

Mark Scheme (Results)

Summer 2016

Pearson Edexcel GCE in Core  
Mathematics 4 (6666/01)

## **Edexcel and BTEC Qualifications**

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at [www.edexcel.com](http://www.edexcel.com) or [www.btec.co.uk](http://www.btec.co.uk). Alternatively, you can get in touch with us using the details on our contact us page at [www.edexcel.com/contactus](http://www.edexcel.com/contactus).

## **Pearson: helping people progress, everywhere**

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: [www.pearson.com/uk](http://www.pearson.com/uk)

Summer 2016

Publications Code 6666\_01\_1606\_MS

All the material in this publication is copyright

© Pearson Education Ltd 2016

## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## PEARSON EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.

### 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol  $\surd$  will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d... or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- \* The answer is printed on the paper or ag- answer given
- $\square$  or d... The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
  - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Core Mathematics Marking

*(But note that specific mark schemes may sometimes override these general principles).*

### **Method mark for solving 3 term quadratic:**

#### **1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$ , where  $|pq| = |c|$ , leading to  $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$ , where  $|pq| = |c|$  and  $|mn| = |a|$ , leading to  $x = \dots$

#### **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

#### **3. Completing the square**

Solving  $x^2 + bx + c = 0$ :  $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$ ,  $q \neq 0$ , leading to  $x = \dots$

### **Method marks for differentiation and integration:**

#### **1. Differentiation**

Power of at least one term decreased by 1. ( $x^n \rightarrow x^{n-1}$ )

#### **2. Integration**

Power of at least one term increased by 1. ( $x^n \rightarrow x^{n+1}$ )

## **Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

## **Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Notes	Marks
<b>1.</b> <b>Way 1</b>	$\left\{ \frac{1}{(2+5x)^3} \right\} (2+5x)^{-3}$	Writes down $(2+5x)^{-3}$ or uses power of $-3$	M1
	$= (2)^{-3} \left( 1 + \frac{5x}{2} \right)^{-3} = \frac{1}{8} \left( 1 + \frac{5x}{2} \right)^{-3}$	$2^{-3}$ or $\frac{1}{8}$	<u>B1</u>
	$= \left\{ \frac{1}{8} \right\} \left[ 1 + (-3)(kx) + \frac{(-3)(-4)}{2!} (kx)^2 + \frac{(-3)(-4)(-5)}{3!} (kx)^3 + \dots \right]$	see notes	M1 A1
	$= \left\{ \frac{1}{8} \right\} \left[ 1 + (-3) \left( \frac{5x}{2} \right) + \frac{(-3)(-4)}{2!} \left( \frac{5x}{2} \right)^2 + \frac{(-3)(-4)(-5)}{3!} \left( \frac{5x}{2} \right)^3 + \dots \right]$		
	$= \frac{1}{8} \left[ 1 - \frac{15}{2}x + \frac{75}{2}x^2 - \frac{625}{4}x^3 + \dots \right]$		
	$= \frac{1}{8} \left[ 1 - 7.5x + 37.5x^2 - 156.25x^3 + \dots \right]$		
	$= \frac{1}{8} - \frac{15}{16}x; + \frac{75}{16}x^2 - \frac{625}{32}x^3 + \dots$ or $\frac{1}{8} - \frac{15}{16}x; + 4\frac{11}{16}x^2 - 19\frac{17}{32}x^3 + \dots$		A1; A1
			<b>6</b>
<b>Way 2</b>	$f(x) = (2+5x)^{-3}$	Writes down $(2+5x)^{-3}$ or uses power of $-3$	M1
	$f''(x) = 300(2+5x)^{-5}, f'''(x) = -7500(2+5x)^{-6}$	Correct $f''(x)$ and $f'''(x)$	B1
	$f'(x) = -15(2+5x)^{-4}$	$\pm a(2+5x)^{-4}, a \neq \pm 1$	M1
		$-15(2+5x)^{-4}$	A1 oe
	$\left\{ \therefore f(0) = \frac{1}{8}, f'(0) = -\frac{15}{16}, f''(0) = \frac{75}{8} \text{ and } f'''(0) = -\frac{1875}{16} \right\}$		
	So, $f(x) = \frac{1}{8} - \frac{15}{16}x; + \frac{75}{16}x^2 - \frac{625}{32}x^3 + \dots$	Same as in Way 1	A1; A1
			[6]
<b>Way 3</b>	$(2+5x)^{-3}$	Same as in Way 1	M1
	$= (2)^{-3} + (-3)(2)^{-4}(5x) + \frac{(-3)(-4)}{2!} (2)^{-5}(5x)^2 + \frac{(-3)(-4)(-5)}{3!} (2)^{-6}(5x)^3$	Same as in Way 1	<u>B1</u>
		Any two terms correct	M1
		All four terms correct	A1
	$= \frac{1}{8} - \frac{15}{16}x; + \frac{75}{16}x^2 - \frac{625}{32}x^3 + \dots$	Same as in Way 1	A1; A1
	<b>Note:</b> Terms can be simplified or un-simplified for B1 2 <sup>nd</sup> M1 1 <sup>st</sup> A1		
<b>Note:</b> The terms in C need to be evaluated So ${}^{-3}C_0(2)^{-3} + {}^{-3}C_1(2)^{-4}(5x) + {}^{-3}C_2(2)^{-5}(5x)^2 + {}^{-3}C_3(2)^{-6}(5x)^3$ without further working is B0 1 <sup>st</sup> M0 1 <sup>st</sup> A0			



Question 1 Notes		
<b>1.</b>	<b>1<sup>st</sup> M1</b>	mark can be implied by a constant term of $(2)^{-3}$ or $\frac{1}{8}$ .
	<b>B1</b>	$\frac{2^{-3}}{8}$ or $\frac{1}{8}$ outside brackets or $\frac{1}{8}$ as candidate's constant term in their binomial expansion.
	<b>2<sup>nd</sup> M1</b>	Expands $(\dots + kx)^{-3}$ , $k = \text{a value} \neq 1$ , to give any 2 terms out of 4 terms simplified or un-simplified, Eg: $1 + (-3)(kx)$ or $\frac{(-3)(-4)}{2!}(kx)^2 + \frac{(-3)(-4)(-5)}{3!}(kx)^3$ or $1 + \dots + \frac{(-3)(-4)}{2!}(kx)^2$ or $\frac{(-3)(-4)}{2!}(kx)^2 + \frac{(-3)(-4)(-5)}{3!}(kx)^3$ are fine for M1.
	<b>1<sup>st</sup> A1</b>	A correct simplified or un-simplified $1 + (-3)(kx) + \frac{(-3)(-4)}{2!}(kx)^2 + \frac{(-3)(-4)(-5)}{3!}(kx)^3$ expansion with consistent $(kx)$ . Note that $(kx)$ must be consistent and $k = \text{a value} \neq 1$ . (on the RHS, not necessarily the LHS) in a candidate's expansion.
	<b>Note</b>	You would award B1M1A0 for $\frac{1}{8} \left[ 1 + (-3)\left(\frac{5x}{2}\right) + \frac{(-3)(-4)}{2!}(5x)^2 + \frac{(-3)(-4)(-5)}{3!}\left(\frac{5x}{2}\right)^3 + \dots \right]$ because $(kx)$ is not consistent.
	<b>Note</b>	<b>Incorrect bracketing:</b> $= \left\{ \frac{1}{8} \right\} \left[ 1 + (-3)\left(\frac{5x}{2}\right) + \frac{(-3)(-4)}{2!}\left(\frac{5x^2}{2}\right) + \frac{(-3)(-4)(-5)}{3!}\left(\frac{5x^3}{2}\right) + \dots \right]$ is M1A0 unless recovered.
	<b>2<sup>nd</sup> A1</b>	For $\frac{1}{8} - \frac{15}{16}x$ ( <b>simplified</b> ) or also allow $0.125 - 0.9375x$ .
	<b>3<sup>rd</sup> A1</b>	Accept only $\frac{75}{16}x^2 - \frac{625}{32}x^3$ or $4\frac{11}{16}x^2 - 19\frac{17}{32}x^3$ or $4.6875x^2 - 19.53125x^3$
	<b>SC</b>	If a candidate <i>would otherwise score</i> 2 <sup>nd</sup> A0, 3 <sup>rd</sup> A0 then <b>allow Special Case 2<sup>nd</sup> A1 for either</b>
		<b>SC:</b> $\frac{1}{8} \left[ 1 - \frac{15}{2}x; \dots \right]$ or <b>SC:</b> $\frac{1}{8} \left[ 1 + \dots + \frac{75}{2}x^2 + \dots \right]$ or <b>SC:</b> $\frac{1}{8} \left[ 1 + \dots - \frac{625}{4}x^3 + \dots \right]$
		<b>SC:</b> $\lambda \left[ 1 - \frac{15}{2}x + \frac{75}{2}x^2 - \frac{625}{4}x^3 + \dots \right]$ or <b>SC:</b> $\left[ \lambda - \frac{15\lambda}{2}x + \frac{75\lambda}{2}x^2 - \frac{625\lambda}{4}x^3 + \dots \right]$
		(where $\lambda$ can be 1 or omitted), where each term in the $[\dots]$ is a simplified fraction or a decimal
	<b>SC</b>	<b>Special case for the 2<sup>nd</sup> M1 mark</b> Award Special Case 2 <sup>nd</sup> M1 for a correct simplified or un-simplified $1 + n(kx) + \frac{n(n-1)}{2!}(kx)^2 + \frac{n(n-1)(n-2)}{3!}(kx)^3$ expansion with their $n \neq -3$ , $n \neq \text{positive integer}$ and a consistent $(kx)$ . Note that $(kx)$ must be consistent (on the RHS, not necessarily the LHS) in a candidate's expansion. <b>Note</b> that $k \neq 1$ .
	<b>Note</b>	Ignore extra terms beyond the term in $x^3$
<b>Note</b>	You can ignore subsequent working following a correct answer.	

Question Number	Scheme							Marks	
	$\frac{x}{y}$	1	1.2	1.4	1.6	1.8	2		
2.	$\frac{x}{y}$	1	1.2	1.4	1.6	1.8	2	$y = x^2 \ln x$	
		0	0.2625	<b>0.659485...</b>	1.2032	1.9044	2.7726		
(a)	{At $x=1.4$ ,} $y = 0.6595$ (4 dp)							0.6595	B1 cao
<b>[1]</b>									
(b)	$\frac{1}{2} \times (0.2) \times [0 + 2.7726 + 2(0.2625 + \text{their } 0.6595 + 1.2032 + 1.9044)]$						Outside brackets $\frac{1}{2} \times (0.2)$ or $\frac{1}{10}$	B1 o.e.	
	{Note: The "0" does not have to be included in [.....]}						For structure of [.....]	M1	
	$\left\{ = \frac{1}{10}(10.8318) \right\} = 1.08318 = 1.083$ (3 dp)				anything that rounds to 1.083			A1	
<b>[3]</b>									
(c) Way 1	$\left\{ I = \int x^2 \ln x dx \right\}, \left\{ \begin{array}{l} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^2 \Rightarrow v = \frac{1}{3}x^3 \end{array} \right\}$								
	$= \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \left( \frac{1}{x} \right) \{dx\}$			Either $x^2 \ln x \rightarrow \pm \lambda x^3 \ln x - \int \mu x^3 \left( \frac{1}{x} \right) \{dx\}$ or $\pm \lambda x^3 \ln x - \int \mu x^2 \{dx\}$ , where $\lambda, \mu > 0$			M1		
				$x^2 \ln x \rightarrow \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \left( \frac{1}{x} \right) \{dx\}$ , simplified or un-simplified			A1		
	$= \frac{x^3}{3} \ln x - \frac{x^3}{9}$			$\frac{x^3}{3} \ln x - \frac{x^3}{9}$ , simplified or un-simplified			A1		
	Area(R) = $\left\{ \left[ \frac{x^3}{3} \ln x - \frac{x^3}{9} \right]_1^2 \right\} = \left( \frac{8}{3} \ln 2 - \frac{8}{9} \right) - \left( 0 - \frac{1}{9} \right)$				<b>dependent on the previous M mark.</b> Applies limits of 2 and 1 and subtracts the correct way round			dM1	
	$= \frac{8}{3} \ln 2 - \frac{7}{9}$				$\frac{8}{3} \ln 2 - \frac{7}{9}$ or $\frac{1}{9}(24 \ln 2 - 7)$			A1 oe cso	
<b>[5]</b>									
(c) Way 2	$I = x^2(x \ln x - x) - \int 2x(x \ln x - x) dx$			$\left\{ \begin{array}{l} u = x^2 \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = \ln x \Rightarrow v = x \ln x - x \end{array} \right\}$					
	So, $3I = x^2(x \ln x - x) + \int 2x^2 \{dx\}$								
	and $I = \frac{1}{3}x^2(x \ln x - x) + \frac{1}{3} \int 2x^2 \{dx\}$			A full method of applying $u = x^2, v' = \ln x$ to give $\pm \lambda x^2(x \ln x - x) \pm \mu \int x^2 \{dx\}$			M1		
				$\frac{1}{3}x^2(x \ln x - x) + \frac{1}{3} \int 2x^2 \{dx\}$ simplified or un-simplified			A1		
	$= \frac{1}{3}x^2(x \ln x - x) + \frac{2}{9}x^3$			$\frac{x^3}{3} \ln x - \frac{x^3}{9}$ , simplified or un-simplified			A1		
				<b>Then award dM1A1 in the same way as above</b>			M1 A1		
<b>[5]</b>									

<b>Question 2 Notes</b>		
2. (a)	<b>B1</b>	0.6595 correct answer only. Look for this on the table or in the candidate's working.
(b)	<b>B1</b>	Outside brackets $\frac{1}{2} \times (0.2)$ or $\frac{1}{2} \times \frac{1}{5}$ or $\frac{1}{10}$ or equivalent.
	<b>M1</b>	For structure of trapezium rule [ ..... ]
	<b>Note</b>	No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate].
	<b>A1</b>	anything that rounds to 1.083
	<b>Note</b>	Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 1.070614704...)
	<b>Note</b>	Full marks can be gained in part (b) for using an incorrect part (a) answer of 0.6594
	<b>Note</b>	Award B1M1A1 for $\frac{1}{10}(2.7726) + \frac{1}{5}(0.2625 + \text{their } 0.6595 + 1.2032 + 1.9044) = \text{awrt } 1.083$
	<b>Bracketing mistake: Unless the final answer implies that the calculation has been done correctly</b>	
	Award B1M0A0 for $\frac{1}{2}(0.2) + 2(0.2625 + \text{their } 0.6595 + 1.2032 + 1.9044) + 2.7726$ (answer of 10.9318)	
	Award B1M0A0 for $\frac{1}{2}(0.2)(2.7726) + 2(0.2625 + \text{their } 0.6595 + 1.2032 + 1.9044)$ (answer of 8.33646)	
<b>Alternative method: Adding individual trapezia</b>		
Area $\approx 0.2 \times \left[ \frac{0+0.2625}{2} + \frac{0.2625+"0.6595"}{2} + \frac{"0.6595"+1.2032}{2} + \frac{1.2032+1.9044}{2} + \frac{1.9044+2.7726}{2} \right] = 1.08318\dots$		
<b>B1</b>	0.2 and a divisor of 2 on all terms inside brackets	
<b>M1</b>	First and last ordinates once and two of the middle ordinates inside brackets ignoring the 2	
<b>A1</b>	anything that rounds to 1.083	
(c)	<b>A1</b>	Exact answer needs to be a two term expression in the form $a \ln b + c$
	<b>Note</b>	Give A1 e.g. $\frac{8}{3} \ln 2 - \frac{7}{9}$ or $\frac{1}{9}(24 \ln 2 - 7)$ or $\frac{4}{3} \ln 4 - \frac{7}{9}$ or $\frac{1}{3} \ln 256 - \frac{7}{9}$ or $-\frac{7}{9} + \frac{8}{3} \ln 2$ or $\ln 2^{\frac{8}{3}} - \frac{7}{9}$ or equivalent.
	<b>Note</b>	Give final A0 for a final answer of $\frac{8 \ln 2 - \ln 1}{3} - \frac{7}{9}$ or $\frac{8 \ln 2}{3} - \frac{1}{3} \ln 1 - \frac{7}{9}$ or $\frac{8 \ln 2}{3} - \frac{8}{9} + \frac{1}{9}$ or $\frac{8}{3} \ln 2 - \frac{7}{9} + c$
	<b>Note</b>	$\left[ \frac{x^3}{3} \ln x - \frac{x^3}{9} \right]_1^2$ followed by awrt 1.07 with no correct answer seen is dM1A0
	<b>Note</b>	Give dM0A0 for $\left[ \frac{x^3}{3} \ln x - \frac{x^3}{9} \right]_1^2 \rightarrow \left( \frac{8}{3} \ln 2 - \frac{8}{9} \right) - \frac{1}{9}$ (adding rather than subtracting)
	<b>Note</b>	Allow dM1A0 for $\left[ \frac{x^3}{3} \ln x - \frac{x^3}{9} \right]_1^2 \rightarrow \left( \frac{8}{3} \ln 2 - \frac{8}{9} \right) - \left( 0 + \frac{1}{9} \right)$
	<b>SC</b>	A candidate who uses $u = \ln x$ and $\frac{dv}{dx} = x^2$ , $\frac{du}{dx} = \frac{\alpha}{x}$ , $v = \beta x^3$ , writes down the correct "by parts" formula but makes only one error when applying it can be awarded Special Case 1 <sup>st</sup> M1.

Question Number	Scheme	Notes	Marks
3.	$2x^2y + 2x + 4y - \cos(\pi y) = 17$		
(a) Way 1	$\left\{ \frac{dy}{dx} \right\} \left( 4xy + 2x^2 \frac{dy}{dx} \right) + 2 + 4 \frac{dy}{dx} + \pi \sin(\pi y) \frac{dy}{dx} = 0$		M1 <u>A1</u> <u>B1</u>
	$\frac{dy}{dx} (2x^2 + 4 + \pi \sin(\pi y)) + 4xy + 2 = 0$		dM1
	$\left\{ \frac{dy}{dx} \right\} = \frac{-4xy - 2}{2x^2 + 4 + \pi \sin(\pi y)} \text{ or } \frac{4xy + 2}{-2x^2 - 4 - \pi \sin(\pi y)}$	Correct answer or equivalent	A1 cso
			[5]
(b)	At $\left( 3, \frac{1}{2} \right)$ , $m_T = \frac{dy}{dx} = \frac{-4(3)(\frac{1}{2}) - 2}{2(3)^2 + 4 + \pi \sin(\frac{1}{2}\pi)} \left\{ = \frac{-8}{22 + \pi} \right\}$	Substituting $x = 3$ & $y = \frac{1}{2}$ into an equation involving $\frac{dy}{dx}$	M1
	$m_N = \frac{22 + \pi}{8}$	Applying $m_N = \frac{-1}{m_T}$ to find a numerical $m_N$ <b>Can be implied by later working</b>	M1
	<ul style="list-style-type: none"> <li><math>y - \frac{1}{2} = \left( \frac{22 + \pi}{8} \right) (x - 3)</math></li> <li><math>\frac{1}{2} = \left( \frac{22 + \pi}{8} \right) (3) + c \Rightarrow c = \frac{1}{2} - \frac{66 + 3\pi}{8}</math> <math>\Rightarrow y = \left( \frac{22 + \pi}{8} \right) x + \frac{1}{2} - \frac{66 + 3\pi}{8}</math></li> </ul> Cuts x-axis $\Rightarrow y = 0$ $\Rightarrow -\frac{1}{2} = \left( \frac{22 + \pi}{8} \right) (x - 3)$	$y - \frac{1}{2} = m_N(x - 3)$ or $y = m_N x + c$ where $\frac{1}{2} = (\text{their } m_N)3 + c$ with a numerical $m_N (\neq m_T)$ where $m_N$ is in terms of $\pi$ and sets $y = 0$ in their normal equation.	dM1
	So, $\left\{ x = \frac{-4}{22 + \pi} + 3 \Rightarrow \right\} x = \frac{3\pi + 62}{\pi + 22}$	$\frac{3\pi + 62}{\pi + 22}$ or $\frac{6\pi + 124}{2\pi + 44}$ or $\frac{62 + 3\pi}{22 + \pi}$	A1 o.e.
			[4]
		9	
(a) Way 2	$\left\{ \frac{dx}{dy} \right\} \left( 4xy \frac{dx}{dy} + 2x^2 \right) + 2 \frac{dx}{dy} + 4 + \pi \sin(\pi y) = 0$		M1 <u>A1</u> <u>B1</u>
	$\frac{dx}{dy} (4xy + 2) + 2x^2 + 4 + \pi \sin(\pi y) = 0$		dM1
	$\frac{dx}{dy} = \frac{-4xy - 2}{2x^2 + 4 + \pi \sin(\pi y)} \text{ or } \frac{4xy + 2}{-2x^2 - 4 - \pi \sin(\pi y)}$	Correct answer or equivalent	A1 cso
			[5]
<b>Question 3 Notes</b>			
3. (a)	<b>Note</b> Writing down <i>from no working</i>		
	<ul style="list-style-type: none"> <li><math>\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi \sin(\pi y)} \text{ or } \frac{4xy + 2}{-2x^2 - 4 - \pi \sin(\pi y)}</math> scores M1A1B1M1A1</li> <li><math>\frac{dy}{dx} = \frac{4xy + 2}{2x^2 + 4 + \pi \sin(\pi y)}</math> scores M1A0B1M1A0</li> </ul>		
	<b>Note</b> Few candidates will write $4xydx + 2x^2dy + 2dx + 4dy + \pi \sin(\pi y)dy = 0$ leading to		
	$\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi \sin(\pi y)}$ or equivalent. This should get full marks.		

<b>Question 3 Notes Continued</b>		
<b>3. (a) Way 1</b>	<b>M1</b>	Differentiates implicitly to include either $2x^2 \frac{dy}{dx}$ or $4y \rightarrow 4 \frac{dy}{dx}$ or $-\cos(\pi y) \rightarrow \pm \lambda \sin(\pi y) \frac{dy}{dx}$ (Ignore $\left(\frac{dy}{dx} = \right)$ ). $\lambda$ is a constant which can be 1.
	<b>1<sup>st</sup> A1</b>	$2x + 4y - \cos(\pi y) = 17 \rightarrow 2 + 4 \frac{dy}{dx} + \pi \sin(\pi y) \frac{dy}{dx} = 0$
	<b>Note</b>	$4xy + 2x^2 \frac{dy}{dx} + 2 + 4 \frac{dy}{dx} + \pi \sin(\pi y) \frac{dy}{dx} \rightarrow 2x^2 \frac{dy}{dx} + 4 \frac{dy}{dx} + \pi \sin(\pi y) \frac{dy}{dx} = -4xy - 2$ will get 1 <sup>st</sup> A1 (implied) as the "= 0" can be implied by the rearrangement of their equation.
	<b>B1</b>	$2x^2 y \rightarrow 4xy + 2x^2 \frac{dy}{dx}$
	<b>Note</b>	If an extra term appears then award 1 <sup>st</sup> A0.
	<b>dM1</b>	<b>Dependent on the first method mark being awarded.</b> An attempt to factorise out <b>all the terms in</b> $\frac{dy}{dx}$ as long as there are <i>at least two terms</i> in $\frac{dy}{dx}$ . ie. $\frac{dy}{dx}(2x^2 + 4 + \pi \sin(\pi y)) + \dots = \dots$
	<b>Note</b>	Writing down an extra $\frac{dy}{dx} = \dots$ and then including it in their factorisation is fine for dM1.
	<b>Note</b>	<b>Final A1 cso:</b> If the candidate's solution is not completely correct, then do not give this mark.
	<b>Note</b>	<b>Final A1 isw:</b> You can, however, ignore subsequent working following on from correct solution.
(a)	<b>Way 2</b>	Apply the mark scheme for Way 2 in the same way as Way 1.
(b)	<b>1<sup>st</sup> M1</b>	M1 can be gained by seeing at least one example of substituting $x = 3$ and at least one example of substituting $y = \frac{1}{2}$ . E.g. "-4xy" $\rightarrow$ "-6" in their $\frac{dy}{dx}$ would be sufficient for M1, unless it is clear that they are instead applying $x = \frac{1}{2}$ , $y = 3$ .
	<b>3<sup>rd</sup> M1</b>	<i>is dependent on the first M1.</i>
	<b>Note</b>	The 2 <sup>nd</sup> M1 mark can be implied by later working. <b>Eg. Award 2<sup>nd</sup> M1 3<sup>rd</sup> M1</b> for $\frac{\frac{1}{2}}{3-x} = \frac{-1}{\text{their } m_r}$
	<b>Note</b>	We can accept $\sin \pi$ or $\sin\left(\frac{\pi}{2}\right)$ as a numerical value for the 2 <sup>nd</sup> M1 mark.  But, $\sin \pi$ by itself or $\sin\left(\frac{\pi}{2}\right)$ by itself are not allowed as being in terms of $\pi$ for the 3 <sup>rd</sup> M1 mark.  The 3 <sup>rd</sup> M1 can be accessed for terms containing $\pi \sin\left(\frac{\pi}{2}\right)$ .

Question Number	Scheme	Notes	Marks
4.	$\frac{dx}{dt} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geq 0$		
(a) Way 1	$\int \frac{1}{x} dx = \int -\frac{5}{2} dt$	Separates variables as shown. dx and dt should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\ln x = -\frac{5}{2}t + c$	Integrates both sides to give <b>either</b> $\pm \frac{\alpha}{x} \rightarrow \pm \alpha \ln x$ <b>or</b> $\pm k \rightarrow \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$\ln x = -\frac{5}{2}t + c$ , including "+c"	A1
	$\{t=0, x=60 \Rightarrow\} \ln 60 = c$ $\ln x = -\frac{5}{2}t + \ln 60 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with <b>no incorrect working seen</b>	A1 cso
			[4]
(a) Way 2	$\frac{dt}{dx} = -\frac{2}{5x}$ or $t = \int -\frac{2}{5x} dx$	<b>Either</b> $\frac{dt}{dx} = -\frac{2}{5x}$ or $t = \int -\frac{2}{5x} dx$	B1
	$t = -\frac{2}{5} \ln x + c$	Integrates both sides to give <b>either</b> $t = \dots$ or $\pm \alpha \ln px; \alpha \neq 0, p > 0$	M1
		$t = -\frac{2}{5} \ln x + c$ , including "+c"	A1
	$\{t=0, x=60 \Rightarrow\} c = \frac{2}{5} \ln 60 \Rightarrow t = -\frac{2}{5} \ln x + \frac{2}{5} \ln 60$ $\Rightarrow -\frac{5}{2}t = \ln x - \ln 60 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with <b>no incorrect working seen</b>	A1 cso
			[4]
(a) Way 3	$\int_{60}^x \frac{1}{x} dx = \int_0^t -\frac{5}{2} dt$	Ignore limits	B1
	$[\ln x]_{60}^x = \left[-\frac{5}{2}t\right]_0^t$	Integrates both sides to give <b>either</b> $\pm \frac{\alpha}{x} \rightarrow \pm \alpha \ln x$ <b>or</b> $\pm k \rightarrow \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$[\ln x]_{60}^x = \left[-\frac{5}{2}t\right]_0^t$ including the correct limits	A1
	$\ln x - \ln 60 = -\frac{5}{2}t \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 cso
			[4]
(b)	$20 = 60e^{-\frac{5}{2}t}$ or $\ln 20 = -\frac{5}{2}t + \ln 60$	Substitutes $x = 20$ into an equation in the form of <b>either</b> $x = \pm \lambda e^{\pm \mu t} \pm \beta$ or $x = \pm \lambda e^{\pm \mu t \pm \alpha \ln \delta x}$ <b>or</b> $\pm \alpha \ln \delta x = \pm \mu t \pm \beta$ or $t = \pm \lambda \ln \delta x \pm \beta$ ; $\alpha, \lambda, \mu, \delta \neq 0$ and $\beta$ can be 0	M1
	$t = -\frac{2}{5} \ln \left(\frac{20}{60}\right)$ $\{= 0.4394449\dots \text{ (days)}\}$ <b>Note: t must be greater than 0</b>	<b>dependent on the previous M mark</b> Uses correct algebra to achieve an equation of the form of either $t = A \ln \left(\frac{60}{20}\right)$ or $A \ln \left(\frac{20}{60}\right)$ or $A \ln 3$ or $A \ln \left(\frac{1}{3}\right)$ o.e. or $t = A(\ln 20 - \ln 60)$ or $A(\ln 60 - \ln 20)$ o.e. ( $A \in \square, t > 0$ )	dM1
	$\Rightarrow t = 632.8006\dots = 633$ (to the nearest minute)	awrt 633 or 10 hours and awrt 33 minutes	A1 cso
	<b>Note: dM1 can be implied by <math>t = \text{awrt } 0.44</math> from no incorrect working.</b>		
			7

Question Number	Scheme	Notes	Marks
4.	$\frac{dx}{dt} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geq 0$		
(a) Way 4	$\int \frac{2}{5x} dx = - \int dt$	Separates variables as shown. $dx$ and $dt$ should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\frac{2}{5} \ln(5x) = -t + c$	Integrates both sides to give <b>either</b> $\pm \alpha \ln(px)$ <b>or</b> $\pm k \rightarrow \pm kt$ (with respect to $t$ ); $k, \alpha \neq 0$ ; $p > 0$	M1
		$\frac{2}{5} \ln(5x) = -t + c$ , including "+c"	A1
	$\{t = 0, x = 60 \Rightarrow\} \frac{2}{5} \ln 300 = c$ $\frac{2}{5} \ln(5x) = -t + \frac{2}{5} \ln 300 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Finds their $c$ and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with <b>no incorrect working seen</b>	A1 <b>cso</b>
			[4]
(a) Way 5	$\left\{ \frac{dt}{dx} = -\frac{2}{5x} \Rightarrow \right\} t = \int_{60}^x -\frac{2}{5x} dx$	Ignore limits	B1
	$t = \left[ -\frac{2}{5} \ln x \right]_{60}^x$	Integrates both sides to give <b>either</b> $\pm k \rightarrow \pm kt$ (with respect to $t$ ) <b>or</b> $\pm \frac{\alpha}{x} \rightarrow \pm \alpha \ln x$ ; $k, \alpha \neq 0$	M1
		$t = \left[ -\frac{2}{5} \ln x \right]_{60}^x$ including the correct limits	A1
	$t = -\frac{2}{5} \ln x + \frac{2}{5} \ln 60 \Rightarrow -\frac{5}{2}t = \ln x - \ln 60$ $\Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 <b>cso</b>
			[4]
<b>Question 4 Notes</b>			
4. (a)	<b>B1</b>	For the correct separation of variables. E.g. $\int \frac{1}{5x} dx = \int -\frac{1}{2} dt$	
	<b>Note</b>	B1 can be implied by seeing <b>either</b> $\ln x = -\frac{5}{2}t + c$ <b>or</b> $t = -\frac{2}{5} \ln x + c$ with or without $+c$	
	<b>Note</b>	B1 can also be implied by seeing $[\ln x]_{60}^x = \left[ -\frac{5}{2}t \right]_0^t$	
	<b>Note</b>	Allow A1 for $x = 60\sqrt{e^{-5t}}$ or $x = \frac{60}{\sqrt{e^{5t}}}$ with no incorrect working seen	
	<b>Note</b>	Give final A0 for $x = e^{-\frac{5}{2}t} + 60 \rightarrow x = 60e^{-\frac{5}{2}t}$	
	<b>Note</b>	Give final A0 for writing $x = e^{-\frac{5}{2}t + \ln 60}$ as their final answer (without seeing $x = 60e^{-\frac{5}{2}t}$ )	
	<b>Note</b>	Way 1 to Way 5 do not exhaust all the different methods that candidates can give.	
	<b>Note</b>	Give B0M0A0A0 for writing down $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with no evidence of working or integration seen.	
(b)	<b>A1</b>	You can apply <b>cso</b> for the work only seen in part (b).	
	<b>Note</b>	Give dM1(Implied) A1 for $\frac{5}{2}t = \ln 3$ followed by $t = \text{awrt } 633$ from no incorrect working.	
	<b>Note</b>	Substitutes $x = 40$ into their equation from part (a) is M0dM0A0	

Question Number	Scheme		Notes	Marks
5.	$x = 4 \tan t, \quad y = 5\sqrt{3} \sin 2t, \quad 0 \leq t < \frac{\pi}{2}$			
(a) Way 1	$\frac{dx}{dt} = 4 \sec^2 t, \quad \frac{dy}{dt} = 10\sqrt{3} \cos 2t$ $\Rightarrow \frac{dy}{dx} = \frac{10\sqrt{3} \cos 2t}{4 \sec^2 t} \quad \left\{ = \frac{5}{2} \sqrt{3} \cos 2t \cos^2 t \right\}$		<b>Either both</b> $x$ and $y$ are differentiated correctly with respect to $t$ <b>or</b> their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ <b>or</b> applies $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
			Correct $\frac{dy}{dx}$ (Can be implied)	A1 oe
	$\left\{ \text{At } P \left( 4\sqrt{3}, \frac{15}{2} \right), t = \frac{\pi}{3} \right\}$			
	$\frac{dy}{dx} = \frac{10\sqrt{3} \cos \left( \frac{2\pi}{3} \right)}{4 \sec^2 \left( \frac{\pi}{3} \right)}$		<b>dependent on the previous M mark</b> <i>Some evidence</i> of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16} \sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$		$-\frac{5}{16} \sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ <b>from a correct solution only</b>	A1 cso
<b>[4]</b>				
(b)	$\left\{ 10\sqrt{3} \cos 2t = 0 \Rightarrow t = \frac{\pi}{4} \right\}$			
	So $x = 4 \tan \left( \frac{\pi}{4} \right), y = 5\sqrt{3} \sin \left( 2 \left( \frac{\pi}{4} \right) \right)$		At least one of either $x = 4 \tan \left( \frac{\pi}{4} \right)$ <b>or</b> $y = 5\sqrt{3} \sin \left( 2 \left( \frac{\pi}{4} \right) \right)$ <b>or</b> $x = 4$ <b>or</b> $y = 5\sqrt{3}$ <b>or</b> $y = \text{awrt } 8.7$	M1
	Coordinates are $(4, 5\sqrt{3})$		$(4, 5\sqrt{3})$ <b>or</b> $x = 4, y = 5\sqrt{3}$	A1
<b>[2]</b>				
<b>6</b>				
<b>Question 5 Notes</b>				
5. (a)	<b>1<sup>st</sup> A1</b>	Correct $\frac{dy}{dx}$ . E.g. $\frac{10\sqrt{3} \cos 2t}{4 \sec^2 t}$ or $\frac{5}{2} \sqrt{3} \cos 2t \cos^2 t$ or $\frac{5}{2} \sqrt{3} \cos^2 t (\cos^2 t - \sin^2 t)$ or any equivalent form.		
	<b>Note</b>	Give the final A0 for a final answer of $-\frac{10}{32} \sqrt{3}$ <b>without reference to</b> $-\frac{5}{16} \sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$		
	<b>Note</b>	Give the final A0 for more than one value stated for $\frac{dy}{dx}$		
(b)	<b>Note</b>	Also allow M1 for either $x = 4 \tan(45)$ or $y = 5\sqrt{3} \sin(2(45))$		
	<b>Note</b>	M1 can be gained by ignoring previous working in part (a) and/or part (b)		
	<b>Note</b>	Give A0 for stating more than one set of coordinates for $Q$ .		
	<b>Note</b>	Writing $x = 4, y = 5\sqrt{3}$ followed by $(5\sqrt{3}, 4)$ is A0.		



Question Number	Scheme	Notes	Marks
5.	$x = 4 \tan t, \quad y = 5\sqrt{3} \sin 2t, \quad 0 \leq t < \frac{\pi}{2}$		
(a) Way 2	$\tan t = \frac{x}{4} \Rightarrow \sin t = \frac{x}{\sqrt{(x^2+16)}}, \quad \cos t = \frac{4}{\sqrt{(x^2+16)}} \Rightarrow y = \frac{40\sqrt{3}x}{x^2+16}$		
	$\left\{ \begin{array}{l} u = 40\sqrt{3}x \quad v = x^2 + 16 \\ \frac{du}{dx} = 40\sqrt{3} \quad \frac{dv}{dx} = 2x \end{array} \right\}$		
	$\frac{dy}{dx} = \frac{40\sqrt{3}(x^2+16) - 2x(40\sqrt{3}x)}{(x^2+16)^2} \left\{ = \frac{40\sqrt{3}(16-x^2)}{(x^2+16)^2} \right\}$	$\frac{\pm A(x^2+16) \pm Bx^2}{(x^2+16)^2}$	M1
		Correct $\frac{dy}{dx}$ ; simplified or un-simplified	A1
	$\frac{dy}{dx} = \frac{40\sqrt{3}(48+16) - 80\sqrt{3}(48)}{(48+16)^2}$	<b>dependent on the previous M mark</b> <i>Some evidence</i> of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ <b>from a correct solution only</b>	A1 cso
			[4]
(a) Way 3	$y = 5\sqrt{3} \sin\left(2 \tan^{-1}\left(\frac{x}{4}\right)\right)$		
	$\frac{dy}{dx} = 5\sqrt{3} \cos\left(2 \tan^{-1}\left(\frac{x}{4}\right)\right) \left(\frac{2}{1+\left(\frac{x}{4}\right)^2}\right) \left(\frac{1}{4}\right)$	$\frac{dy}{dx} = \pm A \cos\left(2 \tan^{-1}\left(\frac{x}{4}\right)\right) \left(\frac{1}{1+x^2}\right)$	M1
		Correct $\frac{dy}{dx}$ ; simplified or un-simplified.	A1
	$\frac{dy}{dx} = 5\sqrt{3} \cos\left(2 \tan^{-1}(\sqrt{3})\right) \left(\frac{2}{1+3}\right) \left(\frac{1}{4}\right) \left\{ = 5\sqrt{3} \left(-\frac{1}{2}\right) \left(\frac{1}{2}\right) \left(\frac{1}{4}\right) \right\}$	<b>dependent on the previous M mark</b> <i>Some evidence</i> of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ <b>from a correct solution only</b>	A1 cso
			[4]

Question Number	Scheme	Notes	Marks
6.	(i) $\int \frac{3y-4}{y(3y+2)} dy, y > 0$ , (ii) $\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)} dx, x = 4\sin^2 \theta$		
(i) <b>Way 1</b>	$\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y-4 = A(3y+2) + By$ $y=0 \Rightarrow -4=2A \Rightarrow A=-2$ $y=-\frac{2}{3} \Rightarrow -6=-\frac{2}{3}B \Rightarrow B=9$	See notes	M1
		At least one of their $A = -2$ or their $B = 9$	A1
		Both their $A = -2$ and their $B = 9$	A1
	$\int \frac{3y-4}{y(3y+2)} dy = \int \frac{-2}{y} + \frac{9}{(3y+2)} dy$ $= -2\ln y + 3\ln(3y+2) \{+c\}$	Integrates to give at least one of <b>either</b> $\frac{A}{y} \rightarrow \pm \lambda \ln y$ or $\frac{B}{(3y+2)} \rightarrow \pm \mu \ln(3y+2)$ $A \neq 0, B \neq 0$	M1
		At least one term correctly followed through from their A or from their B	A1 ft
		$-2\ln y + 3\ln(3y+2)$ or $-2\ln y + 3\ln\left(y + \frac{2}{3}\right)$ with correct bracketing, simplified or un-simplified. Can apply isw.	A1 cao
			[6]
(ii) (a) <b>Way 1</b>	$\{x = 4\sin^2 \theta \Rightarrow\} \frac{dx}{d\theta} = 8\sin\theta\cos\theta$ or $\frac{dx}{d\theta} = 4\sin 2\theta$ or $dx = 8\sin\theta\cos\theta d\theta$		B1
	$\int \sqrt{\frac{4\sin^2 \theta}{4-4\sin^2 \theta}} \cdot 8\sin\theta\cos\theta \{d\theta\}$ or $\int \sqrt{\frac{4\sin^2 \theta}{4-4\sin^2 \theta}} \cdot 4\sin 2\theta \{d\theta\}$		M1
	$= \int \underline{\tan\theta} \cdot 8\sin\theta\cos\theta \{d\theta\}$ or $\int \underline{\tan\theta} \cdot 4\sin 2\theta \{d\theta\}$	$\sqrt{\left(\frac{x}{4-x}\right)} \rightarrow \pm K \tan\theta$ or $\pm K \left(\frac{\sin\theta}{\cos\theta}\right)$	<u>M1</u>
	$= \int 8\sin^2 \theta d\theta$	$\int 8\sin^2 \theta d\theta$ including $d\theta$	A1
	$3 = 4\sin^2 \theta$ or $\frac{3}{4} = \sin^2 \theta$ or $\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$ $\{x=0 \rightarrow \theta=0\}$	Writes down a correct equation involving $x = 3$ leading to $\theta = \frac{\pi}{3}$ and no incorrect work seen regarding limits	B1
(ii) (b)	$= \{8\} \int \left(\frac{1-\cos 2\theta}{2}\right) d\theta \quad \left\{ = \int (4-4\cos 2\theta) d\theta \right\}$	Applies $\cos 2\theta = 1 - 2\sin^2 \theta$ to their integral. (See notes)	M1
	$= \{8\} \left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right) \quad \left\{ = 4\theta - 2\sin 2\theta \right\}$	For $\pm \alpha\theta \pm \beta\sin 2\theta, \alpha, \beta \neq 0$	M1
		$\sin^2 \theta \rightarrow \left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)$	A1
	$\left\{ \int_0^{\frac{\pi}{3}} 8\sin^2 \theta d\theta = 8 \left[ \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \right]_0^{\frac{\pi}{3}} \right\} = 8 \left( \left( \frac{\pi}{6} - \frac{1}{4} \left( \frac{\sqrt{3}}{2} \right) \right) - (0+0) \right)$		
	$= \frac{4}{3}\pi - \sqrt{3}$	"two term" exact answer of e.g. $\frac{4}{3}\pi - \sqrt{3}$ or $\frac{1}{3}(4\pi - 3\sqrt{3})$	A1 o.e.
			[4]
			15

Question 6 Notes		
6. (i)	<b>1<sup>st</sup> M1</b>	Writing $\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)}$ and a complete method for finding the value of at least one of their A or their B.
	<b>Note</b>	M1A1 can be implied <i>for writing down</i> either $\frac{3y-4}{y(3y+2)} \equiv \frac{-2}{y} + \frac{\text{their } B}{(3y+2)}$ or $\frac{3y-4}{y(3y+2)} \equiv \frac{\text{their } A}{y} + \frac{9}{(3y+2)}$ with no working.
	<b>Note</b>	Correct bracketing is not necessary for the penultimate A1ft, but is required for the final A1 in (i)
	<b>Note</b>	<b>Give</b> 2 <sup>nd</sup> M0 for $\frac{3y-4}{y(3y+2)}$ going directly to $\pm \alpha \ln(3y^2+2y)$
	<b>Note</b>	<b>...but allow</b> 2 <sup>nd</sup> M1 for either $\frac{M(6y+2)}{3y^2+2y} \rightarrow \pm \alpha \ln(3y^2+2y)$ or $\frac{M(3y+1)}{3y^2+2y} \rightarrow \pm \alpha \ln(3y^2+2y)$
6. (ii)(a)	<b>1<sup>st</sup> M1</b>	Substitutes $x = 4\sin^2 \theta$ and their $dx$ (from their correctly rearranged $\frac{dx}{d\theta}$ ) into $\sqrt{\left(\frac{x}{4-x}\right)} dx$
	<b>Note</b>	$dx \neq \lambda d\theta$ . For example $dx \neq d\theta$
	<b>Note</b>	Allow substituting $dx = 4\sin 2\theta$ for the 1 <sup>st</sup> M1 after a correct $\frac{dx}{d\theta} = 4\sin 2\theta$ or $dx = 4\sin 2\theta d\theta$
	<b>2<sup>nd</sup> M1</b>	Applying $x = 4\sin^2 \theta$ to $\sqrt{\left(\frac{x}{4-x}\right)}$ to give $\pm K \tan \theta$ or $\pm K \left(\frac{\sin \theta}{\cos \theta}\right)$
	<b>Note</b>	Integral sign is not needed for this mark.
	<b>1<sup>st</sup> A1</b>	Simplifies to give $\int 8\sin^2 \theta d\theta$ including $d\theta$
	<b>2<sup>nd</sup> B1</b>	Writes down a correct equation involving $x = 3$ leading to $\theta = \frac{\pi}{3}$ and no incorrect work seen regarding limits
(ii)(b)	<b>M1</b>	Writes down a correct equation involving $\cos 2\theta$ and $\sin^2 \theta$ <b>E.g.:</b> $\cos 2\theta = 1 - 2\sin^2 \theta$ or $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$ or $K \sin^2 \theta = K \left(\frac{1 - \cos 2\theta}{2}\right)$ and <i>applies</i> it to their integral. <b>Note:</b> Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	<b>M1</b>	Integrates to give an expression of the form $\pm \alpha \theta \pm \beta \sin 2\theta$ or $k(\pm \alpha \theta \pm \beta \sin 2\theta)$ , $\alpha \neq 0, \beta \neq 0$ (can be simplified or un-simplified).
	<b>1<sup>st</sup> A1</b>	Integrating $\sin^2 \theta$ to give $\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta$ , un-simplified or simplified. Correct solution only. Can be implied by $k \sin^2 \theta$ giving $\frac{k}{2}\theta - \frac{k}{4}\sin 2\theta$ or $\frac{k}{4}(2\theta - \sin 2\theta)$ un-simplified or simplified.
	<b>2<sup>nd</sup> A1</b>	A <i>correct solution in part (ii)</i> leading to a “two term” exact answer of e.g. $\frac{4}{3}\pi - \sqrt{3}$ or $\frac{8}{6}\pi - \sqrt{3}$ or $\frac{4}{3}\pi - \frac{2\sqrt{3}}{2}$ or $\frac{1}{3}(4\pi - 3\sqrt{3})$
	<b>Note</b>	A decimal answer of 2.456739397... (without a correct <b>exact</b> answer) is A0.
	<b>Note</b>	Candidates can work in terms of $\lambda$ (note that $\lambda$ is not given in (ii)) and gain the 1 <sup>st</sup> three marks (i.e. M1M1A1) in part (b).
	<b>Note</b>	If they incorrectly obtain $\int_0^{\frac{\pi}{3}} 8\sin^2 \theta d\theta$ in part (i)(a) (or correctly guess that $\lambda = 8$ ) then the final A1 is available for a correct solution in part (ii)(b).

	Scheme	Notes	Marks
<b>6. (i) Way 2</b>	$\int \frac{3y-4}{y(3y+2)} dy = \int \frac{6y+2}{3y^2+2y} dy - \int \frac{3y+6}{y(3y+2)} dy$		
	$\frac{3y+6}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y+6 = A(3y+2) + By$	See notes	M1
	$y=0 \Rightarrow 6=2A \Rightarrow A=3$	At least one of their $A=3$ <b>or</b> their $B=-6$	A1
	$y=-\frac{2}{3} \Rightarrow 4=-\frac{2}{3}B \Rightarrow B=-6$	Both their $A=3$ <b>and</b> their $B=-6$	A1
	$\int \frac{3y-4}{y(3y+2)} dy$ $= \int \frac{6y+2}{3y^2+2y} dy - \int \frac{3}{y} dy + \int \frac{6}{(3y+2)} dy$ $= \ln(3y^2+2y) - 3\ln y + 2\ln(3y+2) \{+c\}$	Integrates to give at least one of <b>either</b> $\frac{M(6y+2)}{3y^2+2y} \rightarrow \pm \alpha \ln(3y^2+2y)$ <b>or</b> $\frac{A}{y} \rightarrow \pm \lambda \ln y$ <b>or</b> $\frac{B}{(3y+2)} \rightarrow \pm \mu \ln(3y+2)$ $M \neq 0, A \neq 0, B \neq 0$	M1
	At least one term correctly followed through	A1 ft	
	$\ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 <b>cao</b>	
			[6]
<b>6. (i) Way 3</b>	$\int \frac{3y-4}{y(3y+2)} dy = \int \frac{3y+1}{3y^2+2y} dy - \int \frac{5}{y(3y+2)} dy$		
	$\frac{5}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 5 = A(3y+2) + By$	See notes	M1
	$y=0 \Rightarrow 5=2A \Rightarrow A=\frac{5}{2}$	At least one of their $A=\frac{5}{2}$ <b>or</b> their $B=-\frac{15}{2}$	A1
	$y=-\frac{2}{3} \Rightarrow 5=-\frac{2}{3}B \Rightarrow B=-\frac{15}{2}$	Both their $A=\frac{5}{2}$ <b>and</b> their $B=-\frac{15}{2}$	A1
	$\int \frac{3y-4}{y(3y+2)} dy$ $= \int \frac{3y+1}{3y^2+2y} dy - \int \frac{\frac{5}{2}}{y} dy + \int \frac{\frac{15}{2}}{(3y+2)} dy$ $= \frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2) \{+c\}$	Integrates to give at least one of <b>either</b> $\frac{M(3y+1)}{3y^2+2y} \rightarrow \pm \alpha \ln(3y^2+2y)$ <b>or</b> $\frac{A}{y} \rightarrow \pm \lambda \ln y$ <b>or</b> $\frac{B}{(3y+2)} \rightarrow \pm \mu \ln(3y+2)$ $M \neq 0, A \neq 0, B \neq 0$	M1
	At least one term correctly followed through	A1 ft	
	$\frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 <b>cao</b>	
			[6]

	Scheme	Notes	
<b>6. (i)</b> <b>Way 4</b>	$\int \frac{3y-4}{y(3y+2)} dy = \int \frac{3y}{y(3y+2)} dy - \int \frac{4}{y(3y+2)} dy$		
	$= \int \frac{3}{(3y+2)} dy - \int \frac{4}{y(3y+2)} dy$		
	$\frac{4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 4 = A(3y+2) + By$	See notes	M1
	$y=0 \Rightarrow 4=2A \Rightarrow A=2$	At least one of their $A=2$ <b>or</b> their $B=-6$	A1
	$y=-\frac{2}{3} \Rightarrow 4=-\frac{2}{3}B \Rightarrow B=-6$	Both their $A=2$ <b>and</b> their $B=-6$	A1
	$\int \frac{3y-4}{y(3y+2)} dy$	Integrates to give at least one of <b>either</b> $\frac{C}{(3y+2)} \rightarrow \pm \alpha \ln(3y+2)$ <b>or</b> $\frac{A}{y} \rightarrow \pm \lambda \ln y$ <b>or</b> $\frac{B}{(3y+2)} \rightarrow \pm \mu \ln(3y+2)$ , $A \neq 0, B \neq 0, C \neq 0$	M1
	$= \int \frac{3}{3y+2} dy - \int \frac{2}{y} dy + \int \frac{6}{(3y+2)} dy$	At least one term correctly followed through	A1 ft
	$= \ln(3y+2) - 2\ln y + 2\ln(3y+2) \{+c\}$	$\ln(3y+2) - 2\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 <b>cao</b>
			[6]
<b>Alternative methods for B1M1M1A1 in (ii)(a)</b>			
<b>(ii)(a)</b> <b>Way 2</b>	$\{x = 4\sin^2 \theta \Rightarrow\} \frac{dx}{d\theta} = 8\sin\theta\cos\theta$	As in Way 1	B1
	$\int \sqrt{\frac{4\sin^2 \theta}{4-4\sin^2 \theta}} \cdot 8\sin\theta\cos\theta \{d\theta\}$	As before	M1
	$= \int \sqrt{\frac{\sin^2 \theta}{(1-\sin^2 \theta)}} \cdot 8\cos\theta\sin\theta \{d\theta\}$		
	$= \int \frac{\sin\theta}{\sqrt{(1-\sin^2 \theta)}} \cdot 8\sqrt{(1-\sin^2 \theta)} \sin\theta \{d\theta\}$		
	$= \int \sin\theta \cdot 8\sin\theta \{d\theta\}$	Correct method leading to $\sqrt{(1-\sin^2 \theta)}$ being cancelled out	M1
	$= \int 8\sin^2 \theta d\theta$	$\int 8\sin^2 \theta d\theta$ including $d\theta$	A1 <b>cso</b>
<b>(ii)(a)</b> <b>Way 3</b>	$\{x = 4\sin^2 \theta \Rightarrow\} \frac{dx}{d\theta} = 4\sin 2\theta$	As in Way 1	B1
	$x = 4\sin^2 \theta = 2 - 2\cos 2\theta, \quad 4-x = 2 + 2\cos 2\theta$		
	$\int \sqrt{\frac{2-2\cos 2\theta}{2+2\cos 2\theta}} \cdot 4\sin 2\theta \{d\theta\}$		M1
	$= \int \frac{\sqrt{2-2\cos 2\theta}}{\sqrt{2+2\cos 2\theta}} \cdot \frac{\sqrt{2-2\cos 2\theta}}{\sqrt{2-2\cos 2\theta}} 4\sin 2\theta \{d\theta\} = \int \frac{2-2\cos 2\theta}{\sqrt{4-4\cos^2 2\theta}} \cdot 4\sin 2\theta \{d\theta\}$		
	$= \int \frac{2-2\cos 2\theta}{2\sin 2\theta} \cdot 4\sin 2\theta \{d\theta\} = \int 2(2-2\cos 2\theta) \cdot \{d\theta\}$	Correct method leading to $\sin 2\theta$ being cancelled out	M1
	$= \int 8\sin^2 \theta d\theta$	$\int 8\sin^2 \theta d\theta$ including $d\theta$	A1 <b>cso</b>

Question Number	Scheme	Notes	Marks
7.	$y = (2x - 1)^{\frac{3}{4}}, \quad x \geq \frac{1}{2}$ passes through $P(k, 8)$		
(a)	$\left\{ \int (2x - 1)^{\frac{3}{4}} dx \right\} = \frac{1}{5}(2x - 1)^{\frac{5}{2}} \{ + c \}$	$(2x \pm 1)^{\frac{3}{2}} \rightarrow \pm \lambda(2x \pm 1)^{\frac{5}{2}}$ <b>or</b> $\pm \lambda u^{\frac{5}{2}}$ where $u = 2x \pm 1; \lambda \neq 0$	M1
		$\frac{1}{5}(2x - 1)^{\frac{5}{2}}$ with or without $+ c$ . Must be simplified.	A1
<b>[2]</b>			
(b)	$\{P(k, 8) \Rightarrow\} 8 = (2k - 1)^{\frac{3}{4}} \Rightarrow k = \frac{8^{\frac{4}{3}} + 1}{2}$	Sets $8 = (2k - 1)^{\frac{3}{4}}$ or $8 = (2x - 1)^{\frac{3}{4}}$ and rearranges to give $k =$ (or $x =$ ) a numerical value.	M1
	So, $k = \frac{17}{2}$	$k$ (or $x$ ) = $\frac{17}{2}$ or 8.5	A1
<b>[2]</b>			
(c)	$\pi \int \left( (2x - 1)^{\frac{3}{4}} \right)^2 dx$	For $\pi \int \left( (2x - 1)^{\frac{3}{4}} \right)^2$ or $\pi \int (2x - 1)^{\frac{3}{2}}$ Ignore limits and $dx$ . Can be implied.	B1
	$\left\{ \int_{\frac{1}{2}}^{\frac{17}{2}} y^2 dx \right\} = \left[ \frac{(2x - 1)^{\frac{5}{2}}}{5} \right]_{\frac{1}{2}}^{\frac{17}{2}} = \left( \left( \frac{16^{\frac{5}{2}}}{5} \right) - (0) \right) \left\{ = \frac{1024}{5} \right\}$ <b>Note:</b> It is not necessary to write the "-0"	Applies $x$ -limits of "8.5" (their answer to part (b)) and 0.5 to an expression of the form $\pm \beta(2x - 1)^{\frac{5}{2}}; \beta \neq 0$ and subtracts the correct way round.	M1
	$\left\{ V_{\text{cylinder}} \right\} = \pi(8)^2 \left( \frac{17}{2} \right) \left\{ = 544\pi \right\}$	$\pi(8)^2$ (their answer to part (b)) $V_{\text{cylinder}} = 544\pi$ implies this mark	B1 ft
	$\left\{ \text{Vol}(S) = 544\pi - \frac{1024\pi}{5} \right\} \Rightarrow \text{Vol}(S) = \frac{1696}{5}\pi$	An exact correct answer in the form $k\pi$ E.g. $\frac{1696}{5}\pi, \frac{3392}{10}\pi$ or $339.2\pi$	A1
<b>[4]</b>			
Alt. (c)	$\text{Vol}(S) = \pi(8)^2 \left( \frac{1}{2} \right) + \pi \int_{0.5}^{8.5} \left( 8^2 - \underline{\underline{(2x - 1)^{\frac{3}{2}}}} \right) dx$	For $\pi \int \dots \underline{\underline{(2x - 1)^{\frac{3}{2}}}}$ Ignore limits and $dx$ .	B1
	$= \pi(8)^2 \left( \frac{1}{2} \right) + \pi \left[ 64x - \frac{1}{5}(2x - 1)^{\frac{5}{2}} \right]_{0.5}^{8.5}$		
	$= \pi(8)^2 \left( \frac{1}{2} \right) + \pi \left( \left( \underline{\underline{64("8.5")}} - \frac{1}{5}(2(8.5) - 1)^{\frac{5}{2}} \right) - \left( \underline{\underline{64(0.5)}} - \frac{1}{5}(2(0.5) - 1)^{\frac{5}{2}} \right) \right)$	as above	M1
	$\left\{ = 32\pi + \pi \left( \left( \underline{\underline{544 - \frac{1024}{5}}} \right) - (32 - 0) \right) \right\} \Rightarrow \text{Vol}(S) = \frac{1696}{5}\pi$		A1
<b>[4]</b>			
<b>8</b>			

Question 7 Notes			
7. (b)	SC	Allow Special Case SC M1 for a candidate who sets $8 = (2k - 1)^{\frac{3}{2}}$ or $8 = (2x - 1)^{\frac{3}{2}}$ and rearranges to give $k =$ (or $x =$ ) a numerical value.	
7. (c)	M1	Can also be given for applying $u$ -limits of “16” ( $2(\text{part (b)}) - 1$ ) and 0 to an expression of the form $\pm\beta u^{\frac{5}{2}}$ ; $\beta \neq 0$ and subtracts the correct way round.	
	Note	You can give M1 for $\left[ \frac{(2x - 1)^{\frac{5}{2}}}{5} \right]_{\frac{1}{2}}^{\frac{17}{2}} = \frac{1024}{5}$	
	Note	Give M0 for $\left[ \frac{(2x - 1)^{\frac{5}{2}}}{5} \right]_0^{\frac{17}{2}} = \left( \left( \frac{16^{\frac{5}{2}}}{5} \right) - (0) \right)$	
	B1ft	Correct expression for the volume of a cylinder with radius 8 and their (part (b)) height $k$ .	
	Note	If a candidate uses integration to find the volume of this cylinder they need to apply their limits to give a correct expression for its volume. So $\pi \int_0^{8.5} 8^2 dx = \pi [64x]_0^{8.5}$ is <b>not sufficient</b> for B1 but $\pi(64(8.5) - 0)$ is <b>sufficient</b> for B1.	
7.	<b>MISREADING IN BOTH PARTS (B) AND (C)</b>		
Apply the misread rule (MR) for candidates who apply $y = (2x - 1)^{\frac{3}{2}}$ to <b>both</b> parts (b) and (c)			
(b)	$\{P(k, 8) \Rightarrow\} 8 = (2k - 1)^{\frac{3}{2}} \Rightarrow k = \frac{8^{\frac{2}{3}} + 1}{2}$	Sets $8 = (2k - 1)^{\frac{3}{2}}$ or $8 = (2x - 1)^{\frac{3}{2}}$ and rearranges to give $k =$ (or $x =$ ) a numerical value.	M1
	So, $k = \frac{5}{2}$	$k$ (or $x$ ) = $\frac{5}{2}$ or 2.5	A1
			[2]
(c)	$\pi \int \left( (2x - 1)^{\frac{3}{2}} \right)^2 dx$	For $\pi \int \left( (2x - 1)^{\frac{3}{2}} \right)^2$ or $\pi \int (2x - 1)^3$	B1
	Ignore limits and $dx$ . Can be implied.		
	$\left\{ \int_{\frac{1}{2}}^{\frac{17}{2}} y^2 dx \right\} = \left[ \frac{(2x - 1)^4}{8} \right]_{\frac{1}{2}}^{\frac{17}{2}} = \left( \left( \frac{4^4}{8} \right) - (0) \right) \{= 32\}$	Applies $x$ -limits of “2.5” (their answer to part (b)) and 0.5 to an expression of the form $\pm\beta(2x - 1)^4$ ; $\beta \neq 0$ and subtracts the correct way round.	M1
	$V_{\text{cylinder}} = \pi(8)^2 \left( \frac{5}{2} \right) \{= 160\pi\}$	$\pi(8)^2$ (their answer to part (b)) Sight of $160\pi$ implies this mark	B1 ft
$\{ \text{Vol}(S) = 160\pi - 32\pi \} \Rightarrow \text{Vol}(S) = 128\pi$	An exact correct answer in the form $k\pi$ E.g. $128\pi$	A1	
			[4]
Note	Mark parts (b) and (c) using the mark scheme above and then working forwards from part (b) deduct two from any A or B marks gained. E.g. (b) M1A1 (c) B1M1B1A1 would score (b) M1A0 (c) B0M1B1A1 E.g. (b) M1A1 (c) B1M1B0A0 would score (b) M1A0 (c) B0M1B0A0		
Note	If a candidate uses $y = (2x - 1)^{\frac{3}{4}}$ in part (b) and then uses $y = (2x - 1)^{\frac{3}{2}}$ in part (c) do not apply a misread in part (c).		

Question Number	Scheme	Notes	Marks
8.	$l_1: \mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$ So $\mathbf{d}_1 = \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$ . $\overline{OA}$ occurs when $\mu = 1$ . $\overline{OP} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$		
(a)	$A(3, 5, 0)$	$(3, 5, 0)$	B1
			[1]
(b)	$\{l_2: \} \mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$	$\mathbf{a} + \lambda \mathbf{d}$ or $\mathbf{a} + \mu \mathbf{d}$ , $\mathbf{a} + t \mathbf{d}$ , $\mathbf{a} \neq 0$ , $\mathbf{d} \neq 0$ with either $\mathbf{a} = \mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ or $\mathbf{d} = -5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ , or a multiple of $-5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$	M1
		Correct vector equation using $\mathbf{r} =$ or $l =$ or $l_2 =$	A1
	$\mathbf{a}_2$ is the direction vector of $l_2$	Do not allow $l_2: \text{or } l_2 \rightarrow \text{or } l_1 =$ for the A1 mark.	[2]
(c)	$\overline{AP} = \overline{OP} - \overline{OA} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$		
	$AP = \sqrt{(-2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$	Full method for finding $AP$	M1
		$2\sqrt{2}$	A1
			[2]
(d)	So $\overline{AP} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$ and $\mathbf{d}_2 = \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$	Realisation that the dot product is required between $(\overline{AP}$ or $\overline{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	M1
	$\{\cos \theta\} = \frac{\overline{AP} \cdot \mathbf{d}_2}{ \overline{AP}   \mathbf{d}_2 } = \frac{\pm \left( \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} \right)}{\sqrt{(-2)^2 + (0)^2 + (2)^2} \cdot \sqrt{(-5)^2 + (4)^2 + (3)^2}}$	<b>dependent on the previous M mark.</b> Applies dot product formula between their $(\overline{AP}$ or $\overline{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	dM1
	$\{\cos \theta\} = \frac{\pm (10+0+6)}{\sqrt{8} \cdot \sqrt{50}} = \frac{4}{-5}$	$\{\cos \theta\} = \frac{4}{5}$ or 0.8 or $\frac{8}{10}$ or $\frac{16}{20}$	A1 cso
			[3]
(e)	$\{\text{Area } APE\} = \frac{1}{2}(\text{their } 2\sqrt{2})^2 \sin \theta$	$\frac{1}{2}(\text{their } 2\sqrt{2})^2 \sin \theta$ or $\frac{1}{2}(\text{their } 2\sqrt{2})^2 \sin(\text{their } \theta)$	M1
	$= 2.4$	$2.4$ or $\frac{12}{5}$ or $\frac{24}{10}$ or awrt 2.40	A1
			[2]
(f)	$\overline{PE} = (-5\lambda)\mathbf{i} + (4\lambda)\mathbf{j} + (3\lambda)\mathbf{k}$ and $PE = \text{their } 2\sqrt{2}$ from part (c)		
	$\{PE^2\} = (-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})^2$	This mark can be implied.	M1
	$\{\Rightarrow 50\lambda^2 = 8 \Rightarrow \lambda^2 = \frac{4}{25} \Rightarrow \lambda = \pm \frac{2}{5}\}$	Either $\lambda = \frac{2}{5}$ or $\lambda = -\frac{2}{5}$	A1
	$l_2: \mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} \pm \frac{2}{5} \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$	<b>dependent on the previous M mark</b> Substitutes at least one of their values of $\lambda$ into $l_2$ .	dM1
	$\{\overline{OE}\} = \begin{pmatrix} 3 \\ 17 \\ 4 \\ 5 \end{pmatrix}$ or $\begin{pmatrix} 3 \\ 3.4 \\ 0.8 \end{pmatrix}$ , $\{\overline{OE}\} = \begin{pmatrix} -1 \\ 33 \\ 16 \\ 5 \end{pmatrix}$ or $\begin{pmatrix} -1 \\ 6.6 \\ 3.2 \end{pmatrix}$	At least one set of coordinates are correct.	A1
		Both sets of coordinates are correct.	A1
			[5]
			15



Question 8 Notes		
8. (a)	<b>B1</b>	Allow $A(3, 5, 0)$ or $3\mathbf{i} + 5\mathbf{j}$ or $3\mathbf{i} + 5\mathbf{j} + 0\mathbf{k}$ or $\begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix}$ or benefit of the doubt $\begin{matrix} 3 \\ 5 \\ 0 \end{matrix}$
(b)	<b>A1</b>	Correct vector equation using $\mathbf{r} = \mathbf{or} \ l = \mathbf{or} \ l_2 = \mathbf{or} \ \text{Line 2} =$ i.e. Writing $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \mathbf{d}$ , where $\mathbf{d}$ is a multiple of $\begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$ .
	<b>Note</b>	Allow the use of parameters $\mu$ or $t$ instead of $\lambda$ .
(c)	<b>M1</b>	Finds the difference between $\overline{OP}$ and their $\overline{OA}$ and applies Pythagoras to the result to find $AP$
	<b>Note</b>	Allow M1A1 for $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$ leading to $AP = \sqrt{(2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$ .
(d)	<b>Note</b>	For both the M1 and dM1 marks $\overline{AP}$ (or $\overline{PA}$ ) must be the vector used in part (c) or the difference $\overline{OP}$ and their $\overline{OA}$ from part (a).
	<b>Note</b>	Applying the dot product formula correctly without $\cos\theta$ as the subject is fine for M1dM1
	<b>Note</b>	<b>Evaluating</b> the dot product (i.e. $(-2)(-5) + (0)(4) + (2)(3)$ ) is not required for M1 and dM1 marks.
	<b>Note</b>	In part (d) allow one slip in writing $\overline{AP}$ and $\mathbf{d}_2$
	<b>Note</b>	$\cos\theta = \frac{-10+0-6}{\sqrt{8}\cdot\sqrt{50}} = -\frac{4}{5}$ followed by $\cos\theta = \frac{4}{5}$ is fine for A1 cso
	<b>Note</b>	Give M1dM1A1 for $\{\cos\theta\} = \frac{\begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -10 \\ 8 \\ 6 \end{pmatrix}}{\sqrt{8}\cdot 10\sqrt{2}} = \frac{20+12}{40} = \frac{4}{5}$
	<b>Note</b>	Allow final A1 (ignore subsequent working) for $\cos\theta = 0.8$ followed by 36.869...°
<b>Alternative Method: Vector Cross Product</b>		
<b>Only apply this scheme if it is clear that a candidate is applying a vector cross product method.</b>		
	$\overline{AP} \times \mathbf{d}_2 = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \times \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} = \left\{ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 0 & 2 \\ -5 & 4 & 3 \end{vmatrix} = -8\mathbf{i} - 4\mathbf{j} - 8\mathbf{k} \right\}$	Realisation that the vector cross product is required between their $(\overline{AP}$ or $\overline{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$
	$\sin\theta = \frac{\sqrt{(-8)^2 + (-4)^2 + (-8)^2}}{\sqrt{(-2)^2 + (0)^2 + (2)^2} \cdot \sqrt{(-5)^2 + (4)^2 + (3)^2}}$	Applies the vector product formula between their $(\overline{AP}$ or $\overline{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$
	$\sin\theta = \frac{12}{\sqrt{8}\cdot\sqrt{50}} = \frac{3}{5} \Rightarrow \cos\theta = \frac{4}{5}$	$\cos\theta = \frac{4}{5}$ or 0.8 or $\frac{8}{10}$ or $\frac{16}{20}$
(e)	<b>Note</b>	Allow M1;A1 for $\frac{1}{2}(2\sqrt{2})^2 \sin(36.869...^\circ)$ or $\frac{1}{2}(2\sqrt{2})^2 \sin(180^\circ - 36.869...^\circ)$ ; = awrt 2.40
	<b>Note</b>	Candidates must use their $\theta$ from part (d) or apply a correct method of finding their $\sin\theta = \frac{3}{5}$ from their $\cos\theta = \frac{4}{5}$

Question 8 Notes Continued		
8. (f)	<b>Note</b>	Allow the first M1A1 for deducing $\lambda = \frac{2}{5}$ or $\lambda = -\frac{2}{5}$ from no incorrect working
	<b>SC</b>	Allow special case 1 <sup>st</sup> M1 for $\lambda = 2.5$ from comparing lengths or from no working
	<b>Note</b>	Give 1 <sup>st</sup> M1 for $\sqrt{(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2} = (\text{their } 2\sqrt{2})$
	<b>Note</b>	Give 1 <sup>st</sup> M0 for $(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})$ or equivalent
	<b>Note</b>	Give 1 <sup>st</sup> M1 for $\lambda = \frac{\text{their } AP = "2\sqrt{2}"}{\sqrt{(-5)^2 + (4)^2 + (3)^2}}$ and 1 <sup>st</sup> A1 for $\lambda = \frac{2\sqrt{2}}{5\sqrt{2}}$
	<b>Note</b>	So $\left\{ \hat{\mathbf{d}}_1 = \frac{1}{5\sqrt{2}} \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} \right\} \Rightarrow$ "vector" = $\frac{2\sqrt{2}}{5\sqrt{2}} \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$ is M1A1
	<b>Note</b>	The 2 <sup>nd</sup> dM1 in part (f) can be implied for at least 2 (out of 6) correct x, y, z ordinates from their values of $\lambda$ .
	<b>Note</b>	Giving their "coordinates" as a column vector or position vector is fine for the final A1A1.
	<b>CAREFUL</b>	Putting $l_2$ equal to A gives $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \lambda = \frac{2}{5} \\ \lambda = 0 \\ \lambda = -\frac{2}{3} \end{pmatrix}$
<b>CAREFUL</b>	Putting $\lambda \mathbf{d}_2 = \overline{AP}$ gives $\lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} \rightarrow \begin{pmatrix} \lambda = -\frac{2}{5} \\ \lambda = 0 \\ \lambda = -\frac{2}{3} \end{pmatrix}$	Give M0 dM0 for finding and using $\lambda = -\frac{2}{5}$ from this incorrect method.
<b>General</b>	You can follow through the part (c) answer of their $AP = 2\sqrt{2}$ for (d) M1dM1, (e) M1, (f) M1dM1	
<b>General</b>	You can follow through their $\mathbf{a}_2$ in part (b) for (d) M1dM1, (f) M1dM1.	

