C4 Practice Paper 5

- The paper is $1^1/_2$ hours long.
- You are advised to show all your working.
- Calculators may be used.

1. The curve C has equation

$$x^2 + 4y^2 - 4x - 12y - 12 = 0$$

a) By differentiation find the gradient of C at the point (6, 3)

b) Find an equation of the tangent to the curve C at the point (6, 3)

2. A curve, C, is given by

$$x = 2t + 3, \quad y = t^3 - 4t,$$

where t is a parameter. The point A has parameter $t = -1$ and the line l is the tangent to C at A. The line l also intersects the curve at B.

a) Show that an equation for l is $2y + x = 7$

b) Find the value of t at B.

3. a) Find $\int x \sin 2x \, \mathrm{d}x$.

b) Given that $y = 0$ at $x = \frac{\pi}{4}$, solve the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}x} = x \sin 2x \cos^2 y$$

4. $$\mathrm{f}(x) \equiv \frac{5 + x}{(1 + 2x)(1 - x)^2}$$

a) Express $\mathrm{f}(x)$ in partial fractions.

b) Given that $|x| < \frac{1}{2}$, expand $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^3.

5. With respect to an origin O, the position vectors of the points L and M are $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $2\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$ respectively.

a) Write down the vector $\overrightarrow{LM}$

b) Show that $|\overrightarrow{OL}| = \overrightarrow{LM}$.

c) Find $\angle OLM$, giving your answer to the nearest tenth of a degree.

6. a) Obtain the first four non-zero terms of the binomial expansion in ascending powers of x of

$(1 - x^2)^{-\frac{1}{2}}$, given that $|x| < 1$.

b) Show that, when $x = \frac{1}{3}$, $(1 - x^2)^{-\frac{1}{2}} = \frac{3}{4}\sqrt{2}$.

c) Substitute $x = \frac{1}{3}$ into your expansion and hence obtain an approximation to $\sqrt{2}$, giving your answer to 5 decimal places.

7. a) Use the identities for $\cos(A + B)$ and $\cos(A - B)$ to prove that

i) $2\cos A \cos B \equiv \cos(A + B) + \cos(A - B)$.

ii) $\cos^2 A \equiv \frac{1}{2}(1 + \cos 2A)$.

b) Find $\int \cos 3x \cos x \, dx$

c) Use the substitution $x = \cos t$ to evaluate

$$\int_0^{\frac{1}{2}} \frac{x^2}{(1 - x^2)^{\frac{1}{2}}} \, dx$$