IYGB GCE

Core Mathematics C4

Advanced

Practice Paper H

Difficulty Rating: 3.3333/1.500

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 10 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

$$\frac{2x^2-3}{(x-1)^2} \equiv A+\frac{B}{x-1}+\frac{C}{(x-1)^2}.$$

a) Determine the value of each of the constants A, B and C. **(4)**

b) Evaluate

$$\int_2^3 \frac{2x^2-3}{(x-1)^2}\, dx,$$

giving the answer in the form $p+\ln q$, where p and q are constants. **(5)**

Question 2

a) Use the trapezium rule with 4 equally spaced strips to find an estimate for

$$\int_0^1 \mathrm{e}^{-x^2}\, dx. \quad \textbf{(4)}$$

b) Use the answer of part (a) to find an estimate for

$$\int_0^1 \mathrm{e}^{-x^2+3}\, dx. \quad \textbf{(2)}$$

Question 3

The volume, V cm^3, of a metallic cube of side length x cm, is increasing at the constant rate of 0.108 cm^{3}s^{-1}.

a) Determine the rate at which the side of the cube is increasing when the side length reaches 3 cm. **(4)**

b) Find the rate at which the surface area of the cube, A cm^2, is increasing when the side length reaches 3 cm. **(3)**

Question 4

The point $P(2,3)$ lies on the curve with equation

$$4x^3 - 6xy + 3^y = 23 .$$

Show that the gradient of the curve at P is

$$\frac{k}{4 - 9\ln 3},$$

where k is a positive integer to be found. **(6)**

Question 5

The point $P(-5,3)$ lies on the curve C with parametric equations

$$x = \frac{a}{t} - 1, \ y = \frac{t + a}{t + 1}, \ t \in \mathbb{R}, \ t \neq 0, -1$$

where a is a non zero constant.

Show that a Cartesian equation of C is

$$y = \frac{2x + 4}{x + 3} . \quad \textbf{(7)}$$

Question 6

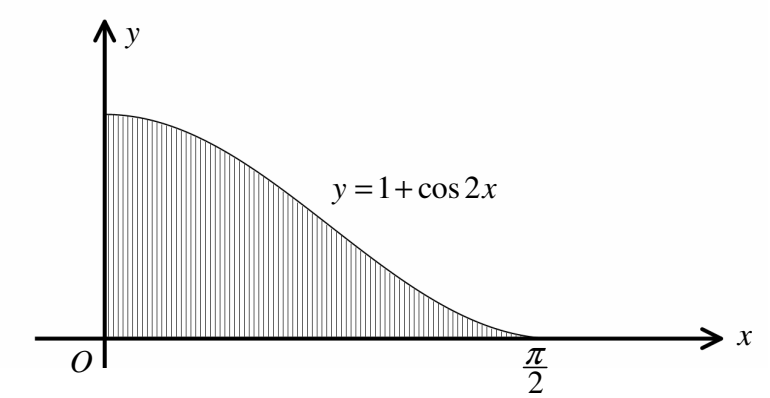
Find a general solution of the differential equation

$$x^2 \frac{dy}{dx} = xy + y, \ x \neq 0, \ y \neq 0,$$

giving the answer in the form $y = f(x)$. **(8)**

[*the final answer may not contain natural logarithms*]

Question 7



The figure above shows the graph of the curve with equation

$$y = 1 + \cos 2x\,,\ 0 \le x \le \frac{\pi}{2}\,.$$

a) Show clearly that

$$(1+\cos 2x)^2 \equiv \frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x\,. \qquad (\mathbf{3})$$

The shaded region bounded by the curve and the coordinate axes is rotated by 2π radians about the x axis to form a solid of revolution.

b) Show that the volume of the solid is

$$\frac{3}{4}\pi^2\,. \qquad (\mathbf{5})$$

Question 8

With respect to a fixed origin O, the straight line l has vector equation

$$\mathbf{r} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix},$$

where a and b are scalar constants and λ is a scalar parameter.

a) If l passes though the point $P(7,3,6)$, find the value of a and the value of b. **(3)**

The point Q lies on l so that OQ is perpendicular to l.

b) Find the coordinates of Q. **(5)**

The point T lies on l where $\lambda = -7$.

c) Find the ratio $PQ : QT$. **(1)**

Question 9

$$f(x) = (1+kx)^{-3}, \ |kx| < 1,$$

where k is a non zero constant.

a) Expand $f(x)$, in terms of k, as an infinite convergent series up and including the term in x^3.

$$g(x) = \frac{6-x}{(1+kx)^3}, \ |kx| < 1.$$

(4)

The coefficient of x^2 in the expansion of $g(x)$ is 3.

b) Find the possible values of k. **(6)**

Question 10

Use an appropriate integration method to find an exact value for

$$\int_0^{\frac{\pi}{3}} 6x\sin 3x \ \ dx\,. \qquad (\mathbf{5})$$
