

June 2005  
6666 Core C4  
Mark Scheme

Question Number	Scheme	Marks
1.	$(4-9x)^{\frac{1}{2}} = 2\left(1 - \frac{9x}{4}\right)^{\frac{1}{2}}$ $= 2\left(1 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{9x}{4}\right)}{1.2} + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{9x}{4}\right)^2}{1.2.3} + \dots\right)$ $= 2\left(1 - \frac{9}{8}x - \frac{81}{128}x^2 - \frac{729}{1024}x^3 + \dots\right)$ $= 2 - \frac{9}{4}x - \frac{81}{64}x^2 - \frac{729}{512}x^3 + \dots$ <p><i>Note</i> The M1 is gained for <math>\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}\left(\dots\right)^2</math> or <math>\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2.3}\left(\dots\right)^3</math></p> <p><i>Special Case</i> If the candidate reaches <math>= 2\left(1 - \frac{9}{8}x - \frac{81}{128}x^2 - \frac{729}{1024}x^3 + \dots\right)</math> and goes no further allow A1 A0 A0</p>	<p>B1</p> <p>M1</p> <p>A1, A1, A1</p> <p style="text-align: right;"><b>[5]</b></p>

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2.	$2x + \left( 2x \frac{dy}{dx} + 2y \right) - 6y \frac{dy}{dx} = 0$ $\frac{dy}{dx} = 0 \Rightarrow x + y = 0$ <p>Eliminating either variable and solving for at least one value of <math>x</math> or <math>y</math>.</p> $y^2 - 2y^2 - 3y^2 + 16 = 0 \quad \text{or the same equation in } x$ $y = \pm 2 \quad \text{or } x = \pm 2$ $(2, -2), (-2, 2)$ <p>Note: <math>\frac{dy}{dx} = \frac{x+y}{3y-x}</math></p> <p><i>Alternative</i></p> $3y^2 - 2xy - (x^2 + 16) = 0$ $y = \frac{2x \pm \sqrt{(16x^2 + 192)}}{6}$ $\frac{dy}{dx} = \frac{1}{3} \pm \frac{1}{3} \cdot \frac{8x}{\sqrt{(16x^2 + 192)}}$ $\frac{dy}{dx} = 0 \Rightarrow \frac{8x}{\sqrt{(16x^2 + 192)}} = \pm 1$ $64x^2 = 16x^2 + 192$ $x = \pm 2$ $(2, -2), (-2, 2)$	<p>M1 (A1) A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[7]</p> <p>M1 A1± A1</p> <p>M1</p> <p>M1 A1</p> <p>A1</p> <p>[7]</p>

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3.	<p>(a) <math display="block">\frac{5x+3}{(2x-3)(x+2)} = \frac{A}{2x-3} + \frac{B}{x+2}</math></p> $5x+3 = A(x+2) + B(2x-3)$ <p>Substituting <math>x = -2</math> or <math>x = \frac{3}{2}</math> and obtaining <math>A</math> or <math>B</math>; or equating coefficients and solving a pair of simultaneous equations to obtain <math>A</math> or <math>B</math>.</p> $A = 3, B = 1$ <p>If the cover-up rule is used, give M1 A1 for the first of <math>A</math> or <math>B</math> found, A1 for the second.</p> <p>(b) <math display="block">\int \frac{5x+3}{(2x-3)(x+2)} dx = \frac{3}{2} \ln(2x-3) + \ln(x+2)</math></p> $\left[ \dots \right]_2^6 = \frac{3}{2} \ln 9 + \ln 2$ $= \ln 54$	<p>M1</p> <p>A1, A1</p> <p>(3)</p> <p>M1 A1ft</p> <p>M1 A1</p> <p>cao A1 (5)</p> <p>[8]</p>

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4.	$\int \frac{1}{(1-x^2)^{\frac{1}{2}}} dx = \int \frac{1}{(1-\sin^2 \theta)^{\frac{1}{2}}} \cos \theta d\theta \quad \text{Use of } x = \sin \theta \text{ and } \frac{dx}{d\theta} = \cos \theta$ $= \int \frac{1}{\cos^2 \theta} d\theta$ $= \int \sec^2 \theta d\theta = \tan \theta$ <p>Using the limits 0 and <math>\frac{\pi}{6}</math> to evaluate integral</p> $[\tan \theta]_0^{\frac{\pi}{6}} = \frac{1}{\sqrt{3}} \left( = \frac{\sqrt{3}}{3} \right)$ <p><i>Alternative for final M1 A1</i></p> <p>Returning to the variable <math>x</math> and using the limits 0 and <math>\frac{1}{2}</math> to evaluate integral</p> $\left[ \frac{x}{\sqrt{1-x^2}} \right]_0^{\frac{1}{2}} = \frac{1}{\sqrt{3}} \left( = \frac{\sqrt{3}}{3} \right)$	<p>M1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>cao A1</p> <p><b>[7]</b></p> <p>M1</p> <p>cao A1</p>

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5.	<p>(a) <math>\int x e^{2x} dx = \frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x} dx</math> Attempting parts in the right direction</p> $= \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x}$ $\left[ \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} \right]_0^1 = \frac{1}{4} + \frac{1}{4} e^2$	<p>M1 A1 A1 M1 A1</p>
	<p>(b) <math>x = 0.4 \Rightarrow y \approx 0.89022</math>  <math>x = 0.8 \Rightarrow y \approx 3.96243</math> Both are required to 5 d.p.</p>	<p>(5) B1 (1)</p>
	<p>(c) <math>I \approx \frac{1}{2} \times 0.2 \times [ \dots ]</math>  <math>\approx \dots \times [ 0 + 7.38906 + 2(0.29836 + .89022 + 1.99207 + 3.96243) ]</math>  <math>\approx 0.1 \times 21.67522</math>  <math>\approx 2.168</math></p> <p><i>Note</i> <math>\frac{1}{4} + \frac{1}{4} e^2 \approx 2.097 \dots</math></p>	<p>B1 M1 A1ft ft their answers to (b) cao A1 (4) [10]</p>

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6.	(a) $\frac{dx}{dt} = -2 \operatorname{cosec}^2 t, \frac{dy}{dt} = 4 \sin t \cos t$ both $\frac{dy}{dx} = \frac{-2 \sin t \cos t}{\operatorname{cosec}^2 t} (= -2 \sin^3 t \cos t)$	M1 A1 M1 A1 <b>(4)</b>
	(b) At $t = \frac{\pi}{4}, x = 2, y = 1$ both $x$ and $y$ Substitutes $t = \frac{\pi}{4}$ into an attempt at $\frac{dy}{dx}$ to obtain gradient $\left(-\frac{1}{2}\right)$ Equation of tangent is $y - 1 = -\frac{1}{2}(x - 2)$ Accept $x + 2y = 4$ or any correct equivalent	B1 M1 M1 A1 <b>(4)</b>
	(c) Uses $1 + \cot^2 t = \operatorname{cosec}^2 t$ , or equivalent, to eliminate $t$ $1 + \left(\frac{x}{2}\right)^2 = \frac{2}{y}$ correctly eliminates $t$ $y = \frac{8}{4 + x^2}$ cao The domain is $x \geq 0$	M1 A1 A1 B1 <b>(4)</b>
	<i>An alternative in (c)</i> $\sin t = \left(\frac{y}{2}\right)^{\frac{1}{2}}; \cos t = \frac{x}{2} \sin t = \frac{x}{2} \left(\frac{y}{2}\right)^{\frac{1}{2}}$ $\sin^2 t + \cos^2 t = 1 \Rightarrow \frac{y}{2} + \frac{x^2}{4} \times \frac{y}{2} = 1$ Leading to $y = \frac{8}{4 + x^2}$	<b>[12]</b> M1 A1 A1

Question Number	Scheme	Marks
7.	(a) <b>k</b> component $2 + 4\lambda = -2 \Rightarrow \lambda = -1$ $\text{Note } \mu = 2$ Substituting their $\lambda$ (or $\mu$ ) into equation of line and obtaining $B$ $B: (2, 2, -2)$ Accept vector forms	M1 A1 M1 A1 <b>(4)</b>
	(b) $\left  \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \right  = \sqrt{18}; \left  \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \right  = \sqrt{2}$	both B1
	$\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = 1 + 1 + 0 (= 2)$	B1
	$\cos \theta = \frac{2}{\sqrt{18}\sqrt{2}} = \frac{1}{3}$	cao M1 A1 <b>(4)</b>
	(c) $\overline{AB} = -\mathbf{i} + \mathbf{j} - 4\mathbf{k} \Rightarrow  \overline{AB} ^2 = 18$ or $ \overline{AB}  = \sqrt{18}$ ignore direction of vector $\overline{BC} = 3\mathbf{i} - 3\mathbf{j} \Rightarrow  \overline{BC} ^2 = 18$ or $ \overline{BC}  = \sqrt{18}$ ignore direction of vector Hence $ \overline{AB}  =  \overline{BC} $ *	M1 M1 A1 <b>(3)</b>
(d) $\overline{OD} = 6\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$	Allow first B1 for any two correct Accept column form or coordinates B1 B1 <b>(2)</b> <b>[13]</b>	

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8.	(a) $\frac{dV}{dt}$ is the rate of increase of volume (with respect to time) $-kV$ : $k$ is constant of proportionality and the negative shows decrease (or loss) giving $\frac{dV}{dt} = 20 - kV$ *      These Bs are to be awarded independently	B1 B1 (2)
	(b) $\int \frac{1}{20 - kV} dV = \int 1 dt$ separating variables $-\frac{1}{k} \ln(20 - kV) = t \quad (+C)$ Using $V = 0, t = 0$ to evaluate the constant of integration $c = -\frac{1}{k} \ln 20$ $t = \frac{1}{k} \ln\left(\frac{20}{20 - kV}\right)$ Obtaining answer in the form $V = A + B e^{-kt}$ $V = \frac{20}{k} - \frac{20}{k} e^{-kt}$ Accept $\frac{20}{k}(1 - e^{-kt})$	M1 M1 A1 M1 M1 A1      (6)
	(c) $\frac{dV}{dt} = 20e^{-kt}$ Can be implied $\frac{dV}{dt} = 10, t = 5 \Rightarrow 10 = 20e^{-kt} \Rightarrow k = \frac{1}{5} \ln 2 \approx 0.139$ At $t = 10, V = \frac{75}{\ln 2}$ awrt 108	M1 M1 A1 M1 A1      (5) [13]
	<i>Alternative to (b)</i> Using printed answer and differentiating $\frac{dV}{dt} = -kB e^{-kt}$ Substituting into differential equation $-kB e^{-kt} = 20 - kA - kB e^{-kt}$ $A = \frac{20}{k}$ Using $V = 0, t = 0$ in printed answer to obtain $A + B = 0$ $B = -\frac{20}{k}$	M1 M1 M1 A1 M1 A1      (6)