C4 PRACTICE PAPER 5 - SOLUTIONS

1. $x^2 + 4y^2 - 4x - 12y - 12 = 0$

a/ $2x + 8y\dfrac{dy}{dx} - 4 - 12\dfrac{dy}{dx} = 0$ M1 A2

$(6,3)$ $12 + 24\dfrac{dy}{dx} - 4 - 12\dfrac{dy}{dx} = 0$ M1

$12\dfrac{dy}{dx} = 8$

$\dfrac{dy}{dx} = \dfrac{2}{3}$ A1

b/ $y - 3 = \dfrac{2}{3}(x-6)$ M1 A1 (7)

2. $x = 2t + 3$ $y = t^3 - 4t$

a/ $\dfrac{dx}{dt} = 2$ $\dfrac{dy}{dt} = 3t^2 - 4$ M1

$\therefore \dfrac{dy}{dx} = \dfrac{3t^2-4}{2}.$ M1 A1

at A $t = -1$ $\dfrac{dy}{dx} = -\dfrac{1}{2}$ $x = 1$ $y = 3$ M1 A1

tangent $y - 3 = -\dfrac{1}{2}(x-1)$ M1

$2y - 6 = -x + 1$

$x + 2y = 7$ A1

b/ intersects curve when $(2t+3) + 2(t^3 - 4t) = 7$ M1

$2t^3 - 6t - 4 = 0$ A1

$(t+1)(t+1)(2t-4) = 0$

$\underline{\underline{t = 2}}$ at B A1 (10)

3. a/ $\displaystyle\int x\sin 2x \, dx$ $u = x$ $\dfrac{dv}{dx} = \sin 2x$

$\dfrac{du}{dx} = 1$ $v = -\dfrac{\cos 2x}{2}.$ M1 A1

$= -\dfrac{x}{2}\cos 2x + \displaystyle\int \frac{\cos 2x}{2} dx$

$= -\dfrac{x}{2}\cos 2x + \dfrac{1}{4}\sin 2x + c$ M1 A1

condone no C

b/ $\dfrac{dy}{dx} = x\sin 2x \cos^2 y$

$\displaystyle\int \sec^2 y \, dy = \int x \sin 2x \, dx$ M1 A1

$\tan y = -\dfrac{x}{2}\cos 2x + \dfrac{1}{4}\sin 2x + c$ M1 A1

$y = 0$ $x = \dfrac{\pi}{4}$ $0 = -\dfrac{\pi}{8}\cos\dfrac{\pi}{2} + \dfrac{1}{4}\sin\dfrac{\pi}{2} + c$ $\Rightarrow$ $c = -\dfrac{1}{4}$ M1

$\therefore \tan y = -\dfrac{x}{2}\cos 2x + \dfrac{1}{4}\sin 2x - \dfrac{1}{4}$ A1 (10)

4. a) $\dfrac{5+x}{(1+2x)(1-x)^2} \equiv \dfrac{A}{1+2x} + \dfrac{B}{1-x} + \dfrac{C}{(1-x)^2}$ B1

$5+x \equiv A(1-x)^2 + B(1+2x)(1-x) + C(1+2x)$ M1

$x=1$ $\quad 6 = 3C \Rightarrow C = 2$ M1 A1

$x=-\frac{1}{2}$ $\quad \frac{9}{2} = \frac{9A}{4} \Rightarrow A = 2.$ A1

coeff x^2 $\quad 0 = A - 2B \Rightarrow B = 1$ A1

$\therefore f(x) = \dfrac{2}{1+2x} + \dfrac{1}{1-x} + \dfrac{2}{(1-x)^2}$

b) $2(1+2x)^{-1} = 2\left(1 + (-1)(2x) + \dfrac{(-1)(-2)(2x)^2}{2!} + \dfrac{(-1)(-2)(-3)(2x)^3}{3!}\right)$

$= 2(1 - 2x + 4x^2 - 8x^3 + \dots)$ M1 A1

$(1-x)^{-1} = 1 + (-1)(-x) + \dfrac{(-1)(-2)(-x)^2}{2!} + \dfrac{(-1)(-2)(-3)}{3!}(-x)^3 + \dots$

$= 1 + x + x^2 + x^3 + \dots$ A1

$2(1-x)^{-2} = 2\left(1 + (-2)(-x) + \dfrac{(-2)(-3)}{2!}(-x)^2 + \dfrac{(-2)(-3)(-4)}{3!}(-x)^3 + \dots\right)$

$= 2(1 + 2x + 3x^2 + 4x^3 + \dots)$ A1

$\therefore f(x) = 2 - 4x + 8x^2 - 16x^3 + 1 + x + x^2 + x^3 + 2 + 4x + 6x^2 + 8x^3 + \dots$ M1 A1 (12)

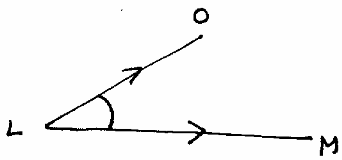
$= 5 + x + 15x^2 - 7x^3 + \dots$

5. $\overrightarrow{OL} = \underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$ $\quad \overrightarrow{OM} = 2\underset{\sim}{i} - 4\underset{\sim}{j} + 2\underset{\sim}{k}.$

a) $\overrightarrow{LM} = \overrightarrow{OM} - \overrightarrow{OL}$ M1 A1

$= \underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}$

b) $|\overrightarrow{OL}| = \sqrt{1+1+9} = \sqrt{11}$ $\quad |\overrightarrow{LM}| = \sqrt{1+9+1} = \sqrt{11}$ $\quad \therefore |\overrightarrow{OL}| = |\overrightarrow{LM}|$ M1 A1

c) 

$\overrightarrow{LO} = -\underset{\sim}{i} + \underset{\sim}{j} - 3\underset{\sim}{k}.$

$\overrightarrow{LO} \cdot \overrightarrow{LM} = -1 - 3 + 3 = -1$ M1 A1

$\cos O\hat{L}M = \dfrac{-1}{\sqrt{11}\sqrt{11}}$ M1

$= \dfrac{-1}{11}$

$\therefore O\hat{L}M = 95.2°$ A1

(8)

6. a) $(1-x^2)^{-1/2} = 1 + \left(-\frac{1}{2}\right)(-x^2) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-x^2)^2}{2} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(-x^2)^3}{3.2}$ M1 A1

$= 1 + \frac{x^2}{2} + \frac{3x^4}{8} + \frac{5x^6}{16} + \ldots$ A2

b) $x = \frac{1}{3}$ $\quad (1-x^2)^{-1/2} = \left(1 - \frac{1}{9}\right)^{-1/2}$

$= \left(\frac{8}{9}\right)^{-1/2}$

$= \left(\frac{9}{8}\right)^{1/2}$ M1

$= \frac{3}{\sqrt{8}}$

$= \frac{3}{2\sqrt{2}}$ A1

$= \frac{3\sqrt{2}}{4}$

c) $x = \frac{1}{3}$ $\quad (1-x^2)^{-1/2} \simeq 1 + \frac{1}{2}\left(\frac{1}{3}\right)^2 + \frac{3}{8}\left(\frac{1}{3}\right)^4 + \frac{5}{16}\left(\frac{1}{3}\right)^6$ M1

$= 1 + 0.0555556 + 0.0046296 + 0.0004287$

$= 1.0606139$ A2

$\therefore \frac{3\sqrt{2}}{4} \simeq 1.0606139$ M1

$\sqrt{2} \simeq \frac{4}{3}(1.0606139)$

$= 1.41415$ A1 (11)

7. a) (i) $\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$

$\cos(A-B) \equiv \cos A \cos B + \sin A \sin B$

ADD $\quad \cos(A+B) + \cos(A-B) \equiv 2\cos A \cos B$ M1 A1

(ii) Let $B = A$ M1

$\cos 2A + \cos 0 \equiv 2\cos^2 A$

$\cos 2A + 1 \equiv 2\cos^2 A$ A1

$\therefore \cos^2 A \equiv \frac{1}{2}(1 + \cos 2A)$

b) $\int \cos 3x \cos x \, dx = \frac{1}{2}\int (\cos 4x + \cos 2x)\, dx$ using a(i) M1 A1

$= \frac{1}{2}\left(\frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right) + C$ M1

$= \frac{1}{8}\sin 4x + \frac{1}{4}\sin 2x + C$ A1 c $\displaystyle\int_0^{1/2} \frac{x^2}{(1-x^2)^{1/2}}\,dx$

$x = \cos t$

$\frac{dx}{dt} = -\sin t$ M1

$x = \frac{1}{2}$ $\cos t = \frac{1}{2} \Rightarrow t = \pi/3$

$x = 0$ $\cos t = 0 \Rightarrow t = \pi/2$

M1

$= \displaystyle\int_{\pi/2}^{\pi/3} \frac{\cos^2 t}{(1-\cos^2 t)^{1/2}}(-\sin t\, dt)$ M1

$= -\displaystyle\int_{\pi/2}^{\pi/3} \frac{\cos^2 t}{\cancel{\sin t}}\cdot \cancel{\sin t}\, dt$ A1

$= \frac{1}{2}\displaystyle\int_{\pi/3}^{\pi/2} (1+\cos 2t)\, dt$ using a(ii) A1

$= \frac{1}{2}\left[t + \frac{\sin 2t}{2}\right]_{\pi/3}^{\pi/2}$ M1 A1

$= \frac{1}{2}\left[\left(\pi/2 + \frac{\sin \pi}{2}\right) - \left(\pi/3 + \frac{\sin 2\pi/3}{2}\right)\right]$ M1

$= \frac{1}{2}\left[\frac{\pi}{2} + 0 - \frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]$

$= \frac{1}{2}\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$

$= \frac{\pi}{12} - \frac{\sqrt{3}}{8}$ A1

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Total : 75