Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Tuesday 28 June 2005 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. Use the binomial theorem to expand

$$\sqrt{(4 - 9x)}, \qquad |x| < \frac{4}{9},$$

in ascending powers of x, up to and including the term in x^3, simplifying each term.

(5)

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2. A curve has equation

$$x^2 + 2xy - 3y^2 + 16 = 0.$$

Find the coordinates of the points on the curve where $\frac{dy}{dx} = 0$.

(7)

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3. (*a*) Express $\frac{5x+3}{(2x-3)(x+2)}$ in partial fractions.

(3)

(*b*) Hence find the exact value of $\int_2^6 \frac{5x+3}{(2x-3)(x+2)}\, dx$, giving your answer as a single logarithm.

(5)

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4. Use the substitution $x = \sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}}\, dx.$$

(7)

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5. **Figure 1**

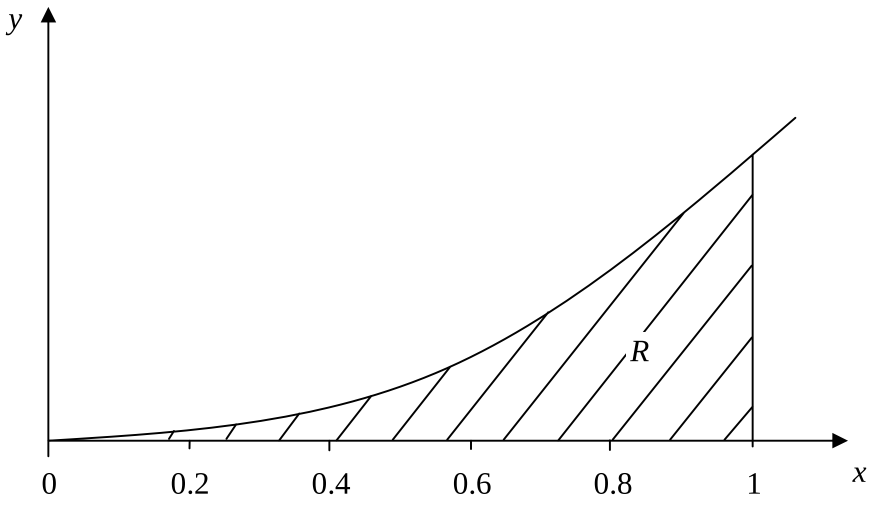

Figure 1 shows the graph of the curve with equation

$$y = xe^{2x}, \qquad x \geq 0.$$

The finite region R bounded by the lines $x = 1$, the x-axis and the curve is shown shaded in Figure 1.

(*a*) Use integration to find the exact value of the area for R.

(5)

(*b*) Complete the table with the values of y corresponding to $x = 0.4$ and 0.8.

x	0	0.2	0.4	0.6	0.8	1
$y = xe^{2x}$	0	0.29836		1.99207		7.38906

(1)

(*c*) Use the trapezium rule with all the values in the table to find an approximate value for this area, giving your answer to 4 significant figures.

(4)

6. A curve has parametric equations

$$x = 2\cot t, \quad y = 2\sin^2 t, \quad 0 < t \leq \frac{\pi}{2}.$$

(*a*) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of the parameter t.

(4)

(*b*) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

(4)

(*c*) Find a cartesian equation of the curve in the form $y = \mathrm{f}(x)$. State the domain on which the curve is defined.

(4)

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7. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix},$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point B and the acute angle between l_1 and l_2 is θ.

(*a*) Find the coordinates of B.

(4)

(*b*) Find the value of $\cos\theta$, giving your answer as a simplified fraction.

(4)

The point A, which lies on l_1, has position vector $\mathbf{a} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.
The point C, which lies on l_2, has position vector $\mathbf{c} = 5\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.
The point D is such that $ABCD$ is a parallelogram.

(*c*) Show that $|\overrightarrow{AB}| = |\overrightarrow{BC}|$.

(3)

(*d*) Find the position vector of the point D.

(2)

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8. Liquid is pouring into a container at a constant rate of 20 $\text{cm}^3\ \text{s}^{-1}$ and is leaking out at a rate proportional to the volume of the liquid already in the container.

(*a*) Explain why, at time t seconds, the volume, V cm^3, of liquid in the container satisfies the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 20 - kV,$$

where k is a positive constant.

(2)

The container is initially empty.

(*b*) By solving the differential equation, show that

$$V = A + B\mathrm{e}^{-kt},$$

giving the values of A and B in terms of k.

(6)

Given also that $\frac{\mathrm{d}V}{\mathrm{d}t} = 10$ when $t = 5$,

(*c*) find the volume of liquid in the container at 10 s after the start.

(5)

TOTAL FOR PAPER: 75 MARKS

END