the cosine of angle ABC.

(4)

(*b*) Find the exact value of the area of triangle ABC.

(3)

The point D has position vector $7\mathbf{i} + 3\mathbf{k}$.

(*c*) Show that AC is perpendicular to CD.

(2)

(*d*) Find the ratio AD:DB.

(2)

6.

Figure 2

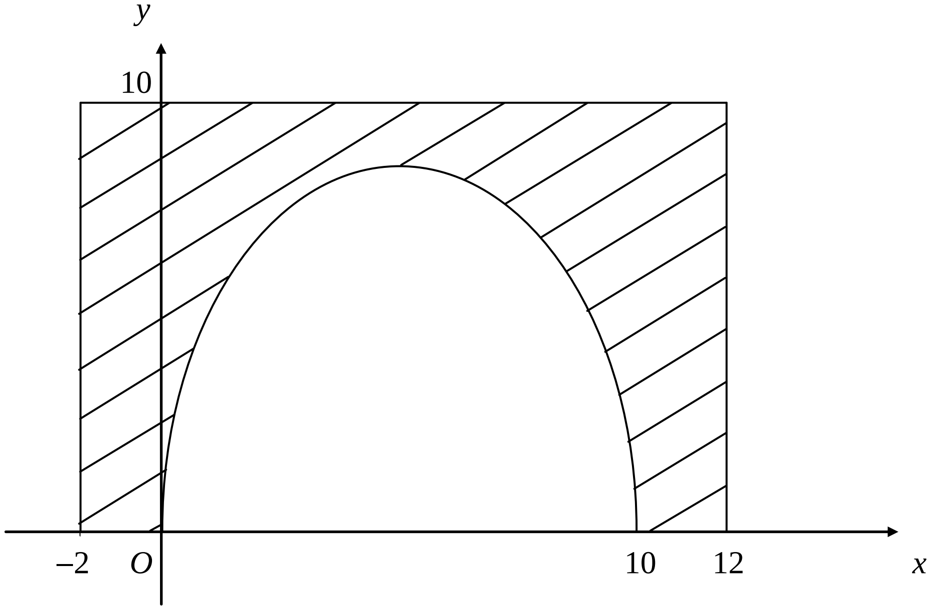

Figure 2 shows the cross-section of a road tunnel and its concrete surround. The curved section of the tunnel is modelled by the curve with equation $y = 8\sqrt{\left(\sin\frac{\pi x}{10}\right)}$, in the interval $0 \leq x \leq 10$. The concrete surround is represented by the shaded area bounded by the curve, the x-axis and the lines $x = -2$, $x = 12$ and $y = 10$. The units on both axes are metres.

(*a*) Using this model, copy and complete the table below, giving the values of y to 2 decimal places.

x	0	2	4	6	8	10
y	0	6.13				0

(2)

The area of the cross-section of the tunnel is given by $\int_0^{10} y \, dx$.

(*b*) Estimate this area, using the trapezium rule with all the values from your table.

(4)

(*c*) Deduce an estimate of the cross-sectional area of the concrete surround.

(1)

(*d*) State, with a reason, whether your answer in part (*c*) over-estimates or under-estimates the true value.

(2)

7. $$\mathrm{f}(x) = \frac{25}{(3+2x)^2(1-x)}, \quad |x| < 1.$$

(*a*) Express f(x) as a sum of partial fractions.

(4)

(*b*) Hence find $\int \mathrm{f}(x)\ \mathrm{d}x$.

(5)

(*c*) Find the series expansion of f(x) in ascending powers of x up to and including the term in x^2. Give each coefficient as a simplified fraction.

(7)

END